ADVANCES IN HEAT AND MASS TRANSFER

by

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Research in heat and mass transfer has received a strong impetus by new engineering developments and has therefore grown considerably in recent years. This is, for instance, evidenced by the fact that the last one of the yearly reviews published in the International Journal of Heat and Mass Transfer contains approximately 500 references selected from more than double the number of published papers (ref. 1). A survey of recent papers and books from the Soviet Union lists 259 references (ref. 2). Consequently, the time available for this lecture permits only to discuss in general terms some highlights in the recent research and in the newer problems presented by engineering developments. The list of references attached to this paper will be helpful for a more detailed information on the subjects discussed. The availability of reference literature (refs. 1 to 3) may also be pointed out.

Heat Conduction

Considerable attention has been directed in recent years to obtain new solutions or to describe new methods of attach on conduction problems. This situation has been created by the fact that new engineering developments required consideration of new and more involved boundary conditions and also that the availability of electronic computers made such solutions possible. The real challenging problems, however, are situations in which heat conduction is interrelated with convection and possibly radiation. An example for such a situation may be discussed with the help of Fig. 1, which is a schematic sketch of an ablation cooling process. In this cooling method, a material which sublimes or decomposes under the influence of heat is used to protect the surface of vehicles re-entering from outer space through the atmosphere to the earth against the heat of friction created in the boundary layer which surrounds the object. Some of the materials used for ablation cooling are composed of a matrix of a temperature-resistant substance like asbestos or ceramics and an ablating substance, for instance, some plastic. Under the influence of a convective heat flux q_c and a radiative heat flux q_r into the surface, the plastic material decomposes and its surface recedes with a velocity v_a, leaving the matrix through which the gas created by the ablation process flows with the velocity v_g. The low heat conductivity of the matrix keeps the heat flow into the interior small. Further cooling of the matrix and a reduction of the convective flow into the surface is provided by the gases created in the ablation process. It is easily seen that in this process conduction, heat convection, and mass transfer processes are interrelated. Additionally, heat sources or sinks are provided by the phase changes and the chemical reactions occurring (ref. 4). In some materials, the radiative heat flux q_r is only gradually absorbed while penetrating into the ablating material. This combination and interrelation of various transfer processes is characteristic for many situations in new applications.

Heat Convection

Channel flows as well as boundary layer flows offer such a variety of boundary conditions that they are far from being completely investigated. An area in channel flow which has recently received attention is connected with ducts of non-circular cross-sections. It was found that the flow and heat transfer characteristics in such a duct are significantly different from those observed in circular pipes, especially when the cross-section contains corners with small opening angle. Fig. 2 shows as an example the results of measurements for fully developed turbulent flow through a duct the cross-section of which has the shape of an isosceles triangle with base to height ratio 1:5 (ref. 5). The measured results indicated by the open circles and triangles are compared with the Nusselt numbers Nu which would be predicted from measurements in a circular pipe of the same hydraulic diameter. It can be recognized that the prediction would overestimate the heat transfer by a factor of two. The actual heat transfer in such a cross-section depends strongly on the boundary condition around the periphery of the duct, for instance, whether a constant wall temperature or a constant heat flux from the duct wall into the fluid is prescribed. The boundary condition for the results in Fig. 2 was between the two extreme cases which have just been measured. Ducts of similar shapes have been used or are considered for coolant passages in nuclear reactors and in gas turbines. The differences between non-circular and circular ducts are less pronounced when the cross-section has only large angle corners (ref. 6).

A situation in boundary layer flow which has found special attention only recently is connected with heat transfer in regions where the flow has separated from the surface. This occurs, for instance, at the downstream part of blunt objects or behind steps in the surface contour. Fig. 3 sketches such a heat transfer situation. The boundary layer which arrives in its downstream movement at the corner of the step separates from the surface and re-attaches again only further downstream. The region between this boundary layer and the surface is filled with a rotating body of fluid (dead water region). A second boundary layer is created between this dead water and the surface. Heat transfer from the main stream to the body surface behind the step has therefore to overcome two resistances in series: one in the separated boundary layer and one in the attached boundary layer. A large variety of flow situations may exist depending on the Reynolds and Mach number of the flow. The boundary layers may be laminar or turbulent, or transition to turbulence may occur in the boundary layers. The body of separated fluid may also be laminar or turbulent or in fluctuating unsteady motion. This complicates an understanding and analysis of the heat transfer process considerably. Nevertheless, analytical approaches for some of the flow conditions were quite successful (refs. 7, 8).

It is the opinion of this lecturer that the most significant advance in the creation of a science of heat transfer was caused by the concept of a constant property fluid introduced by Wilhelm Nusselt in 1916 and that teaching of heat transfer has to be based extensively on the model of a constant property fluid. New engineering developments, on the other hand, have created many situations where large property variations exist. Such variations cause no principal difficulties in laminar flow but make the equations describing this situation much more complicated. Only the advent of electronic computers made solutions to such problems tractable. For turbulent flow the question arises whether the turbulent transport properties, for instance the diffusivities for mementum and heat, are changed in the presence of local property variations. From the results of analyses it appears that this is not the case even in the presence of strong property variations. Fig. 4 compares the results of experiments with theoretical predictions for heat transfer connected with turbulent flow of carbon dioxide in the critical and super-critical region through a tube (ref. 9). The properties involved in the heat transfer process vary very strongly in the neighborhood of the critical region. The parameter on the curves is a measure of the intensity of the property variations and the good agreement between analysis and measurement supports strongly the assumption that the transfer properties were not influenced by the property variations connected with these temperature differences. This does not hold any more in the immediate neighborhood of the critical state. Investigations on free convection heat transfer with a Zehnder-Mach interferometer, on a vertical plate exposed to carbon dioxide within one degree to its critical state and with temperature differences of order 0.01°C, made it possible to calculate the thermal conductivity (ref. 10). The remarkable conclusion has to be drawn from the results that the thermal conductivity does not only depend on temperature and pressure, but also on the intensity of the heat flux. Similar observations had been made before but had been attributed to convection effects which can be excluded in the present investigation. This is only one indication that an understanding of the critical state is still almost non-existent.

Through many years, convective heat transfer had been studied almost exclusively for steady state situations. This is justified by the fact that very rapid changes of the boundary conditions are required to produce heat transfer coefficients which are significantly different from steady state values. Nevertheless, new applications have raised the question on the limit for the use of steady state values. Fig. 5 shows the results of another interferometric investigation of free convection heat transfer to a vertical place under the condition that suddenly a locally uniform heat flux from the surface into the surrounding water is started (ref. 11). The measurements essentially indicate that for the initial period, heat is transferred into the fluid by unsteady conduction and that the subsequent transition period to steady state is quite short.

Heat transfer connected with boiling or condensation is an area which is still understood only partially in spite of the intense research effort which has continuously been devoted to this process. The problems and research attempts in this area are actually so large that they cannot even be sketched in this lecture. For a discussion of the physical processes involved and of the analytic attempts which have been published, the reader is referred to the attached references 12 and 13. It is the feeling of this lecturer that the creation of concise and consistent models is still lacking in the analytic investigations. Boiling and condensation of liquid metals, for instance of mercury, and the influence of gravity on free convection boiling are subjects which deserve special attention.

