A Course in Momentum Transport

FLUID DYNAMICS

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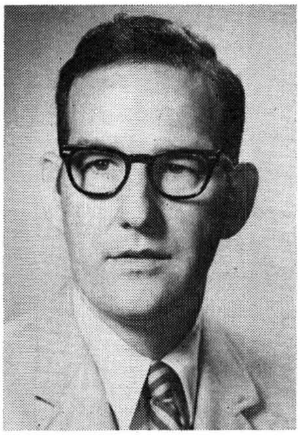
Fluid dynamics plays a central role in many problems of interest to chemical engineers. Because of this, the semester course in the area presented by the ChE Division of the University of Illinois has been one of the most durable offerings in its graduate curriculum. I have taught this course since 1953 and one similar to it had existed many years prior to my involvement.

My goal is to present a unified treatment of fluid dynamics to students who have had courses in differential equations and transport phenomena and who have some knowledge of vector notation. The content reflects the philosophy within the Division that the more advanced and more current aspects of any field are covered in our Special Topics courses and in our seminars. (For example, in recent years I have conducted seminars on turbulence, hydro-dynamic stability, continuum mechanics, water waves, numerical solutions of the equations of fluid mechanics, rheology and in modern aspects of boundary layer theory.) As a result, the fluid dynamics course is largely based on material available in a number of textbooks. It is intended that it be a starting point for advanced studies of the current literature, which are best done in an informal fashion.

At present, the course is in transition because of the introduction by most schools of more advanced topics in fluid dynamics at the undergraduate level. One of my chief difficulties is to assess properly the background of the students since I find that the courses offered at different universities in transport phenomena vary considerably in content and depth. Therefore, at the risk of losing the interest of the better prepared students, I give a rapid treatment of key physical notions that should be covered in a basic course in transport phenomena. I also give the students a chance to do some reviewing of their own by assigning at the beginning of the course a number of problems from the book "Transport Phenomena" by Bird, Stewart, and Lightfoot.



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I have been experimenting with the contents of the course and, as a result, the outline accompanying the article is meant to represent the types of topics treated and not the total material covered in a semester. For example, this past year I did nothing with topic 10 and only partially covered the notes I have prepared under topic 9. One of the ideas that is not sufficiently developed is that of hydrodynamic stability. I am currently giving some thought to ways of working it into the course.

BASIC EQUATIONS

The course is initiated by reviewing the concept of a fluid particle and of a continuum and, in particular, by pointing out circumstances, such as the settling of fine particles, where the continuum model is invalid. It is then indicated that an Eulerian framework is usually more convenient than a Lagrangian framework for solving fluid dynamical problems. The second law of motion and the first law of thermodynamics are reformulated so that they are applicable to an arbitrary fixed volume in space rather than to a fixed mass. Difficulties that sometime can be encountered in applying thermodynamics, which is formulated for an equilibrium system, to flow fields are pointed out.

UNIDIRECTIONAL VISCOUS FLOWS

The concepts of a shear stress and the sign convention to be associated with it are introduced by considering fully developed flow in a pipe. Newton's law of viscosity is initially presented by assuming that the shear stress is directly proportional to the velocity gradient. Through the momentum theorem it is pointed out that the shear stress may also be interpreted as a flux of momentum. Through this concept kinetic theory can be used to interpret fluid viscosity. Flow between circular cylinders is considered because it is a convenient way to introduce concepts which are used later to extend Newton's law of viscosity to three-dimensions. It is pointed out that viscous effects will not depend on that portion of the velocity gradient that gives rise to solid body rotation but will depend on that portion which distorts the shape of fluid area elements.

NON-NEWTONIAN FLUIDS

Experiments are described which illustrate non-Newtonian behavior and which yield definite properties characterizing the rheological behavior of fluids. These include nonlinear dependence on the rate of strain of the fluid, normal stress effects caused by shear flow, and elastic effects. The Rabinowitsch equation is developed either in class or in a homework problem because of its importance in interpreting nonlinear effects in steady flows. Methods for calculating normal stress coefficients and the use of small amplitude sinusoidal oscillations and relaxation tests to determine elastic properties are discussed. The Maxwell equation for linear viscoelastic fluids is applied to a few simple flow problems.

