ChE views and opinions

On what sort of place, if any, THEORETICAL AND MATHEMATICAL studies should have in graduate CHEMICAL ENGINEERING RESEARCH*

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Four questions on the suitability of mathematical studies are raised; namely, whether they have a place as training, whether they are practical, whether their methods are suitable and what is the role of rigour.

Question: On what sort of place, if any, theoretical and mathematical studies should have in graduate chemical engineering research.

In order to contain our subject within reasonable bounds we propose to treat it under four points of inquiry:

I. whether mathematical studies have any place in the chemical engineer's training;

II. whether mathematical and theoretical studies are practical or contemplative;

III. whether the methods of mathematical and theoretical studies are conformable to those of engineering;

IV. whether mathematical rigor is a notion from which the engineer may profitably be dispensed.

ARTICLE I.

Have mathematical studies any place in the chemical engineer's training?

1. It would seem that mathematical studies have no place in the training of a chemical engineer whose true nature is fulfilled in making and doing things. Now the end of mathematics is not to do or make anything. Therefore mathematical studies are alien to the engineer's training.

2. Moreover, there are more important subjects in the engineering curriculum, which is already a full and difficult one. Therefore mathematics should have little part in the curriculum.

3. Again, the training of the engineer should be directed towards developing those faculties that he will need in industry. The principal faculty of an engineer is to make responsible judgments and correct decisions on the basis of information which is always incomplete and often faulty. Now there is no analogous situation in mathematics and therefore it adds nothing to the engineer's training.

On the other hand, mathematics is where the art of generalization is best seen and learned and where the formal object of any science is best displayed. Simple examples free from the compounded difficulties of the material objects of chemical engineering are often didactically desirable.

REPLY

1. Even if the formal object of mathematics is not to make any material thing it is nevertheless often the efficient cause of a good design. Moreover the engineer takes pride in doing for a penny what any fool may do for a dollar. Now if each problem must be considered *de novo* as it arises without the generalizing power of mathematics he is certainly not practicing the economy of which he is so rightly proud. Fur-

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ther it may be questioned whether mathematical studies are so ineffective for does not the poet write[1]

> We are the music makers,
> We are the dreamers of dreams,
> Wandering by lone sea-breakers,
> And sitting by desolate streams;
> World-losers and world-forsakers,
> On whom the pale moon gleams:
> Yet we are the movers and shakers
> Of the world for ever, it seems.

2. To the second objection we may reply by asking what constitutes a more important subject. Certainly mathematics must take its place along with chemistry, physics and engineering science in competition for the student's attention and not arrogate to itself a primacy which in this context it does not possess. But it may be argued where, if not at the university, has the opportunity to become acquainted with both the utility of the methods and beauty of the notions of modern mathematics; as the poet[2] says, *'Omne tulit punctum qui miscuit utile dulci.'*[2]

3. Though it be true that the engineer's task is to make decisions on the basis of incomplete information, yet he can gain part of the experience on which this ability is founded by the study of mathematical models. For example, the formal nature of an exothermic reaction is to possess at each composition a temperature at which the reaction rate is greatest, which temperature is somewhat lower than the equilibrium temperature for the same composition. And this may be learned, not only by experience with the material objects of chemical engineering, namely reacting chemicals and reactors, but more generally from experience with the formal objects of kinetics, namely the kinds of reaction rate expression that are found to be useful. The engineer may thus supplement his incomplete information on a particular reaction by an understanding of the general behavior of a whole class of reactions.

ARTICLE II.

Are mathematical studies practical or contemplative?

1. It would seem that mathematical studies are not part of any practical science, for Aristotle says "a practical science is that which ends in action."[3] Now mathematics is one of the formal sciences whose truth depends on their internal consistency and not upon any relation to the observable world as is the case with the factual sciences.

2. In the hierarchy of the sciences the factual sciences occupy an intermediate position between the formal and the applied. Now engineering is clearly an applied science and practical, therefore mathematics is far removed from it and wholly contemplative.

On the other hand it is generally acknowledged that the purist attitude of such eminent mathematicians as the late G. H. Hardy (who, in his *Mathematician's Apology*,[4] crowns the theory of numbers as queen of the mathematical sciences because of its [her] complete uselessness) is an extreme one. Mathematical thought is diverse in its ramifications and penetrates to other disciplines.

REPLY

1. Nothing is more practical than a good theory, for to be good it must have clear indication within itself of how it may be proved, i.e., tested. Again, theory comes before experiment since some vision is needed to correctly formulate any experiment. Therefore theoretical studies are of practical value.

2. Moreover, although the division of sciences into formal, factual and applied is a useful one, it would be a mistake to interpret it rigidly and refuse to recognize the interpenetration of ideas throughout them. For though applications of such a subject as number theory are rare, this does not mean that they may not be found nor will the theory be defiled if they are.

ARTICLE III.

Are the methods of mathematics conformable to those of engineering?

1. It would seem that the methods of the mathematician have little or nothing in common with those of the engineer. For mathematics is an axiomatic science proceeding by strict deduction, and engineering lore is founded on direct experience and proceeds by trial and error.