Mass Transfer

A discussion of mass transfer should certainly start with the analogy between heat and mass transfer processes which has originally been pointed out by Wilhelm Nusselt in 1916. This analogy permits to predict mass transfer situations solely from information on an analogous heat transfer process especially when the mass transfer rates and the temperature differences involved are small. Recently this analogy has been extended by Russian and American scientists to cover situations with large mass transfer rates, large temperature differences, chemical reactions in the fluid, and also especially at high temperature levels (ref. 14). The analogy includes processes in which heat and mass transfer are interrelated. They are especially useful for gases and will be discussed on the example of mass transfer from a surface into a gas flowing over it, thus creating a two-component mixture in the laminar boundary layer. Eq. (1) in Fig. 6 describes a mass flux vector $\dot{m}_i$ for the component i in the mixture, that is, the fass flux per unit time and unit area of a plane arbitrarily located within the two-component fluid. The first term on the right-hand side of the equation describes the mass flux by diffusion as a consequence of a gradient w_i of the mass fraction of the component i. D_{12} is the mass diffusion coefficient and the mixture density. The second term describes the convective transport of the component i with a density $_i$ as a consequence of a movement of the mixture with the velocity V through the plane under consideration. Mass transfer is also created by coupled effects like thermal diffusion, pressure diffusion, or diffusion as a consequence of body forces. These effects are generally small and are not spelled out in Eq. (1). A heat flux vector q (heat flow per unit time through a unit area) will also exist and is described by Eq. (2). The first term on the right-hand side of this equation describes heat transfer by conduction as a consequence of a temperature gradient T. The second term describes the transport of enthalpy h_i connected with the diffusion mass flux of the components involved. The third term describes transport of the enthalpy h of the mixture as a consequence of the mixture mass flow V through the plane under consideration. Additional terms appear again as a consequence of coupled effects described by irreversible thermodynamics like thermodiffusion. Eq. (2) can be re-written in a different form when the gradient of the temperature T is replaced by the gradient of the mixture enthalpy h (Eq. (3)). This equation is especially useful for a fluid with a Lewis number Le equal to one, because the second term drops out in this case. The Eqs. (1) to (3) have been written in a form which is most useful for mass transfer situations where conduction and convection occur simultaneously.

The boundary layer Eqs. (4) and (5) of Fig. 6 are obtained by a mass balance of the individual components and by an energy balance on a volume element located within the boundary layer. Chemical reactions occuring within the boundary layer destroy or create one or the other component and appear, therefore, in the mass balance of Eq. (4) as a source term K_i. The energy Eq. (5) has been written in terms of the total or stagnation enthalpy h_s (containing kinetic energy as well as internal energy). The first term on the right-hand side of this equation describes essentially heat transport by conduction; the second term, heat generation by viscous dissipation which becomes important in high velocity flow; and the third term is concerned with enthalpy transport by mass diffusion. From Eqs. (4) and (5) one comes to the similarity considerations by two important steps: The first one entails writing the mass balance for the chemical elements involved instead of the two components; thus w_i may indicate the mass fraction of the element i in the mixture regardless whether the element appears as such or in the chemical compound. No chemical element is created or destroyed in a chemical process, and as a consequence the source term K_i vanishes in Eq. (4) when it is written for the chemical element. The second step assumes that, for the fluid mixture under consideration, the Prandtl number Pr as well as the Lewis number Le are both equal to one. Eqs. (4) and (5) simplify then to Eqs. (6) and (7) which can be recognized to be completely similar. As a consequence, analysis of mass transfer processes and of combined mass and heat transfer processes becomes much simpler because the most difficult elements in such an analysis can often be taken over from known solutions of an analogous heat transfer situation. Proper boundary conditions have of course to be considered in such an analysis which may include chemical processes occurring at the surface. It should be mentioned that the mass and energy conservation equations alone do not describe the transfer problem completely. A momentum equation and equations for the thermodynamic and transfer properties have to be added. The analogy, however, holds independent of these. It has become especially useful in an analysis of problems like combustion or heat transfer to re-entering vehicles as mentioned in the section on Conduction. Approximate relations have also been developed which extend the analysis to situations with Prandtl and Lewis numbers different from one (ref. 14).

Coupled effects have been neglected in the discussion up to now because they are unimportant in many mass transfer situations. Recent studies (refs. 15, 16), however, have demonstrated that one has to be careful in this respect. This will be discussed with the help of Fig. 7 which presents the results of the following experiments (ref. 16). A cylinder with porous surface was exposed on its outside to a flow of air in axial direction at a Reynolds number which created a turbulent boundary layer. Helium was injected from the inside of the cylinder through the porous surface into this boundary layer. A difference in the temperature with which the helium was fed into the cylinder and the air temperature T_∞ outside of the boundary layer could be adjusted by pre-heating or pre-cooling of the helium. Fig. 7 presents the heat flux q_w through the porous surface as a function of the difference between the wall surface temperature T_w and the air temperature T_∞

with the specific mass flow $\dot{m}$ of the helium as parameter. One series of measurements made with air instead of helium injection is also entered as dashed line. The heat flux q_w is defined as the sum of the first two terms in Eq. (2). The striking feature in this figure is the observation that the heat flux becomes zero at a finite temperature difference $T_w - T_\infty$ and it is believed that this is a consequence of the fact that concentration differences within the boundary layer appear through thermodiffusion as driving force in addition to conduction. Another consequence of this interplay of driving potentials is the fact that a finite difference between the wall surface temperature T_w and the air temperature T_∞ exists when the helium is admitted into the porous cylinder at a temperature T_∞ equal to the temperature in the outside air flow. This situation is marked in the figure by crosses and it can be recognized that the wall temperature may be up to almost 30 degrees higher than both the helium and the air temperatures, depending on the injection rate of the helium. Similar effects have also been observed in laminar boundary layers for forced and free convection.

An area in which investigations have recently started are transfer processes in a gas plasma. In such a plasma, the temperature is so high that dissociation and ionization occur. Transport processes in such a situation are therefore most involved because mass transfer processes are interrelated with heat transfer, chemical reactions occur, at least three-component mixtures of neutral atoms, ions, and electrons are involved, and electric as well as magnetic body forces influence the flow. An example of a recent experimental investigation in this area is shown in Fig. 8 which presents and analyzes the local heat flux distribution into a water-cooled anode of an electric arc burning in argon (ref. 17). It can be recognized that only a small portion of the specific heat flux q into the anode surface is caused by convection of the atom gas. Convection of the electron gas, which is generated at the cathode and absorbed by the anode, contributes approximately an equal fraction and the rest is due to energy released when the electrons enter the anode material (similar to a heat of condensation). The heat flux q_c indicates the electrical energy which is converted into heat within the current tube ending at the anode location under consideration. It may be recognized that the majority of the electrically generated heat enters the anode surface. This is the reason for the many burnouts occurring in electrically heated plasma generators at the anode surface. The heat fluxes q occurring at the anode surface on spots at which the arc strikes are among the largest known in any engineering application.

Radiative Heat Transfer

An important tool in all radiative heat transfer calculations is the shape or angle factor. Graphical, mechanical, and optical means have been described, in addition to analytical methods for its calculation. The analysis can, in many cases, be considerably simplified by converting an area integral describing the shape factor into a line integral (ref. 18).

The network method for the solution of radiative heat transfer problems in enclosures, which lumps emitted radiation together with the reflected parts, is of such advantage that it has been introduced into practically all recent books on heat transfer. The analogy to electric circuits, illustrated in Fig. 9, gives in many situations without analysis a feeling for the heat flux distributions occurring in such a transfer process. The network method is applicable to enclosures the surfaces of which emit radiation according to Lambert's cosine law and which reflect diffusely. Many engineering materials, on the other hand, have surfaces, the reflection of which comes closer to a specular than to a diffuse character. Some measured directional distribution curves of reflected radiation are shown in Fig. 10. In enclosures with specularly reflecting surfaces, the analytical approach has to be different and has to consist of summation of the first, second, third, and so on reflections. The analysis is simplified when one introduces the optical images as shown in Fig. 11 in which 1 (3), 2 (3), and 4 (3)denote the optical images of the diffuse surfaces 1, 2, and 3 created by the specularly reflecting surface 3 (ref. 19). This image method can be combined with the network method for enclosures consisting partially of diffusely and partially of specularly reflecting surfaces (ref. 20). The network method is actually an approximation to the integral equations which in principle describe radiative transfer processes. It is important to obtain exact solutions to the integral equations for a few simple situations in order to get a feeling for the errors which may be introduced in the network method. Several recent papers have started to formulate radiative heat processes in this form including the scattering mechanism in a radiating-absorbing medium filling the enclosure (ref. 21). Engineering analyses usually attack problems in which radiative energy occurs simultaneously with other transfer mechanisms like conduction or convection in such a way that the various contributions are calculated separately and that the total energy transfer is obtained by a summation of the individual parts. In reality, situations are encountered in which the various transfer processes interact. Such interactions have been studied for a few cases in the recent past (ref. 22).

