

## PLANNING EXPERIMENTS FOR ENGINEERING KINETIC DATA

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Since chemical kinetics is not as yet a predictive science, one of the tasks commonly faced by the engineer in process development is the accumulation of rate data to be used in reactor design. This paper will discuss some of the principles and techniques which can be used to establish useful design data for complex reactions of obscure mechanism which occur under ill-defined conditions and yield incompletely characterized products -- in other words, the usual case of practical interest.

It is a truism so obvious it is usually not taught in physical chemistry courses that one should first establish the stoichiometry of the reaction he is studying. Yet in process development this is often difficult. Analytical methods for the mixtures being produced may be time-consuming or unavailable and expensive to develop. There is a strong temptation to determine the principal product and most troublesome byproduct and ignore the rest. Gross material balances can often be made on the basis of elemental analyses without detailed knowledge of molecular composition. Yet these incomplete analyses often fail to give adequate kinetic data. When reactions are not well understood, it is not unusual to discover the appearance of a new product when conditions are changed. If the design has been based on data taken in ignorance of this product and operation extrapolated beyond the pilot plant range, serious trouble can ensue.

The first step, therefore, is a qualitative survey of the reaction stoichiometry over as wide a range of conditions as possible. The objective of this survey is to establish the main features of the reaction mechanism. For design purposes, the molecular mechanism will never need to be known in detail, but enough of its properties must be determined to formulate a kinetic model of the reaction for the range of conditions of design interest. The more nearly this model reflects the actual mechanism, the more confidently can it be applied over a wide range of conditions. Nevertheless, at some stage, the engineer must be satisfied to work with the data he has, recognizing that he has not established a mechanism fully.

Is the principal reaction product the ultimate product of reaction; or does it disappear in side or subsequent reactions under some conditions? Are the byproducts formed as or from intermediates en route to the main products, or are they formed by independent reaction routes? Do some or all of the products reach equilibrium or steady-state concentrations which are insensitive to residence time? Is there a phase separation in the course of the reaction? Not every autoclave that is loaded with a homogeneous solution and delivers a homogeneous product solution has had homogeneous contents throughout the course of the run. Are mass or heat transfer rates comparable to or slower than the chemical reaction rates? These are all questions that can receive qualitative answers by comparing the results of a few well-planned runs.

Consider a hypothetical example in which Q-acid is made by catalytic conversion of electamine and carbon dioxide in a fluid bed. Under reaction conditions, Q-acid is volatile in an atmosphere of carbon dioxide but it is found that organic matter accumulates on the catalyst to a degree depending on the temperature and feed ratio of electamine to CO<sub>2</sub>. Some undesirable electaminic acid is found both in the product vapor and in the organic residue on the catalyst, which, however, is largely unidentified material. The acid could be formed by carboxylation of the amine, but there is no evidence for the actual mechanism.

First experiments might be to vary the gas residence time in the converter, determining the space-time yield and purity of the electamine product. For experimental convenience in these survey runs, the process is not run in continuous steady-state. Instead, electamine is added to the catalyst to a predetermined loading at a temperature below that at which Q-acid is formed. The temperature is then raised and Q-acid stripped off in a stream of CO<sub>2</sub> sufficient to fluidize the bed. The concentration of Q-acid in the product stream is followed during the stripping process. It was found that the pounds per hour of product recovered is directly proportional to the CO<sub>2</sub> flow rate, other conditions being fixed, but is nearly independent of the electamine remaining on the catalyst. This was established by a set of runs at two temperatures in which CO<sub>2</sub> rate and initial electamine loading were varied.

At each temperature, the partial pressure of Q-acid in the off-gas was nearly constant throughout each run. However, with high initial loading of electamine, this partial pressure was less than with low initial loading. With other conditions comparable, the partial pressure of Q-acid was higher at higher temperature. Because of initial transients during the stripping runs as the temperature was being raised, the earliest steady data could be obtained only after 20% to 40% of the Q-acid had been stripped off. These results give a strong presumption that Q-acid is being formed in vapor-solid equilibrium with the organic matter on the catalyst, since as much conversion was obtained in a run as in runs with twice the residence time. However, runs which differ in initial loading gave somewhat different apparent vapor pressures. Hence, we must conclude that there is additional complexity in the mechanism. An additional complication in a fluid bed is the possibility of poor contacting of vapor reactant with catalyst when large "bubbles" can form. Since this bypassing increases with gas velocity, a lower yield at high gas velocity would be expected from this cause alone if it were operative, even though the dense phase reaches equilibrium.

