

# CANON AND METHOD

## IN THE ARTS AND SCIENCES\*

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In a famous passage of his Gifford lectures, "The Nature of the Physical World", Sir Arthur Eddington compared the mathematician's and the poet's view of waves generated on water by the wind. In the first, two expressions relate the surface forces to the constants of the waveform leading to the conclusion that a wind of less than half a mile per hour will leave the surface un-ruffled, capillary waves appear at one mile per hour and gravity waves at two. For contrast Eddington quotes the beautiful sestet of the fourth sonnet in Rupert Brooke's cycle '1914'.

There are waters blown by changing winds to laughter  
And lit by the rich skies, all day. And after,  
Frost, with a gesture, stays the waves that dance  
And wandering loveliness. He leaves a white  
Unbroken glory, a gathered radiance,  
A width, a shining peace, under the night.

The comparison is most sensitively drawn and its rapier ring makes some of the more recent exchanges in the conflict of the cultures sound like the clang of clashing cutlasses. Eddington had previously shown how farfetched is the physicist's picture of the real world—"it is not reality but the skeleton of reality"<sup>1</sup>—and he goes on to contrast 'symbolic knowledge' with its analytical techniques with the 'intimate knowledge' that defies codification. This is not the place to pursue or defend Eddington's epistemology, but the example provides a delicate statement of the problem of the relation of the sciences to the humanities.

It is hard to resist the feeling that here is a matter of deep significance to which the scientist and engineer should be increasingly sensitive. We are fortunate at Minnesota to have an exceptionally fine course in our Humanities department that makes this issue a matter of lively

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concern. This course, initiated and taught with more than ordinary verve and perception by my colleague Mischa Penn, opened my eyes to the depth and subtlety of the problem and I confess that I find it difficult and elusive to a degree — far more difficult to get to grips with than the more mundane research that I pursue in the context of chemical engineering science. It is not that the latter is a banausic enterprise, uncongenial to the atmosphere of a university, for in fact — at any rate in the department in which I have the good fortune to be a member — it has much of the spirit of natural philosophy in the sense which that term acquired in the 17th century and in which it is understood — when it is understood — today. One aspect of the difficulty can perhaps be illustrated in one of the words of my title.

Used in a mathematical context, the word 'canon', or more usually 'canonical form', must be defined precisely and all deviations rigidly excluded. Thus the Jordan canonical form of a matrix is a unique presentation of it and can be determined by a finite sequence of operations. But used in a literary context — even in one so humble as a title — the word 'canon' immediately recalls rich overtones. The original word in Greek was for a reed when used as a tool and later a tool whether made of reed or not. Most often it is the tool of the builder or carpenter, used to measure length or check level and direction. Besides being straight it had to be inflexible and was often provided with a scale. From this come the metaphorical meanings: (i) written laws or standards of ethics or behaviour; (ii) the exemplary man; (iii) the rules of philosophers and grammarians; (iv) an ordinance fixing tribute; (v) a list or index (derived from the marks on a scale); (vi) the canon of the mass (derived from the associated lists of

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saints). There are a number of quite special meanings such as the ear of a bell, a size of type and mode of musical composition and there is the normal christian usage, current since the second century,

ὁ κανὼν τῆς πίστεως,

or the 'regula fidei'. Of course in the title the word means a standard of judgment, but the point is that the literary use immediately evokes a whole spectrum of meaning in a way that the scientific does not.

This difference between the arts and sciences is however a superficial one and the bonds that unite scholars from all disciplines are far stronger and more significant than the divisive influences. Moreover it seems of vital importance that engineers should retain a lively appreciation of this, both in industry and the university. Without it, there will be no vision among the captains of industry and the people will surely perish: without it, the university will certainly degenerate into that atrocious artifact of the administrative mind, the "multiversity". I would like to suggest that a sense of craftsmanship and a feeling for form and structure are foremost among the sympathies that will keep the sciences and humanities together, however diverse their expressions of these may be. The historian and philosopher, just as often as the physicist or mathematician, must have wished, whilst listening to a symphony of Mozart's or a quartet of Beethoven's, that he could write just one paper of comparable quality, that he could present the key thesis of each section with that kind of clarity, develop it with like finesse, interweave it with the other threads of his argument as subtly and recapitulate with such power.