EQUATIONS OF MOTION

The differential equations describing the three-dimensional flow of a Newtonian fluid are now developed in a cartesian coordinate system. This is not done in general curvilinear coordinates because the small number of new physical concepts introduced by this generalization does not seem to warrant the added complexity. The equation of conservation of mass and the momentum theorem are applied to a differential volume. The use of cartesian tensors to simplify the notation is introduced. The physical interpretation of the substantial derivative arising from the momentum terms is presented. It is pointed out that in order to describe the force acting on the surface of a volume the location as well as the orientation of each surface element is needed. The problem of representing these surface forces is greatly simplifid by showing that the stress on any arbitrarily oriented surface element can be described in terms of the nine stress components needed to specify the stress vectors acting on three mutually perpendicular planes. Cauchy's equation of motion can then be derived. The properties of the stress components are now explored in order to facilitate the development of constitutive relations. It is shown that the nine stress components are a second order symmetrical cartesian tensor. The concepts of principle axes and principle stresses are introduced. The invariants of the stress tensor are defined.

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CONSTITUTIVE RELATIONS

The constitutive equations relating the stress components to the velocity field are now developed for a Newtonian fluid. The problem of doing this for more general fluids is discussed briefly. The velocity gradient is shown to be a second order cartesian tensor which can be represented as the sum of a symmetric and an antisymmetric tensor. The stress components can be related only to the symmetric part of the stress tensor (rate of strain tensor) since this is the part that gives rise to the distortion of fluid volume elements. Newton's law of viscosity is generalized to three dimensions by assuming that the components of the stress tensor are linearly related to the components of the symmetric part of the velocity gradient tensor. This produces eighty-one coefficients of viscosity. Only two of these coefficients are independent, because the fluid is isotropic and because the stress and rate of strain tensors are symmetric. The assumption of a linear relation between the stress tensor and the rate of strain tensor is now relaxed and the most general relation is developed for an isotropic fluid for which the components of the stress tensor are only functions of the components of the rate of strain tensor. This relation predicts normal stress effects and non-Newtonian behavior for steady flow in a channel. The work of Oldroyd, Rivlin and Erickson, and Colman and Noll aimed at developing constitutive relations which exhibit elastic effects as well as nonlinearity and normal stresses is discussed only qualitatively.

The Navier-Stokes equations are now derived by introducing the constitutive equations for a Newtonian fluid into the Cauchy equation. The solutions of these equations for the case of an incompressible fluid are developed for three very general assumptions: creeping flow, perfect fluids, and boundary layers.

CREEPING FLOWS

According to the creeping flow approximation the inertia terms of the equations of motion can be neglected at small Reynolds numbers. Creeping flow around a sphere is worked out in much detail since it illustrates the use of a stream function, the method of eliminating the pressure term from the equations of motion, and the calculation of body forces and skin friction on a solid body. The case of flow around a bubble is worked out in a homework assignment. It is shown that the creeping flow approximation does a good job in predicting the force on a sphere but a poor job in predicting the flow field at distances far from the sphere. For flow around a cylinder it doesn't even predict the force on the cylinder. Regular perturbation methods are shown to yield poor higher order approximations to the creeping flow solutions. The method of Oseen is discussed. The singular perturbation method, as outlined by Proudman and Pearson, is then shown to be a proper way of getting higher order approximations.

PERFECT FLUIDS

A perfect fluid is defined as one for which the viscosity and thermal conductivity are zero and for which the entropy of fluid particles is constant. The Euler equations then describe the flow field. The conditions under which one might expect an irrotational flow field are discussed. The velocity vector is then describable as the gradient of a potential function and the velocity field is given by the equation of conservation of mass. The integration of the Euler equation using the assumption of irrotational flow yields the Bernoulli equation. The Bernoulli equation can also be obtained for rotational flows by applying the momentum theorem to flow of a perfect fluid along a stream tube. The constant of integration then varies from stream tube to stream tube. The solution for the flow of an irrotational perfect fluid around a sphere is obtained. The predicted pressure distribution around the surface is discussed. The concept of virtual mass is introduced by considering the unsteady motion of a sphere in a perfect fluid and is applied to some problems of interest. Two-dimensional flows of irrotational perfect fluids are then considered. A stream function can be defined for a two-dimensional flow from the equation of conservation of mass and complex variable theory can be used to solve flow problems. A number of examples are considered including the lift of solid bodies and free streamlines. The treatment of small amplitude waves at an interface is one of the more successful applications of the theory for irrotational perfect fluids. Some of the problems considered in this area are progressive two-dimensional waves, standing waves, group velocity, wave resistance, Kelvin-Helmholtz instability, and Taylor instability.