At the end of our discussion we will return to the molecular and convective transport processes with a brief review of the similarity between radiative and molecular transport (ref. 23). We can consider radiative transport as caused by the movement of photons in a similar way as energy, mass, or momentum transport is caused by the movement of molecules. Fig. 12 illustrates this similarity by considering Couette flow or heat conduction in a rarefied gas between two parallel walls on one hand, and radiative energy transfer in a radiating and absorbing medium between two parallel walls which are non-transmitting on the other hand. The temperature or velocity variation between the two walls follow a straight line, as indicated in Fig. 12a, as long as the ratio of the mean free path length is very small compared to the distance L of the two walls. With increasing path length, temperature or velocity still exhibits the linear variation within the gas; however, a slip of the velocity or a temperature jump can be observed in the immediate neighborhood of both wall surfaces (fig. 12b). For situations, on the other hand, in which the mean free path length is large compared to the distance L, velocity or temperature in the gas is uniform (Fig. 12d). The terms for the corresponding regimes are indicated above the figures. Completely analogous situations exist for the radiative transfer process. The black-body emissive power e_b has now to be considered instead of the velocity or temperature. This emissive power drops linearly in the absorbing and radiating medium as long as the free path length of the photons is small compared to the distance L. Jumps near the surface of the two walls occur with increasing photon path length. The variation in the absorbing medium itself decreases towards zero when the photon path length gets larger and larger. The terms for these regimes are listed below the figures. This similarity is very helpful in unifying the concepts for transfer processes and such a unification and interrelation of the concepts I would consider as one of the most essential requirements of a good course in transfer processes.