Four more runs under strictly continuous steady operation should settle the qualitative nature of the mechanism. In these, temperature and feed composition are fixed but feed rate and bed height are varied as follows:

Run	1	2	3	4
V (cu.ft./hr.)	Q	2Q	2Q	4Q
H (ft.)	L	L	2L	2L
$\tau$ (hr.)	AL/Q	AL/2Q	AL/Q	AL/2Q

Conversion of electamine to Q-acid product is the measured response. The following logic ensues:

Sequence Compare	If	Then
A 1. Runs 1 and 2	Same conversion	Equilibrium is attained
B 1. Runs 1 and 2	Different conversion	Either kinetic control or by-passing
B 2. Runs 1 and 3	{ Same conversion { Different conversion	{ No bypassing { Bypassing occurs

From this logic, Run 4 appears superfluous. However, adding it gives a 2 x 2 factorial experiment in bed-height and residence time.

H $\tau$	L	2L
AL/2Q	2	4
AL/Q	1	3

In this balanced design the following effects are measurable.

$$\text{Kinetics: } K = Y_1 - Y_2 + Y_3 - Y_4$$

$$\text{Bed Height: } H = Y_4 - Y_2 + Y_3 - Y_1$$

$$\text{Bypassing: } B = Y_1 - Y_2 - Y_3 + Y_4$$

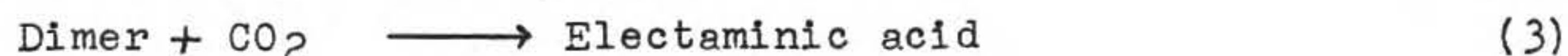
The bypassing effect appears as an interaction between the two main effects and the techniques of statistical analysis can be used to get the most out of the data. At the cost of an extra run, considerable additional confidence can be obtained in the conclusions.

In this example, we see the diagnostic value of a few well-chosen runs. Even more insight can be gained by abandoning the pilot reactor and studying the reaction in an altogether different configuration. Electamine and carbon dioxide might be loaded into a pressure cell adapted to an ultraviolet spectrometer. The product Q-acid vapor as well as the byproduct, electaminic acid could be followed readily by its UV absorption. In this small batch reactor,

the approach to steady vapor pressure of Q-acid could be followed conveniently at times close to the onset of reaction and at a series of temperatures. By interrupting the run at a given time and analyzing the catalyst and its organic contents, it is possible to associate the composition of organic solids with the progress of the main reaction.

In this way, the puzzling dependence of rate on feed composition can be resolved. In the case on which this hypothetical example is based, it was found that a non-volatile dimeric product of electamine forms rapidly on the catalyst with evolution of  $\text{CO}_2$ . In a second step, dimer reacts with  $\text{CO}_2$  and forms a solid, "pre-Q", which rapidly develops a steady vapor pressure of Q-acid. At the same time dimer reacts slowly with  $\text{CO}_2$  to form the byproduct electaminic acid. Thus in the initial runs in which the loading of electamine was low, little dimer was formed and the conversion to Q-acid proceeded rapidly, being governed by the rate of evaporation of pre-Q. In the runs in which the loading of electamine was high, dimer formed rapidly and the conversion of dimer to Q-acid was slower, being governed by the rate of conversion of dimer to pre-Q.

Thus a kinetic model might be written:



Reaction 1 is supposed reversible but not instantaneous. Reaction 2 is irreversible and comparable in speed to reaction 1. Reaction 3 is irreversible and slow. Reaction 4 is reversible and very rapid.