The motivation of the natural philosopher (be he mathematician, pure or applied, chemist, physicist, engineer or what have you) is surely the compelling desire to see the structure of his subject and the longing to carve out an understanding of some part of it that will be significant in content and beautiful in form. To this end he will use the canons of his craft — *rigour*, *elegance*, *seriousness* and *universality* — as may be illustrated by considering one of the elementary theorems of the theory of numbers.

The Greeks were well acquainted with the integers and with rational numbers, but they also had equations like  $x^2 = 2$  for the ratio of the length of the diagonal to the side of a square. What is more they had the penetration to ask the question, "Is the square root of 2 a rational number?" The proof that it is not is commonly attributed to Pythagoras and, as a simple exemplar of the canons I have mentioned, it can scarcely be improved upon. For suppose there are mutually prime integers such that  $p/q = \sqrt{2}$ , then  $p^2 = 2q^2$ . But since the factors of  $p^2$  are just those of  $p$  duplicated and 2 is a factor of  $p^2$ , it must also be a factor of  $p$ . Let  $p = 2r$ , then  $p^2 = 4r^2 = 2q^2$  and  $q^2 = 2r^2$ . But now the argument can be repeated to show that 2 is a factor of  $q$  and this is contrary to the hypothesis that  $p$  and  $q$  had no common factor. It therefore follows that there are no integers such that  $p^2 = 2q^2$ . There are pairs of integers such as 1,414,213,562 and 1,000,000,000 that will suffice for any practical purpose, but none that will satisfy the equation perfectly.

The canons of rigour, elegance, seriousness and universality are fully exemplified here. Rigour is maintained by the precise logic of the demonstration. There has been neither looseness of thought nor approximation in number. Elegance is seen in the spare economy of the proof and in the classic beauty of the 'modus tollendo tollens'. The notion of seriousness, as Hardy calls it in his "Mathematician's Apology",<sup>2</sup> is more difficult to define, but it is clearly present here in the way in which the class of object we have called numbers is enlarged. The theorem tells us that close packed though the rational numbers are, they are not the scales of leviathan and an irrational can come between them. Finally, its universality is seen in the fundamental importance of the number system, pervading much of mathematics and most of science.

Now the same canons surely apply in litterae humaniores. The rigour of the mathematician is mirrored in the formal constructions of the arts, in the logic of a philosophical argument or the build up of evidence in an exposition of history. Admittedly it is the fashion in some of the arts today to break down the form. At one time we used to be told that an artist could only

safely take to the abstract mode after he had first mastered the traditional disciplines of his craft. His breaking down of the form was then held to be an extension of it to new modality and meaning. Nowadays we are not often encouraged to seek meaning in art and the cramping effect of discipline on creativity is held to be so serious that it can be safely dispensed with. Yet a large body of art remains to show us that form does not destroy creativity — the poetry of the Divine Comedy is not diminished by Dante's acceptance of the restrictions of *terza rima*, rather it is enhanced by his mastery of it. Stephen Spender in a most interesting essay on "The Making of a Poem"<sup>3</sup> speaks of the terrifying challenge of poetry. "Can I think out the logic images?" he asks. "How easy it is to explain

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here the poem that I would have liked to write! How difficult it would be to write it. For writing it would imply living my way through the imaged experience of all these ideas, which here are mere abstractions, and such an effort of imaginative experience requires a lifetime of patience and watching."

Again, is it not the principle of economy, which is the hall mark of scientific elegance, also a keynote of humanistic thought? Ockham's razor was propounded in a philosophy dominated by metaphysics: it was adopted and adapted by the natural philosophers — "we are to admit," says Newton, "of no more causes of natural things than are both true and sufficient to explain their appearances; for nature is simple and affects not the pomp and superfluous causes."<sup>4</sup> In letters or in verse we commonly deplore excess verbiage and for a writer to be told some of his words are not bearing any weight is damaging criticism indeed.