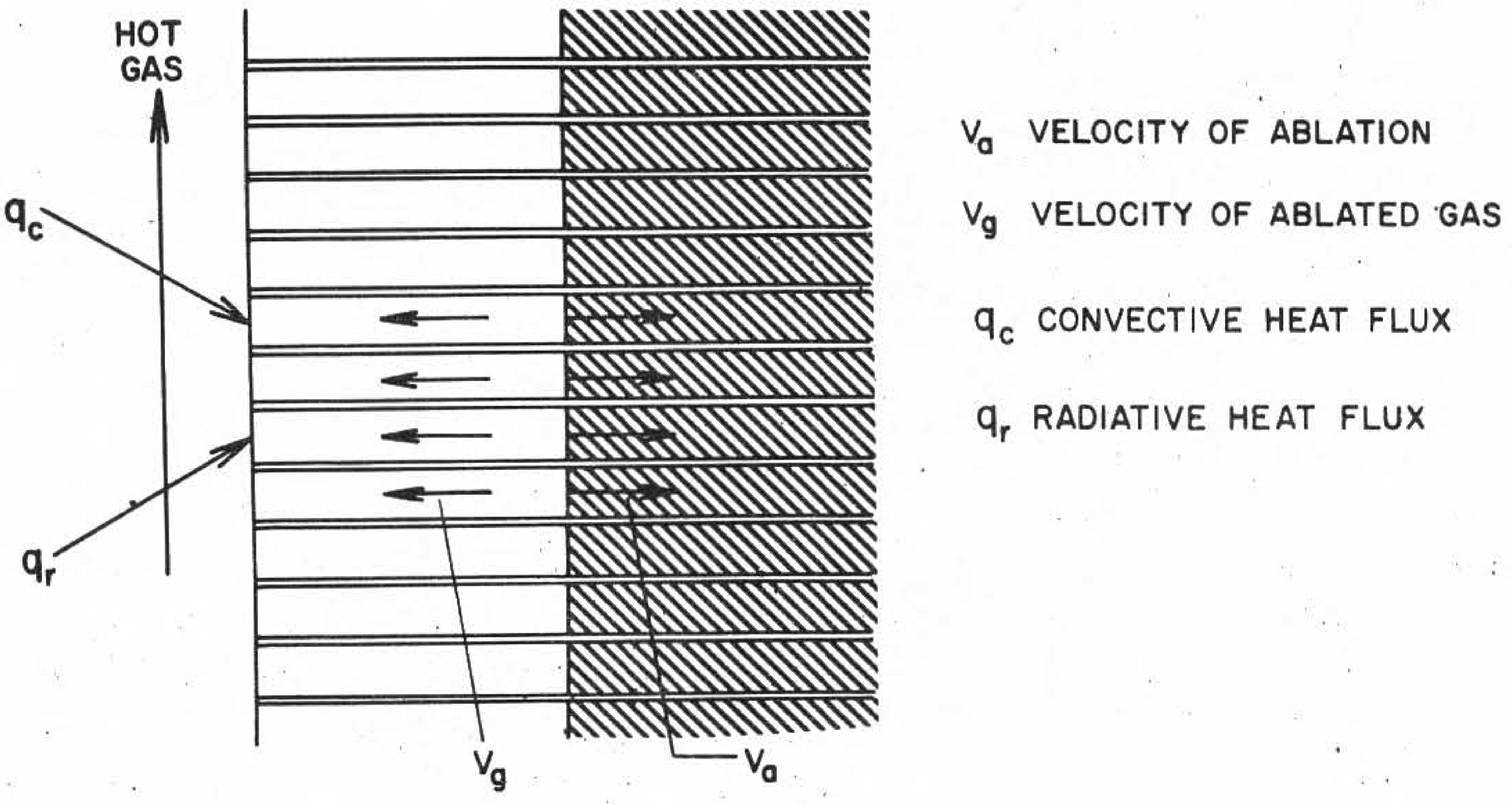

FIG. 1

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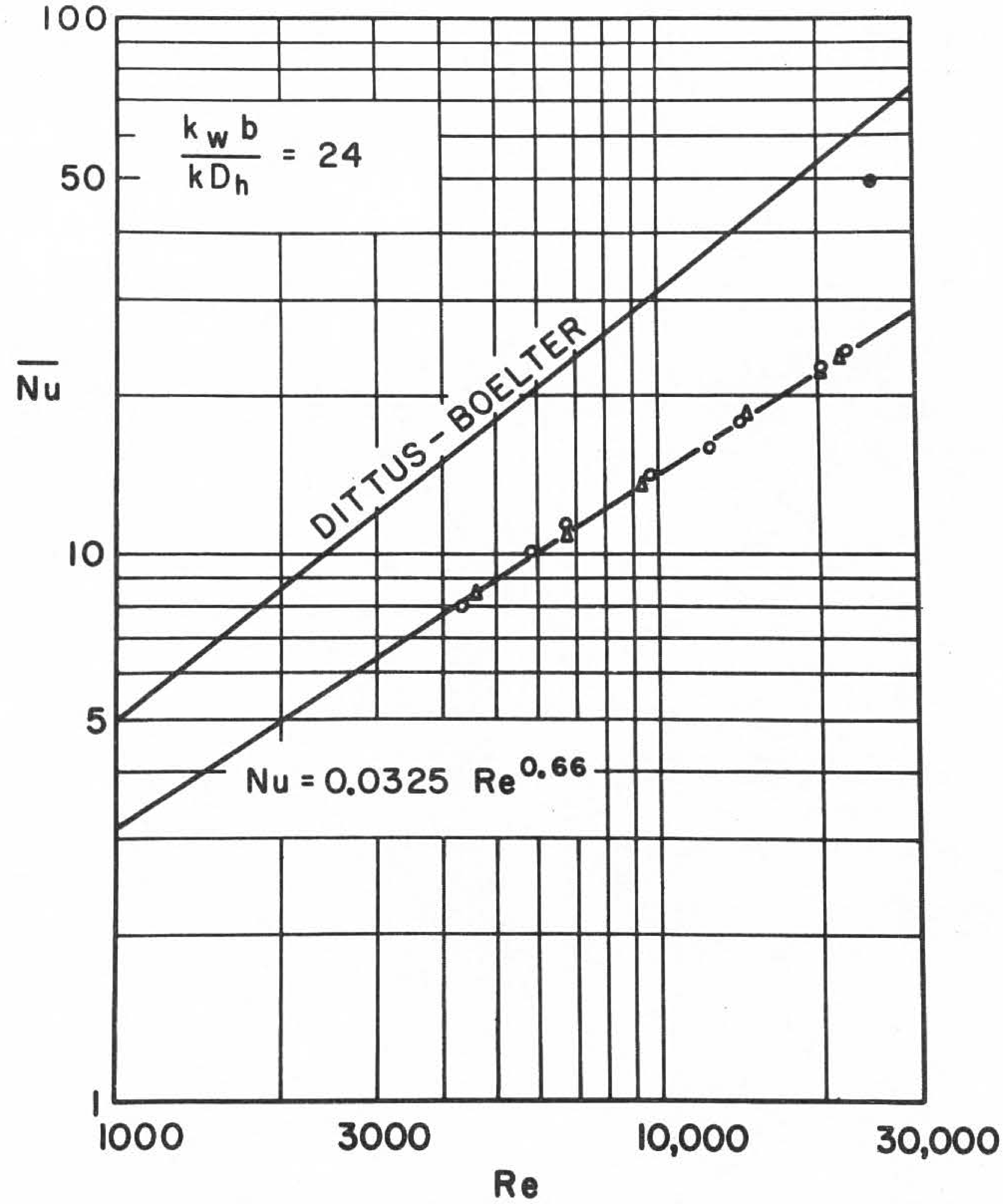
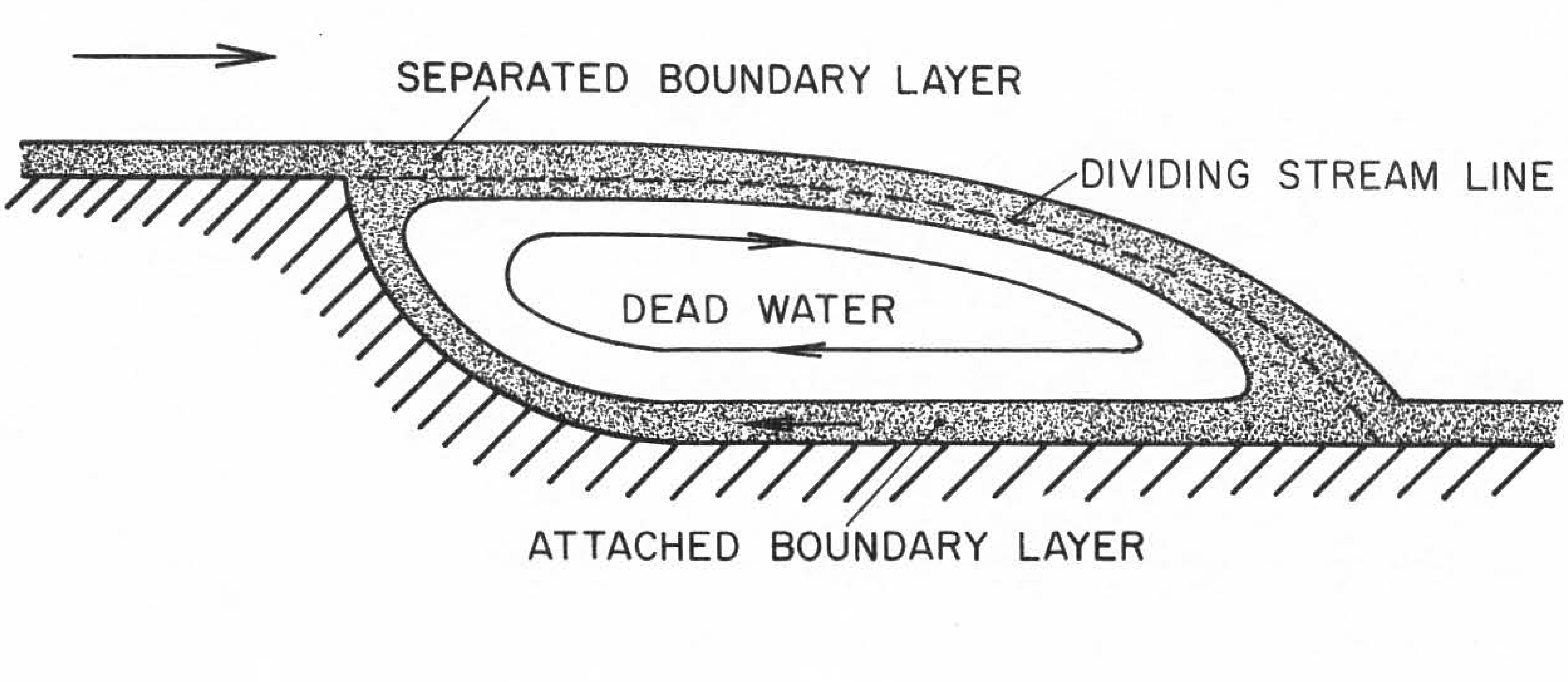


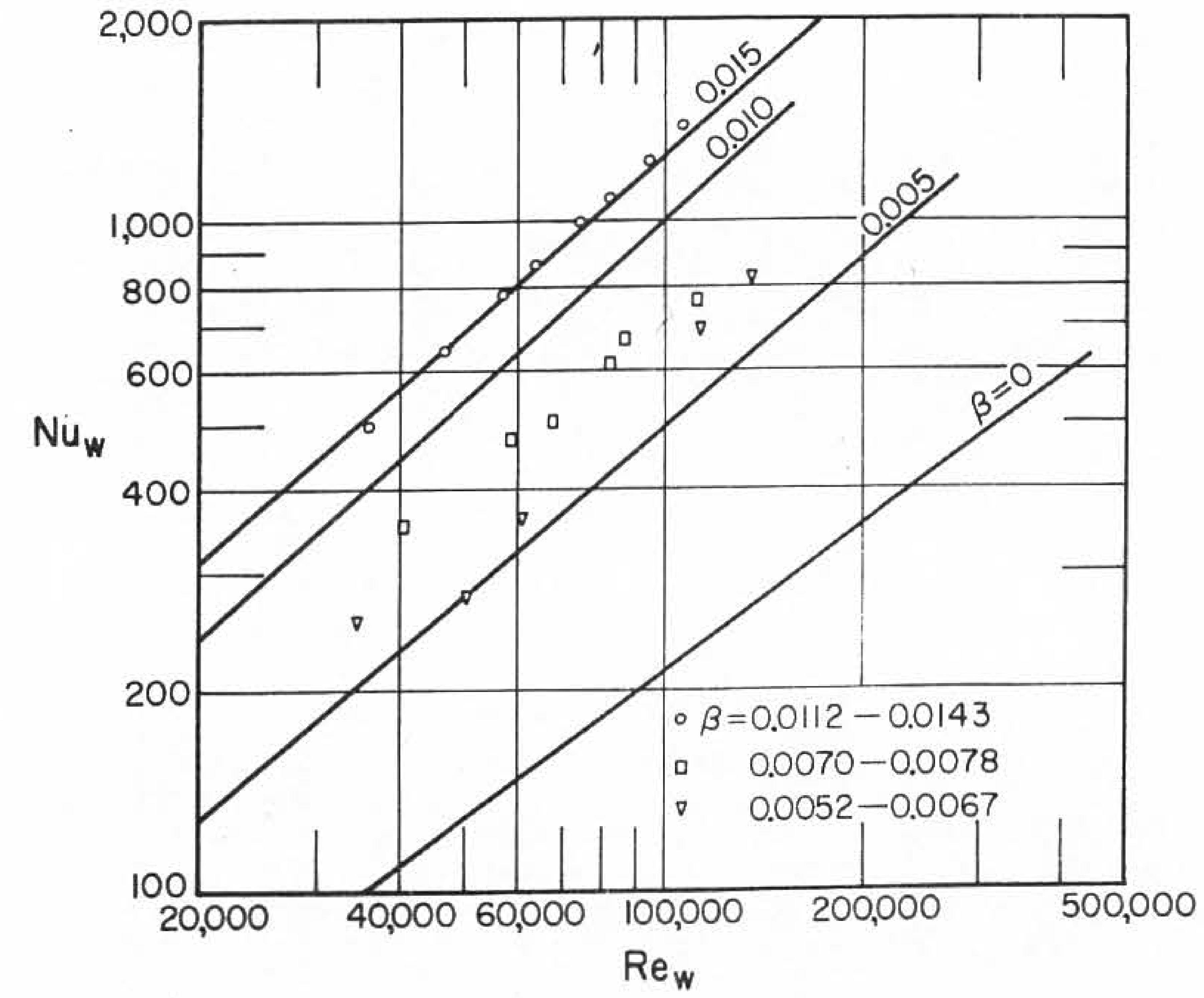
100
50
10
5
1
$\overline{Nu}$
$\frac{k_w b}{k D_h} = 24$
DITTUS - BOELTER
Nu = 0.0325 Re$^{0.66}$
1000
3000
10,000
30,000
Re