Thus far, although rates have been measured, no use has been made of their quantitative magnitudes. The arguments have depended upon relative values, the shapes of time-concentration curves and similar qualitative data. As a result, however, a reaction model has been reached in terms of which rate data can be quantitatively analyzed.

In many cases, such a model is previously known or is sufficiently probable that one can proceed to the quantitative phase with few preliminaries. In such a case, however, the experimental design should permit testing the fit of the data to the model as well as evaluation of the rate constants and activation energies.

When a model has been proposed as the basis for further kinetic study, the methods of statistical experimental design can greatly reduce the number of runs required to determine the rate constants and reaction orders. These methods are now quite readily available in the literature<sup>3</sup> and it is not proposed to discuss them in detail here. They must be used with insight, however, and are no substitute for thought. Some examples will illustrate the power and some of the precautions necessary in this approach.

Srini Vasan and the writer<sup>4</sup> studied the kinetics of the water-gas shift reaction over a commercial iron oxide catalyst. The literature and previous experience suggested a kinetic model, due to Temkin:<sup>7</sup>

$$R = - \frac{dp_{\text{CO}}}{dt} = k p_{\text{CO}}^a \left( \frac{p_{\text{H}_2\text{O}}}{p_{\text{H}_2}} \right)^b \quad (5)$$

Although the reaction is reversible, conditions were chosen to minimize the extent of back-reaction. Taking logarithms,

$$\log R = \log A - \frac{E}{RT} + a \log p_{\text{CO}} + b \log p_{\text{H}_2\text{O}} - b \log p_{\text{H}_2} \quad (6)$$

This is a linear form in  $1/T$  and the  $\log p_i$ . Standard methods for the design and analysis of experiments permit the determination of best values for the parameters  $E/R$ ,  $\log A$ ,  $a$ , and  $b$  as well as a test of the goodness-of-fit of Equation (6) if fairly general conditions on the errors of measurement are satisfied. At no increased labor, the model could be generalized to include all of the possible components with arbitrary exponents:

$$R = k p_{\text{CO}}^a p_{\text{H}_2\text{O}}^b p_{\text{H}_2}^c p_{\text{CO}_2}^d$$

The geometrical interpretation of the experimental design is quite helpful. We may think of  $R$  as a function of  $1/T$  and the  $\log p_i$  which can be plotted in  $n$ -dimensional Euclidean space with the independent variables as coordinate axes. We seek a mathematical representation of the hypersurface  $\log R(1/T, \log p_i)$ . If Equation (6) is such a representation,  $\log R$  will be a plane parallel to the axis of  $\log p_{CO_2}$ . Furthermore, its intercepts on the  $\log p_{H_2O}$  and  $\log p_{H_2}$  axis will be equal but opposite in sign. If Equation (7) holds, the  $\log R$  surface will still be planar, but there will be no a priori constraints on its orientation. The experimental design problem is now to test whether  $\log R$  is indeed a plane, and, if so, to find its equation. With five factors, a 5-plane in six-dimensional space is determined by five points. By determining  $\log R$  at additional points, the deviation from planarity can be tested. In the example cited, sixteen points were determined and it was decisively shown that Equation (6) could not represent the data within the reproducibility of experiments.

If the  $\log R$  surface is not planar, what shape is it? Standard methods now exist to determine the best quadratic approximation to  $\log R$ . It is tempting to use this approximation, since the experiments already done to test the planar hypothesis are the core of the design which determines the quadratic approximation. Before succumbing to temptation, however, one should consider his objectives carefully. Equation (6) or (7) has a theoretical interpretation in that a molecular mechanism can be formulated which leads to it. Of course, the possibility of deriving a kinetic equation from a mechanism does not necessarily make it valid. However, the general quadratic form is not derivable from any mechanism and hence cannot possibly be valid except as an empirical interpolation formula. If the design studies which are contemplated will never go outside the range of the data taken, then an interpolation formula may be a sufficient representation of the kinetics. However, if extrapolation is necessary, much greater confidence can be had in kinetic models based on the best mechanism which can be proposed.