The canon of seriousness in science has nothing to do with possible application to the useful arts any more than seriousness in the humanities has to do with solemnity. Hopkins' sonnet<sup>5</sup>

"I caught this morning morning's minion,  
kingdom of daylight's dauphin, dapple-dawn-  
drawn Falcon . . ."

is serious enough, as its superscription "To

Christ our Lord" shows, but it banishes all notion of solemnity in a burst of holy hilarity. There is plenty of verse and art that is solemn enough, but which it is more than a little difficult to take seriously.

Finally we look for some note of universality in humanistic work of real significance. We value the Aeneid, pace the quondam Professor of Poetry at Oxford, not because the adventures of Aeneas were superior to those of other wanderers, but because in recounting them Virgil has touched on so many themes of human experience with that terseness and penetration which is one of the chief glories of the Latin tongue. It is this quality of universality that made it possible for Ronald Knox to use couplets from the Aeneid to illumine an altogether different

wandering and adventure.<sup>6</sup>

But if the canons of their several arts should tend to bring together the humanist and scientist, must they not be forced apart by the diversity of their methods? Here again I would plead that there is as much, if not more, in common than there is to divide, and that a lively appreciation of each others methods would promote a valuable sympathy between scientist and humanist. The genesis of a poem or work of art, a critical essay or philosophical discourse, a mathematical discovery or an engineering invention lies in an idea or problem and the act of creation can only begin with the recognition of it. The literary critic is the engineer of the world of letters for he is concerned to bring out into the light and into action the work of the author just as the engineer seeks to apply the discovery of the scientist. This does not mean that there is not a creative, or recreative, element in good engineering or in good criticism, but criticism is, in a sense, a derivative activity. The "Diary of Anne Franck" lies in paperback alongside a dozen gripping and even perceptive books of the second world war and many have been moved by the reading of it. Yet if John Berryman is correct, no one has really perceived the masterpiece that it is, nor got down to the critical problems that a worthy analysis of it would present. Here is

the recognition of a problem at the root of the work of a humanist. It is comparable to the recognition of an idea at the root of a work of art. Among humanists, the poet is par excellence the opener of eyes, showing us the significance of some matter. In the realm of the sciences the mathematician is par excellence the refiner of concepts, turning and shaping them until they are precisely true to experience. Each, in his way, sits like a diamond cutter over a stone, seeking the cleavage plane of truth along which the slightest blow will open up the rough gem and reveal the perfection of its intrinsic beauty. Each however has the problem of recognizing the true worth of the matter beneath its rough, amorphous exterior. This first phase of recognition may include the inspiration of the moment in which the artist conceives the idea that he wishes to bring to birth according to his metier, but may be distinguished from the moment of illumination, in which the resolution of a difficulty may appear, or the moment of vision in which the toilsome ascent of a Pisgah is suddenly rewarded.

But, granted the recognition of the problem or idea, there follows for both scientist and humanist the gestative period of cogitation. Ideas and images, many of them unfruitful and inappropriate, are mulled over and mixed together, taken to pieces and reassembled. Stephen Spender speaks of concentration as the sine qua non of creative writing. He distinguishes it from "the kind of concentration required for working out a sum. It is the focusing of the attention in a special way, so that the poet is aware of all the implications and possible developments of his idea, just as one might say that a plant was not concentrating on developing mechanically in one direction, but in many, towards the warmth and light with its leaves, and towards the water with its roots, all at the same time".<sup>3</sup> Perhaps this is different in kind from the concentration required for "working out a sum" by a routine method, but it is precisely the sort of concentration that is required for fruitful original work in the sciences.

Some, it would seem, are gifted with the ability to work out a complete structure in their heads, as Mozart is said to have composed much of his music. Others like Beethoven have to feel their way through draft after draft towards a final statement. From the mine of his memory or the recesses of the subconscious where the

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composition had been going on, Mozart was able to set down the overture to Don Giovanni in a single night but the main theme of the first movement of the Beethoven's 7th Symphony emerged only after six pages of "changing, reflecting, and testing", as he himself described it.