FIG. 2



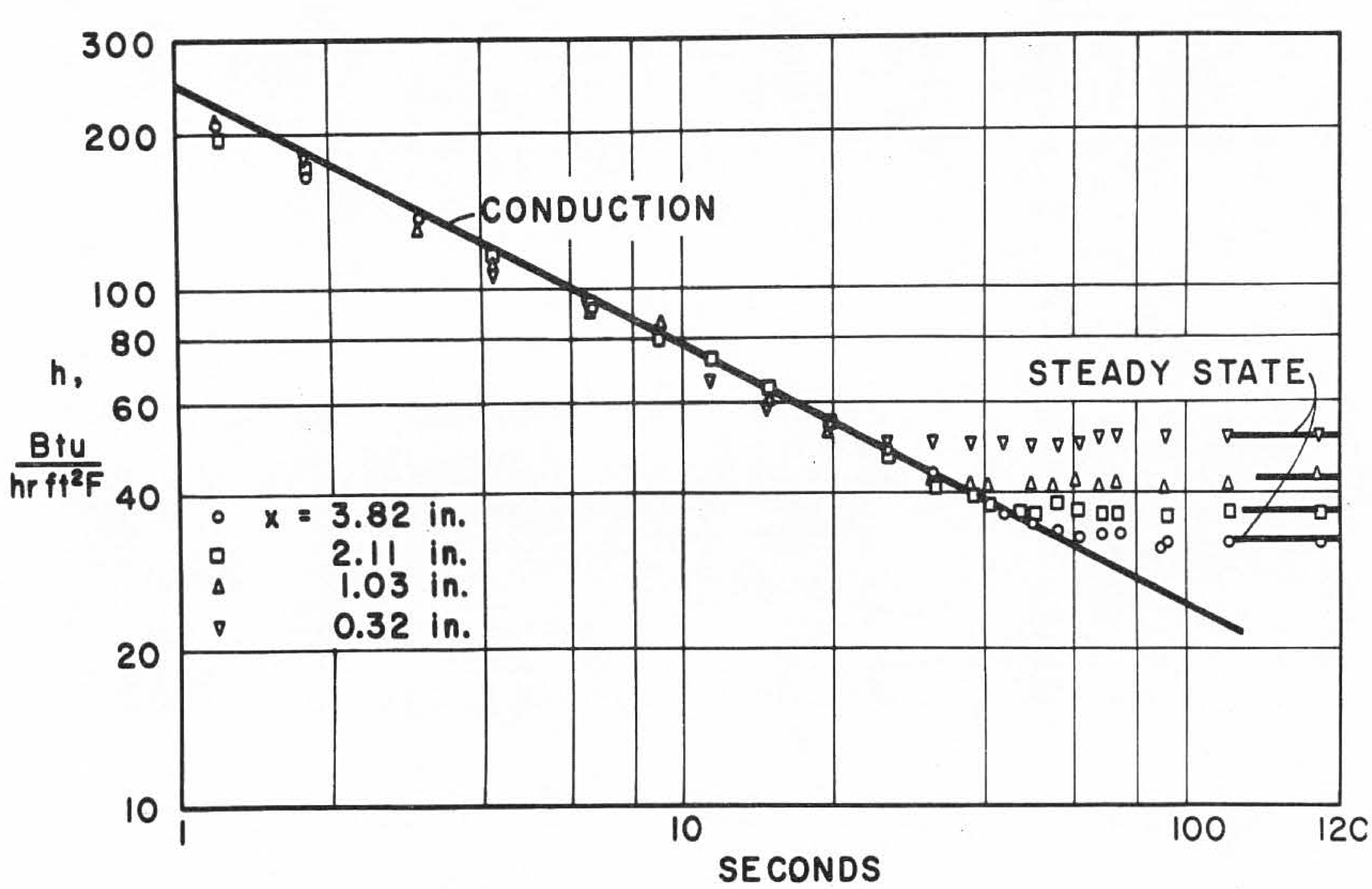
SEPARATED BOUNDARY LAYER
DIVIDING STREAM LINE
DEAD WATER
ATTACHED BOUNDARY LAYER

FIG. 3



2,000
1,000
800
600
400
200
100
Nu_w
20,000
40,000 60,000 100,000
200,000
500,000
Re_w
0.015
0.010
0.005
β=0
β=0.0112 — 0.0143
0.0070—0.0078
0.0052—0.0067

FIG. 4



300
200
100
80
60
40
20
10
h, Btu/hr ft²F
1
10
100
12C
SECONDS
CONDUCTION
STEADY STATE
x = 3.82 in.
2.11 in.
1.03 in.
0.32 in.

FIG. 5

MASS AND HEAT TRANSFER EQUATIONS:

$$\dot{m}_i = -\rho D_{12}\nabla w_i + \rho_i V + \text{COUPLED EFFECTS} \qquad (1)$$

$$\dot{q} = -k\nabla T - \rho D_{12}\Sigma h_i \nabla w_i - \rho V h + \text{" \quad "} \qquad (2)$$

$$= -\frac{k}{c_p}\nabla h - (Le-1)\Sigma h_i \nabla w_i - \rho V h + \text{" \quad "} \qquad (3)$$