A second consideration is often important, however. Complex mechanisms involve many parameters. The general Langmuir mechanism <sup>5</sup> for the catalytic irreversible reaction  $A + B \longrightarrow \text{Products}$  has the form

$$R = \frac{kp_A^a p_B^b}{[1 + K_A p_A + K_B p_B]^n} \quad (8)$$

Each of the parameters  $k$ ,  $K_A$ , and  $K_B$  is exponentially temperature dependent:

$$k_j = A_j e^{-E_j/RT} \quad (9)$$

There are thus nine parameters to determine. Usually they will differ widely in magnitude, but theory will not predict in advance which ones may turn out to be negligible.

It is a fact of the imperfect world that the more parameters that must be determined, the more difficult it usually is to estimate them. When the models are linear in the parameters, experimental designs may be found which will, in theory, allow good estimates to be obtained. The upper limit on precision is determined largely by the magnitude of the experimental error, the number of runs to be made and the experimental range of the variables. In practice, however, the operable region may be such that balanced designs are made impossible by interdependence of the supposedly independent variables. That is, the process may not remain operable unless a change in one variable is compensated for in part by a change in another. In this way correlations tend to creep in to reduce the precision of the estimates as the number of parameters and variables increase. The only partial counter measures are (a) the difficult course of attempting to reduce the magnitude of the experimental errors or (b) an increase in the range of the variables. Chemical processes, however, always have finite restrictions on the operable range of the design variables. The workable temperature range is finite; permissible feed compositions may be limited by phase changes or explosion limits.

When models are non linear in the parameters, the same difficulties exist except that correlations between estimated parameters are more apt to exist. First the optimum theoretical designs in an unhampered experimental region which would maximize the precision of the estimates are usually extremely difficult to find mathematically and secondly, the nature of the function itself may make a high dependence of the estimates unavoidable. Hence in any actual case, there will be a maximum number of kinetic parameters which can be determined with precision from experimental data. The more careful and precise the data, the larger this number becomes, but it rarely exceeds five or six. One must, therefore, scrutinize complex theoretical mechanisms to discover which of the many parameters are likely to be buried in the experimental error.

The quadratic empirical surface can be of great help in this process. Thus, if Equation (8) is rewritten and expanded into the form of Equation (7), it will be discovered that  $K_A$  and  $K_B$  occur only in quadratic terms involving  $1/T$  and the  $\log p_A$  and  $\log p_B$ , respectively. Hence, if the empirical quadratic surface lacks terms in  $1/T \times \log p_A$  and  $\log p_A \times \log p_B$ , one can be confident that  $p_A$  can be omitted from the denominator of Equation (8) without worsening the fit. In fact,  $K_A$  could not be determined with precision from the data that fixed the quadratic surface. In this way, complex mechanisms can be rationally simplified without overstepping the limits of reliability of the data.

One may find, of course, that the data can be more simply represented by a new choice of coordinates. Thus Equation (8) is more naturally represented in the form

$$(1/R)^{1/n} = \frac{1}{k_1 p_A^{a/n} p_B^{b/n}} \frac{K_A}{k_1 p_A^{a/n-1} p_B^{b/n}} \frac{K_B}{k_1 p_A^{a/n} p_B^{b/n-1}} \quad (10)$$

This suggests  $1/p_A$  and  $1/p_B$  as better variables than  $\log p_A$  and  $\log p_B$  for testing this mechanism. In this space, the original experimental points which are well-spaced in  $\log p_i$  may be poorly placed to give the best determination of the  $R^{-1/n}$  surface. New data may be required to determine the parameters of Equation (10) with precision.

The availability of high-speed computing capacity modifies some of these possibilities. It may no longer be necessary to linearize the rate expression for computational reasons. Techniques for non-linear estimation<sup>2</sup> permit working directly with the proposed model. However, the number of parameters which can be determined simultaneously is limited as before and most theoretical mechanisms must be simplified to make non-linear estimation feasible.

