food for thought

"Food for Thought" explores the relationship between food/drink and chemical engineering processes/concepts.

NO SWEAT?

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Here's an in-class exercise that I like to do with first-year students to give them an opportunity to practice their estimation skills in the context of conservation of mass and energy, learning a tiny bit of heat transfer along the way. And it gives me a chance to re-experience something that gives me a little shiver of wonder each time it happens — that by using reasonable estimates and applying first principles, sometimes you can derive the existence of something you see in the real world. In this case, sweat and Gatorade™ (or your favorite similar sports drink). It is an entirely amazing thing that the laws of conservation apply not just to systems we build, like potato chip factories and jet fuel refineries, but also to our own bodies.

First — some ground rules. We're going to calculate this Fermi-problem style, using round estimates, the simplest forms of equations, and assuming pretty much everything in the problem has the properties of water. This could be made into a much more sophisticated model, appropriate for upper-level classes, but what I'm describing here is made for first-years who are only co-enrolled in calculus and physics and not any in ChE-major courses. That being said, I think there's still quite a lot to be learned. An additional note — when I do this, I have students propose the characteristics of the system (weight of the person, type of exercise, surrounding temperature), while here I'll show them as set values. If you do this, it is much more fun in class when none of us know exactly the answer because the students have picked a combination of factors that I didn't work out before class!

Our problem: A 70 kg person (our system) is doing some substantial exercise — let's say running for a full hour. How much (and what substance) should they drink to offset the sweat that they lose? While this is phrased as a mass balance question, I find it more interesting to first attack it as an energy balance question. According to the Mayo Clinic,[1] that person burns 606 kcal (or about 2500 kJ) of chemical energy previously consumed as food and drink. Because this is a food column, we're going to connect back to food in a moment, but first let's follow that forward and see where the energy *goes*.

Energy, being conserved, has to go somewhere. I encourage the students to volunteer ideas for where it goes — a popular guess is it becomes kinetic energy of the runner.

$$KE = \frac{1}{2}mv^2$$

The answer to that is "yes" but…less than the students are expecting. Let's say that the runner is training for a marathon and can maintain a speed of 10 minutes per mile. After a pile of unit conversions, we arrive at

$$KE = \frac{1}{2}70kg * (10mi/hr * 1hr/3600s * 1609m/mile)^2 = 699J$$

(Side note: when doing this with first-years, I absolutely leave unit conversions **in** as part of the in-class problem, asking students to call out conversion factors and collecting them on the board for all to use; and yes, while they prefer

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to allow a website to do the conversions, I try to get the class to do at least one with a calculator.)

The kinetic energy of our runner is about 700 J, which clocks in as less than even a single kilocalorie. More of the energy goes into *work* moving muscles, which is a bit beyond the scope of first-year ChemE efforts, but we mostly don't need to go that far because where nearly all of *that* muscular work enters the surroundings is through *heat*. In fact, as a first order approximation (remember our ground rules!), it works pretty well to call all of that caloric expenditure heat.

I haven't forgotten food! We are getting there. First, I like to have the students reflect upon how critical heat transfer is to continued human existence. For example, what would happen to the runner if they were adiabatic? In this case, I ask the students to approximate the human body as water and ask "What temperature change would happen to a person with 2500 kJ of heat energy?"

$$Q = 2500\,kJ = mC_p\,(T_{final} - T_{initial})$$

$$Q = 70\,kg * 4.184\,kJ/kg°C * (T_{final} - 37\,°C)$$

Take a moment to work that out — I find that our 70 kg person's temperature would go up by nearly 9 ºC. Yikes! This is clearly unacceptable, so it's a good thing that people shed heat energy. I ask the class to consider how much energy can leave the system by two mechanisms — first, by convection to air from skin, then by evaporative cooling (another fun thing to consider would be heat loss through breathing, which I think I will work into this problem next time around). For convection, we're going to make a number of assumptions, including that it's at steady state over the course of the one-hour run with a constant value of the heat transfer coefficient and that the outdoor temperature is 22 ºC, thus arriving at

$$Q = hA\,(T_{surroundings} - T_{system})$$

A lively discussion generally ensues about what makes for a fair value of A, the surface area for heat transfer. I generally accept whatever the students can agree on that's in the 1 - 2m^2 range.

$$Q = 10\,W/m^2\,°C * 2m^2 * (22\,°C - 37\,°C) = -300W$$

For 2m^2, and a convection coefficient of low-forced-convection of 10 Wm^{-2} ºC^{-1}, we get a convective loss of 300 W, or 1080 kJ for the whole hour. That's not quite half of the energy budget, meaning without additional sources of energy transfer, our runner is going to overheat. Thus, let's consider how much water would need to evaporate in order to remove the remaining 1420kJ from the system. Assuming sweat has the enthalpy of vaporization of pure water, 580g of water must be evaporated.

$$1420kJ = \Delta H^{vap} m \longrightarrow m = \frac{1420kJ}{2500kJ/kg} = 0.58kg$$

Which, at long last, brings us to food. A typical 20 oz bottle of a sports drink such as Gatorade™ has just a bit more mass than our calculated water loss! It's almost like someone planned it that way. *In fact, they did*. Gatorade™ in particular came about as a way to address heat stress among the University of Florida's football team back in 1965.[2] The original recipe was very much based on a mass balance to support the energy balance we just did — if football players are losing liters of sweat every hour,[3] then let's replace it! Initial trials, where the recipe didn't differ appreciably from someone's sweat salt and water content (with some added sugar for taste), did not go well (i.e., did not remain, um, part of the system, so to speak). Looking at the original patent for Gatorade™,[4] the amount of sodium added in a liter is just about exactly the mid-range sodium content per liter of sweat found by Godet et al. for football players.[3] Yum! No wonder the original patent included so much sugar!

I like to bring this problem to a close by asking students to review some of their assumptions and what impact relaxing those might have. One that comes up is that sweat isn't, in fact, pure water. The dissolved salts in sweat drive up the enthalpy of vaporization so that less fluid is needed to disperse the energy, which is good. Some other questionable assumptions were the low surrounding temperature and the large amount of body-surface area for heat transfer, both of which will drive up the needed amount of sweat. And every once in a while, a student catches that we've assumed all of the sweat evaporates, which may or may not be a good assumption based on the local relative humidity. Which is all why I like to do this problem with first-year students — it gives them so much in the curriculum to look forward to!*

*For more first/second-year problems along these lines, see Felder, Rousseau, and Bullard, *Elementary Principles of Chemical Processes*, 4th Ed., problems 7.3, 7.34, and 8.36!

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