BOUNDARY LAYER EQUATIONS:

$$\rho\left(u\frac{\partial w_i}{\partial x} + v\frac{\partial w_i}{\partial y}\right) = \frac{\partial}{\partial y}\left(\rho D_{12}\frac{\partial w_i}{\partial y}\right) + K_i \qquad (4)$$

$$\rho\left(u\frac{\partial h_s}{\partial x} + v\frac{\partial h_s}{\partial y}\right) = \frac{\partial}{\partial y}\left(\frac{\mu}{Pr}\frac{\partial h_s}{\partial y}\right) + \frac{\partial}{\partial y}\left[\mu\left(1-\frac{1}{Pr}\right)\frac{\partial}{\partial y}\left(\frac{u}{2}\right)^2\right]$$

$$+ \frac{\partial}{\partial y}\left[\rho D_{12}\left(1-\frac{1}{Le}\right)\Sigma h_i\frac{\partial w_i}{\partial y}\right] \qquad (5)$$

$$\rho\left(u\frac{\partial \tilde{w}_i}{\partial x} + v\frac{\partial \tilde{w}_i}{\partial y}\right) = \frac{\partial}{\partial y}\left(\mu\frac{\partial \tilde{w}_i}{\partial y}\right) \qquad (6)$$

$$\rho\left(u\frac{\partial h_s}{\partial x} + v\frac{\partial h_s}{\partial y}\right) = \frac{\partial}{\partial y}\left(\mu\frac{\partial h_s}{\partial y}\right) \qquad (7)$$