2. Again the technique of the chemical engineer is to scale up his apparatus by stages, and wisely so for he deals with processes that are rarely understood in any complete sense. This procession of scales has no part in mathematics.

3. Moreover, the mathematical approach will often start with so drastic an idealization as to

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take it one remove from engineering realities at the outset.

On the other hand, rational thought must permeate all purposeful human activity, including that of the chemical engineer. Now rational thought is to be seen par excellence in mathematics and so learned there.

REPLY

1. It is false to assume that, because the ultimate presentation of mathematical results is deductive, the context of discovery is also purely deductive. In fact mathematics requires as much imagination as it does logic, and proceeds by the pattern of conjecture and confirmation or refutation familiar in all sciences. Moreover it has its own experiences of theory building which are just as valuable to the engineer as the more banausic experiences of plant building.

2. Even though scale-up has no direct counterpart in mathematical theory it is served by the notions of dimensional analysis. The correlations of empirical data by means of dimensionless groups need to be informed by a proper understanding of the underlying equations or most spurious results can be generated (cf. P. N. Rowe's illustration[5]). Although we are still some distance from completely a priori design of chemical reactors, the progress of scaling up is now often greatly expedited by proper mathematical modelling at an early stage.

3. Furthermore, though a problem is often idealized considerably when formulated in mathematical terms, this does not necessarily take it completely out of touch with reality. There is a hierarchic and iterative aspect to idealization. A situation may be grossly oversimplified at first and yet its analysis can form the basis for an interconnected set of problems in which more complete solution is built up by successively including more refinements and evaluating their effects.

ARTICLE IV

Cannot the engineer be dispensed from the notion and habit of mathematical rigor?

1. The mathematician's notions of rigor would seem to be wholly out of place in the context of engineering applications. For rigor implies exactitude and we have admitted that a mathematical model is always an idealization and its exactitude is therefore an artifact of analysis and does not belong to the real world.

2. Moreover in any model the constants and parameters will be imperfectly determined. It seems therefore foolish for the engineer to worry about existence theorems, strict deduction, necessity and sufficiency and the like, when his basic quantities are always open to error.

3. With the power of modern computers numerical solutions can often be obtained faster than any general properties of the solution can be rigorously proved and a numerical answer is what the engineer wants. On the contrary, "In physical theory, mathematical rigor is of the essence" (so Truesdell[6]).

REPLY

1. If a mathematical model of a real situation is constructed, its utility will be judged by the conformability of its consequences to what may be observed. Now if these consequences have not been rigorously deduced, a comparison between them and observations that may be made is devoid of meaning and has nothing to say to the validity of the model. The whole endeavor of theoretical analysis is thereby rendered futile and this because it is not true to itself. For though the notion of rigor exists only formally in the mind, it exists fundamentally in an instance of mathematical analysis. A sloppy analysis is therefore not 'a good enough approximation for engineering purposes'; it is a deformed creature that has repudiated its own essence. This does not mean that it is always possible for a given person, or even any person, to provide a full, complete and rigorous demonstration of all propositions. But this failing must be honestly recognized as a fault, which may be corrected later by the person himself or by someone more able than he. It is only a snare when it is overlooked. When it is derisively ignored it is not only a snare but a corruption.

2. What we have said above sufficiently answers the second objection, but it may further be pointed out that a proper mathematical analysis can evaluate the effect of the errors that there will be in the estimation of constants. It can also suggest ways in which these constants may be determined more accurately and prescribe confidence limits for them.

3. Nowhere is the need for rigorous mathematical theory better seen than in the present day use of the computer. Without an existence theorem there is no assurance that the numbers ground out by the numerical solution of an equation have any meaning. There may be some intuitive presumption of their meaningfulness but let this be honestly recognized. The particular virtues of the digital computer are its speed, "careful attention" and "indefatigable assiduity"[7]. These may be exploited, but they need also to be controlled by a rationality which is too easily sacrificed in a culture which appreciates it so superficially. It is seldom wise and never desirable to start computing before obtaining a good qualitative feel for the form of a solution; the ability to do this is one of the fruits of mathematical training.

We therefore conclude that there is a valid place for theoretical and mathematical studies in chemical engineering research, provided that their virtues and limitations are properly understood and held in balance. When unwarranted claims or unnecessary derogations are made from either direction, then the whole temper and spirit of natural philosophy is vitiated. Better soap may be made, but better living is not attained if cleanliness has become the first rather than the second virtue.

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Dr. Rutherford Aris was born in England in 1929, studied mathematics in the University of Edinburgh and taught it to engineers there. He has degrees from the University of London (B.Sc. (Math); Ph.D. (Math. and Chem. E.); D.Sc.). He worked a total of seven years in industry, but since 1958 he has been in the Chemical Engineering Department at the University of Minnesota enjoying the liveliness of its interests, both technical and cultural, and endeavouring to contribute to this vitality and communicate it to his students.

Summer School for Teachers of Chemical Engineering which was held at Michigan State University last June. For that reason, we have on hand a certain amount of material that will be published during the year. But we would also like to include in each issue one or two articles on chemical engineering education that have been submitted to us by people in the universities and in industry. Accordingly, your contributions are definitely solicited.

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