In some cases the ultimate design problem may be able to accommodate an integral kinetic form rather than a differential one. If, for instance, it is clear that a batch reactor will be used with no internal concentration gradients, only the total volume or residence time will be required for the design basis. Rather than design a differential reactor or differentiate integral data, one may then propose a kinetic model in which time is an explicit factor. Our previous remarks about the maximum complexity of a useful model are especially pertinent.

These points are illustrated in the case of a study undertaken recently in connection with the purification steps of a commercial process. A minor impurity is removed by precipitation with aqueous ammonia. Complication arises out of the base catalyzed hydrolysis of the principal product, which represents a loss. The kinetic study was designed to locate the conditions under which yield at required purity could be maximized. However, since in design it might become necessary to modify some of the conditions, a kinetic model valid over a range of conditions was required. The change in concentration of the impurity is small and small hydrolysis of the principal product is expected, even though a large single stage integral converter is ultimately envisioned.

As is often the case, some older data were available for which the experimental precision was only moderate. New data in both small and large reactors were obtained to test the assumed independence of yields on reactor size. In all, four sets of data were available:

1. New small reactor data
2. Old small reactor data
3. New large reactor data
4. Old large reactor data

Four independent variables had been studied:

1. Temperature (T)
2. Weight per cent product initially in the impure mixture (P)
3. Weight per cent ammonia initially (N)
4. Reaction time ( $\theta$ )

The response in each run was the precipitate produced, measured as y, per cent of the initial product P.

Since it was desired to make use of the unplanned older data, an orthogonal factorial design or central composite second order design was not possible. As a preliminary survey, a full quadratic model in four variables was fitted by least squares to four groups of data:

- a. Sets 1 and 2
- b. Sets 1, 2 and 3
- c. Sets 1, 3 and 4
- d. Sets 1, 2, 3, and 4

An additional block variable was added for data in sets 3 and 4 to test the effect of reactor size. This effect proved to be non-significant and comparison of residual mean squares of each group of data showed no significant difference between groups. Hence all of the data was used in the final analysis.

A full quadratic model in four variables has fifteen coefficients, four of which only serve to locate the origin with respect to which linear terms vanish. By rotating axes about this origin, the six cross-terms can be eliminated, leaving only the four squared terms and the constant. This reduction to canonical form is done by proper choice of four new independent linear combinations of the independent variables. The results of this analysis are:

$$y = .65362 + .49854 z_1^2 - .09111 z_2^2 - .02671 z_3^2 - .00083 z_4^2 \quad (11)$$

where

$$z_1 = .95990 T' - .27634 P' + .02143 N' + .04214 \theta' \quad (12)$$

$$z_2 = .08732 T' + .37390 P' + .92333 N' - .00672 \theta' \quad (13)$$

$$z_3 = .26183 T' + .88518 P' - .38296 N' + .03520 \theta' \quad (14)$$

$$z_4 = .04916 T' + .01703 P' - .01892 N' - .99847 \theta' \quad (15)$$

Here the primes refer to scaled values of the independent variables.

For example,

$$T' = (T(^{\circ}\text{C.}) - 141.85)/27 \quad (16)$$

From Equation (11) we see that  $z_1$  is by far the most important term. By Equation (12),  $z_1$  is nearly independent of N and  $\theta$  and measures T and P most strongly. Reaction time,  $\theta$ , is almost identical with  $z_4$ . Hence we feel justified in treating its small effect independently of the other variables. Since many of the twenty-one coefficients are small and probably non-significant, one suspects that there are many alternatives to the quadratic form which fit the data as well.

Therefore, a reasonable mechanism was postulated as a guide to a simpler kinetic expression. Suppose the hydrolysis to be catalyzed by hydroxyl ion. Then one might have



$$\frac{d[\text{POH}^-]}{d\theta} = k_1 \frac{[P][\text{OH}^-]}{[\text{OH}^-]} \quad (19)$$

$$\frac{[\text{OH}^-]}{[\text{NH}_4^+]} = K_2 \frac{[\text{NH}_3]}{[\text{NH}_4^+]} = K_2 \frac{N_0 - [\text{OH}^-] - [\text{POH}^-]}{[\text{OH}^-] [\text{POH}^-]} \quad (20)$$