FIG. 6

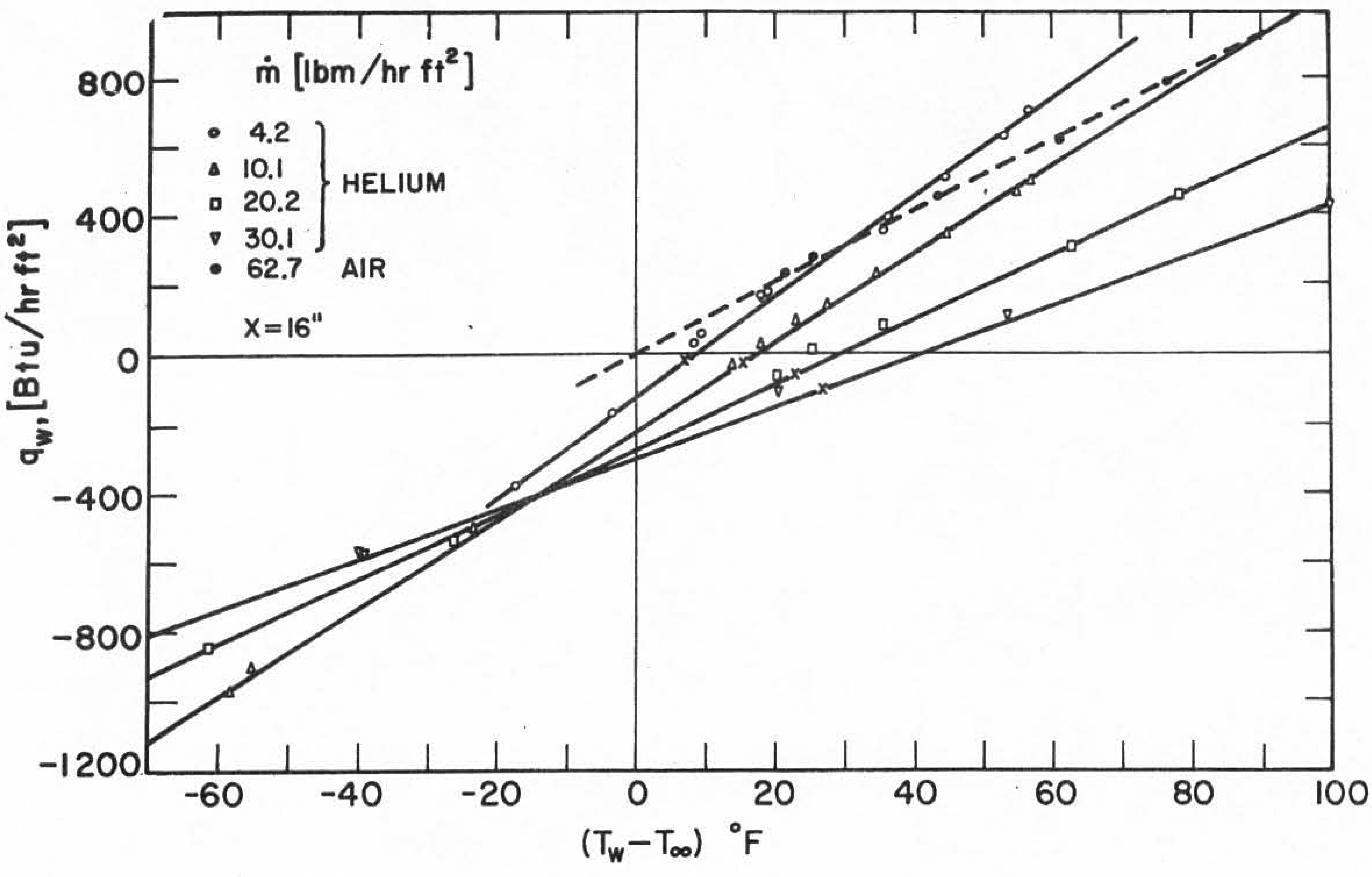


FIG. 7

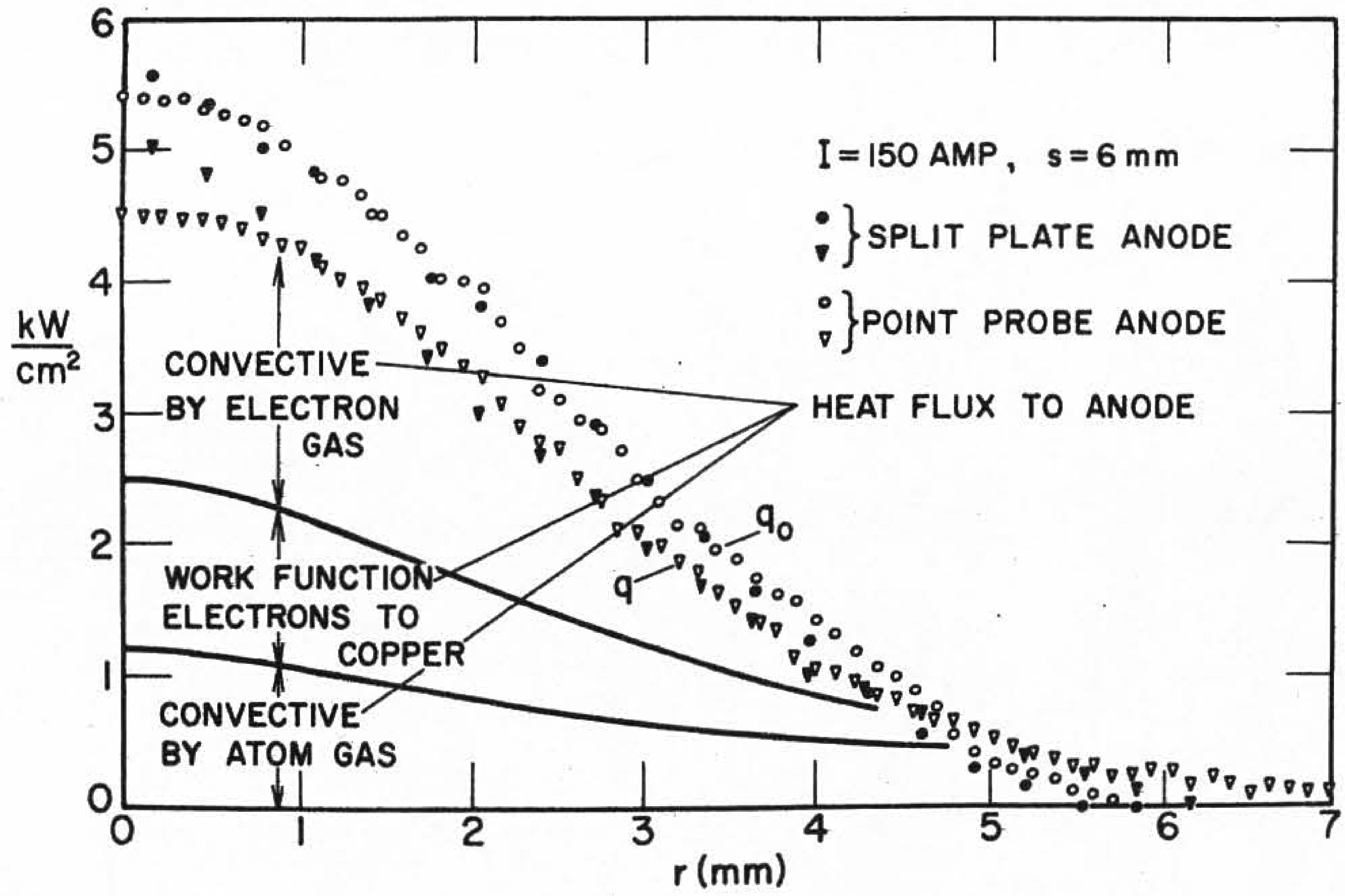


FIG. 8

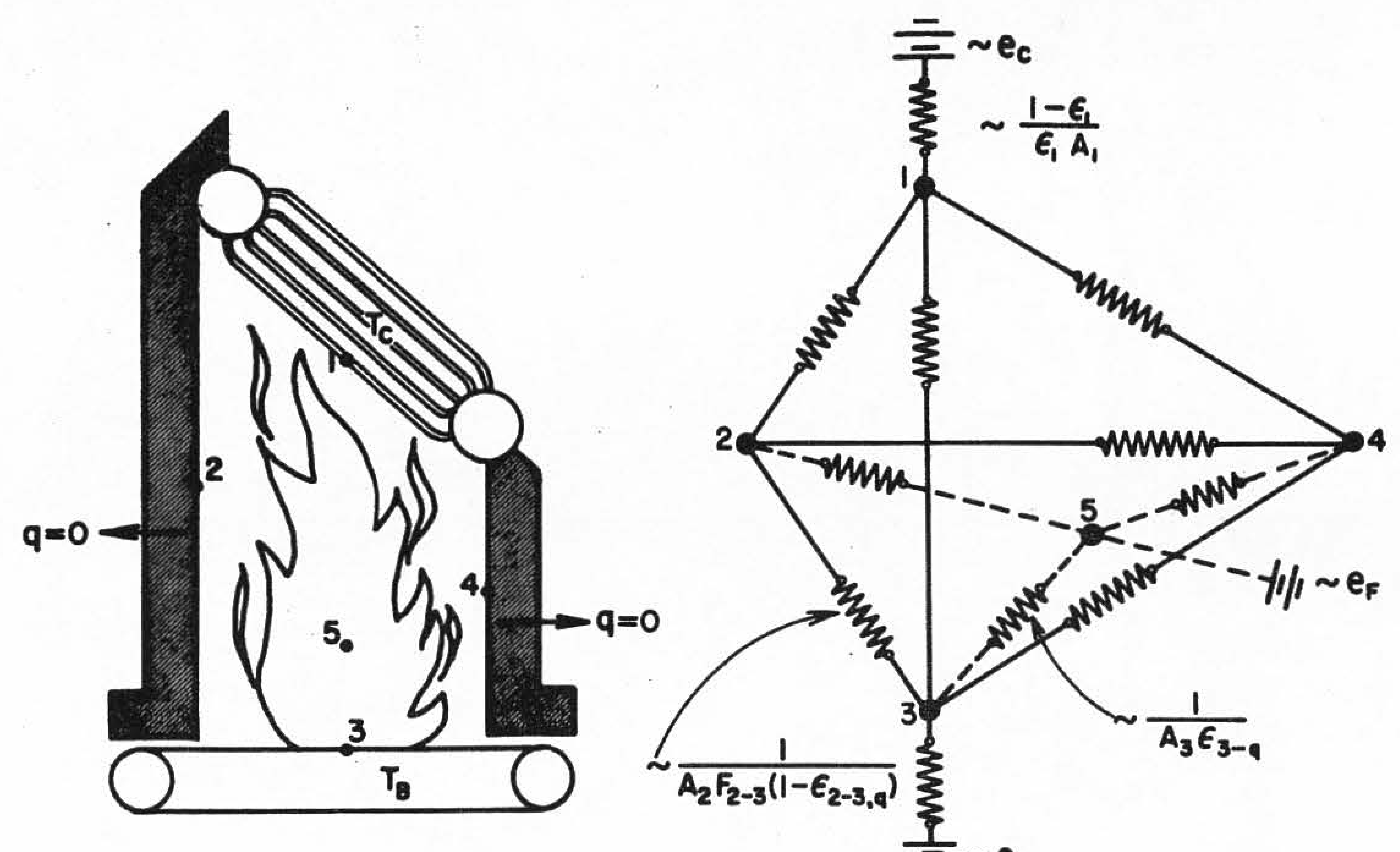
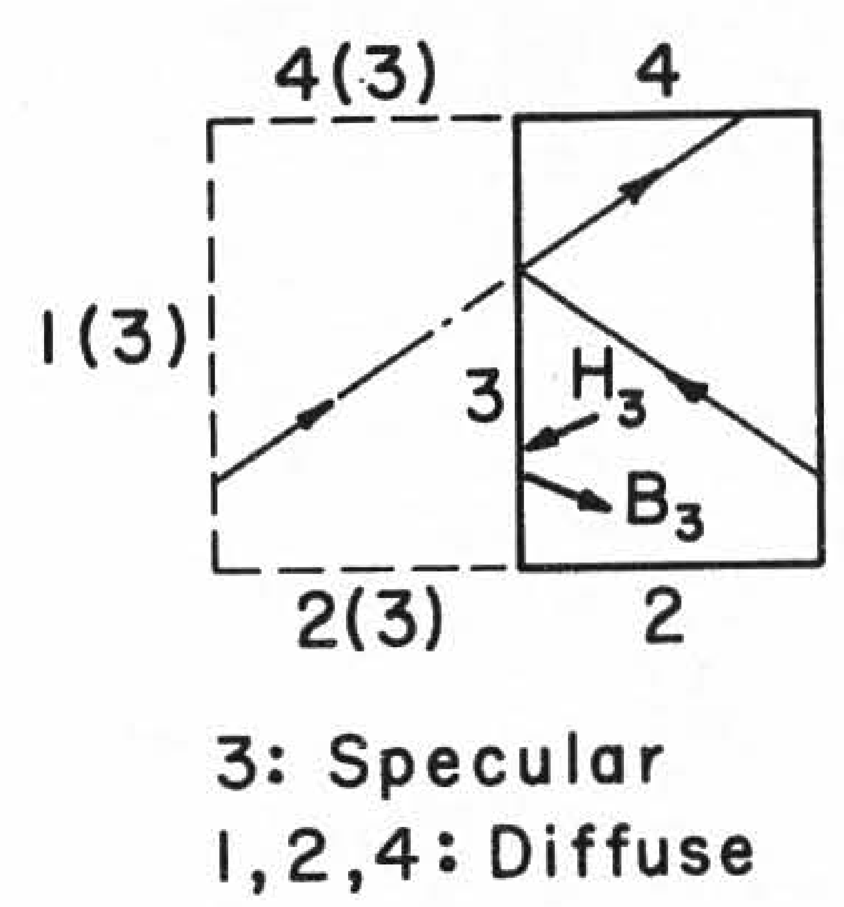


FIG. 9



$$B_3 = \epsilon_3 \sigma T_3^4 + \rho_3 H_3$$

$$H_3 = B_1 F_{3-1} + B_2 F_{3-2} + B_4 F_{3-4}$$

$$B_4 = \epsilon_4 \sigma T_4^4 + \rho_4 H_4$$

$$H_4 = B_1 (F_{4-1} + \rho_3 F_{4-1(3)}) + B_2 (F_{4-2} + \rho_3 F_{4-2(3)}) + \epsilon_3 \sigma T_3^4 F_{4-3}$$