Into the art of heuristic in a mathematical context George Polya has given most valuable insights by his work on mathematical discovery.<sup>7</sup> He shows very vividly how the problem may be tackled, how one works from both ends in searching out the pattern of the solution and how induction and analogy play their role in plausible reasoning. In plausible reasoning the formal modes of demonstrative logic become tentative. For example, the modus tollendo tollens must be replaced by "A implies B, but B is unlikely, therefore A is less credible." This is the kind of reasoning which is used, not only in feeling out the way to the solution of a problem, but also in understanding a demonstrative argument and in gaining confidence in it. Indeed Polya concludes the second of the Princeton volumes with the remark that "we are led to suspect that a good part of our reliance on demonstrative reasoning may come from plausible reasoning."

This emphasis on the process of creation is not to deny the importance of inspiration and the flash of illumination. The classic examples the Poincaré gives in his "Science et Méthode" are so well known that they need not be repeated here. They show, as he himself said, that sudden illumination is a manifest sign of previous subconscious work perhaps over a long period.<sup>8</sup> There must surely be an analogy here with the resolution of "Problems" as they may arise in humanistic scholarship and creative art. At times the several stages of the creative process seem to have been fused into one incandescent period of intense activity. One thinks of Handel completing the "Messiah" in little over three weeks between August 22 and September 12 of 1741, of Schubert writing no less than eight songs on October 15, 1815 or of his sending his song "The Trout" to Josef Huettnerbrenner, calling it "another one which I have just written here at Anselm Huettnerbrenner's at twelve o'clock midnight". These are the exceptions that

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prove the rule that the beauty of creative work in the sciences or the arts is more the shine of "plough down sillion" than "the hurl and gliding" that rebuffs "the big wind".<sup>5</sup> Often too the moment of luminence in literature or philosophy cannot come without the laboured argument or prior discipline. "He who has been instructed thus far in the science of Love, and has been led to see beautiful things in their due order and rank", says Diotima, "*When he comes toward the end of his discipline, will suddenly catch sight of a wondrous thing, beautiful with the absolute Beauty*".<sup>9</sup> The main body of the 15th chapter of St. Paul's first letter to the Corinthians is a lengthy discussion of the reality of the resurrection. But then comes a pause — a reticence of holy writ, as St. Peter Damian has it, "wherein silence itself cries out that some greatness is at hand" — before the incomparable majesty of "Behold I show you a mystery; we shall not all sleep, but we shall all be changed; . . ." I am not for a moment suggesting that this is mere rhetoric — it is vastly more — but, if it has the divine qualities of revelation, it has also the human beauties of a great work of art.

The final stage of polishing or verification is of equal importance though perhaps calmer than the others. The imaginative leap having been made, logic takes over to tighten up each part and to ensure that the connections are sound. The kind of imagination needed here is that which is capable of keeping the whole structure — poem, paper or prelude — in its proper portion and scale. As any editor of a scientific or technical journal will testify this is an aspect of the presentation of research that receives all too little attention, and perhaps scholarly journals in other fields suffer in the same way.

But surely in this craft of our common language should lie the first and final bond between scholars of all disciplines, for all have the same interest in maintaining a sound currency of words. Perhaps the breakdown of commerce between the arts and sciences, whenever it obtains, is a reflection of the inflation of the domestic currency within each camp. The great words of the tradition of western civilization — liberal, intellectual, rational, humane — are in danger of

becoming a paper currency with no backing, deprived of their buying power as effectively by academic verbicides as some of the words of our common life — trust, friends, gracious — have been abused by the writers of newspaper headlines and advertising copy. There was a time when Latin was the lingua franca of the educated world but, serviceable enough though it still would be, there is little hope of reinstating it. We may have to learn to read two or three other languages in order to keep up with the literature of our professions, but we rarely attempt to write in anything but our native tongue. All the more reason therefore that we should cultivate this to the best of our ability, perhaps to find through this medium, not a massage, but the common empathy that is needed if our universities are to remain centres of liberal learning.

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8. See J. Hadamard's "Psychology of invention in the mathematical field" for a considerable discussion of this. Graham Wallas in "The Art of Thought" (1926) comments on the experience of Poincare and the physicist Helmholtz.
9. Plato. Symposium 211. Translation is Bridges' in his anthology "The Spirit of Man." (1915).

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