$$[P] = P_0 - [\text{POH}^-] \quad (21)$$

$$y = [\text{POH}^-] / P_0 \quad (22)$$

whence

$$\frac{dy}{d\theta} = \frac{1}{P_0} \frac{d[\text{POH}^-]}{d\theta} = k_1 k_2 \frac{1 - ([\text{OH}^-]/N_0) - (y P_0/N_0)}{[\text{OH}^-]/N_0 + (y P_0/N_0)} (1-y) \quad (23)$$

since  $y$  remains much less than unity throughout all the tests, and initially

$$[\text{OH}^-]_0/N_0 = K_2^{1/2} / N_0^{1/2} \quad (24)$$

which is also small, Equation (23) can be simplified to

$$\frac{dy}{d\theta} = \frac{k_1 K_2 N_0 (1-y P_0/N_0)}{P_0 y} \quad (25)$$

$$= \frac{k_1 K_2 N_0}{P_0 y} - k_1 K_2 \quad (26)$$

When  $y$  is very small, the first term dominates and the initial conversion should be given approximately by

$$y = (k_1 K_2 N_0 / P_0)^{1/2} \theta \quad (27)$$

This suggests a slightly generalized empirical kinetic expression

$$\log (y - C) = \log A - \frac{\theta}{T} p \log P + n \log N + \alpha \log \theta \quad (28)$$

A preliminary estimate of the coefficients with their confidence limits gave the data in Case I of Table I. This confirms our earlier conclusion that  $\theta$  could be treated independently as shown in Equation (27). The data were then refitted fixing  $\alpha = -p = n = 0.5$ , their theoretical values in Equation (27). This gave Case II in Table I.

Table I

	Case I		Case II	
	Coefficient	95% Conf. Limits	Coefficient	95% Conf. Limits
ln A	+ 16.64	± 4.75	+ 16.16	± 4.46
$\theta$	+ 8257.87	± 1889.65	+ 7648.17	± 1928.25
p	- 0.562	± 0.344	(-0.50)	± 0
n	+ 0.261	± 0.567	(+0.50)	± 0
$\alpha$	+ 0.978	± 0.279	(+0.50)	± 0

There is no significant difference in the fit between Equation 28 with four constants (Case I) and Equation 11 with fifteen.

Note that while  $n$  and  $p$  are not significantly different from their theoretical values,  $\alpha$  is significantly larger than 0.5. Nevertheless, the fit forced with  $\alpha = 0.5$  (Case II) is not tremendously worse.

Examination of the residuals shows that the fit is poorest at low  $N_0$ . In fact, some hydrolysis occurs even in the absence of added ammonia. The hypothetical mechanism does not allow for this. The empirical models based on it force the fit by averaging up the reaction order with respect to ammonia. A more realistic model might arise by adding a term to Equation (26) which is proportional to  $1-y$ . This, when integrated, leads to

$$y = c_1 \theta + \frac{c_2 N_0}{P_0} \ln \left( 1 + \frac{N_0 y}{c_2 P_0} \right) \quad (29)$$

where  $c_1$ ,  $c_2$  and  $c_3$  might each have exponential temperature dependence. The difficulty of fitting Equation (29) is much increased by its non-linear form. For small  $y$ , Equation (29) reduces to Equation (28). Hence it will be essential to use the non-linear form if any improvement is to be expected.

In this example, an empirical quadratic form has again given insight into what mechanistic terms should be retained in formulating a kinetic model with fewer constants to determine. It should be clear that there are a large number of kinetic models that will represent a given set of data. Unless these data are of very high precision, the fact that the engineer has found one such set lends very little support to the corresponding mechanistic interpretation. However, qualitative features can be discerned and more sensitive experiments are suggested by the analysis which may test the mechanistic assumptions in a less equivocal way. Nevertheless, the mechanistically inspired empirical kinetic form will usually be simpler and reliable over a wider range of variables than a pure linear or quadratic form in the original experimental variables. Thus the requirements of engineering data for design purposes can be met without sacrificing the best theoretical knowledge available.

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