FIG. 11

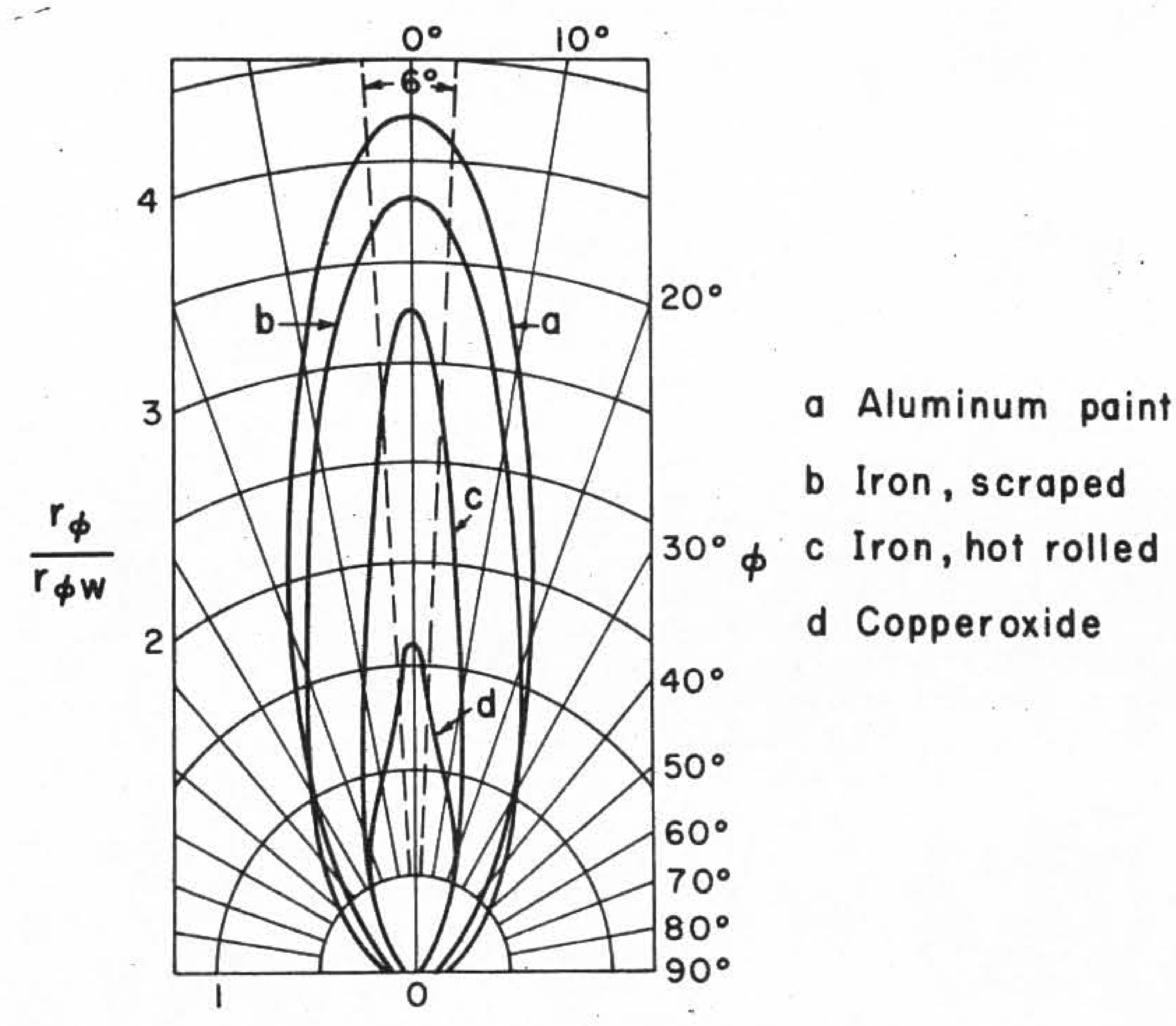
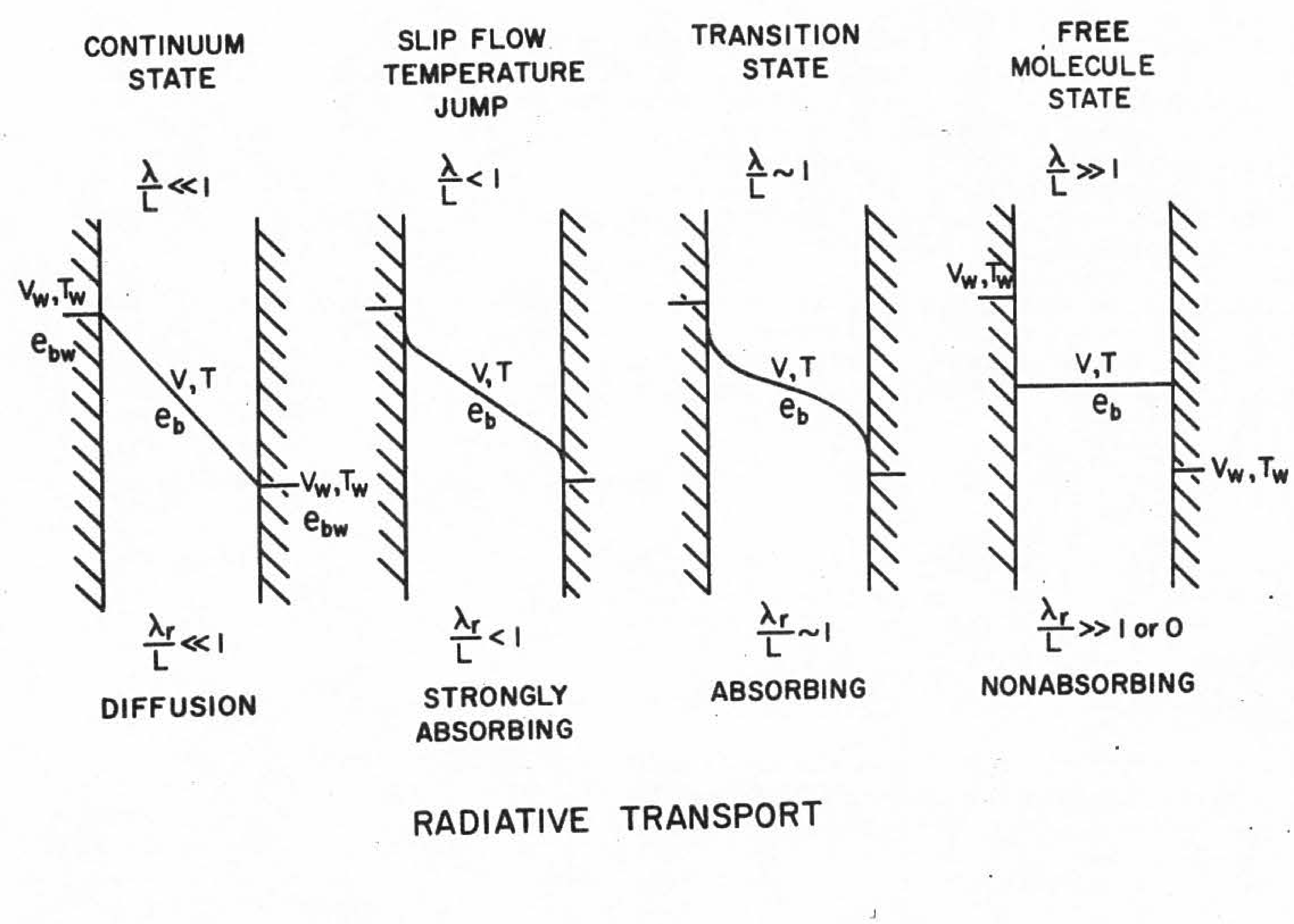


FIG. 10

MOLECULAR TRANSPORT



RADIATIVE TRANSPORT

FIG. 12