BOUNDARY LAYERS

Boundary layer theory attempts to correct perfect fluid theory for viscous effects by assuming the existence of a viscous layer close to a solid surface. If this viscous layer is thin compared to the dimensions of the body a simplified version of the Navier-Stokes equation applicable to boundary layers on flat plates and curved surfaces is obtained. The concept of separation is discussed and it is pointed out that boundary layer theory is only applicable up to the point where the boundary layer separates from the solid surface. The difficulties of applying the theory are discussed, and, in particular, the problems associated with determining the external inviscid flow or the pressure distribution around the body are emphasized. Dimensional analysis is applied to the boundary layer equations to determine the functional relation between the skin friction coefficient and the Reynolds number. Similarity solutions are briefly discussed. The series methods of Blasius and of Görtler and the integral methods of Pohlhausen and of Bohlen and Walz for solving the boundary layer equations for some general pressure distribution are introduced. I find that the best way to present these methods is to give a homework problem which requires the application of boundary layer theory to a solid body for which the pressure distribution is known.

TURBULENCE

This treatment of boundary layers concludes my discussion of laminar flows. It might seem anomolous that even though most flows in nature are turbulent I don't introduce the topic of turbulence until this point in the course. The reason for this is simply that a theory for turbulent flows is still to be developed. Therefore my goal in treating turbulent flows is to introduce some of the language used in correlating turbulence measurements and in characterizing the turbulent field. The increased apparent shear stress in turbulent flows is explained in terms of the momentum transport caused by the fluctuating velocity field. The concepts of eddy viscosity and mixing length are introduced. They are found not to be as useful for turbulent flows as were molecular viscosity and mean free path for laminar flows because the scale of the turbulent motion responsible for transport is of the same order as the dimensions of the field. A general discussion is given on the character of fully developed velocity profiles and, in particular, on the roles of fluid viscosity and of the viscous sublayer. The variation of the average velocity and the eddy viscosity with location is correlated through dimensional analysis and the "law of the wall", the "velocity defect law", and the "overlap law". It is then shown how the definition of eddy conductivities for heat transfer are useful in explaining measurements and in particular the effect of Prandtl number on temperature profiles. The interpretation of fully developed velocity profiles and Cole's "law of the wake" are used to develop predictive methods for general turbulent boundary layer flows. Taylor's treatment of point source diffusion in homogeneous turbulent fields is presented as one of the few successful theories in turbulence. It is used to explain the gross aspects of turbulent mixing and to interpret the observed variation of eddy conductivities. Statistical methods for describing turbulent flows are discussed and in particular the concepts of correlation, scale, and spectrum are introduced. A very brief summary is given of theoretical attempts to deal with turbulence through the use of the concept of isotropy and the definition of a turbulence structure.

COMPRESSIBLE FLOWS

The last topic in the outline is a one-dimensional treatment of compressible flows. It is usually presented after the material on perfect fluid theory but appears here in my outline because in recent offerings of the course I have deleted it. Most of this material with the exception of that on finite amplitude waves and shock tubes are more properly treated in undergraduate courses.

OTHER APPROACHES?

I should conclude this article by saying that the course that I have discussed is only one way, and not necessarily the best way, of introducing graduate students in chemical engineering to basic concepts in fluid dynamics. My own introduction to and interest in fluid dynamics developed from a course in reactor design.

FLUID MECHANICS COURSE OUTLINE

1. Basic Equations
2. Unidirectional Viscous Flows
3. Non-Newtonian Fluids
4. Equations of Motion for a Viscous Fluid
5. Constitutive Relations
6. Creeping Flow Approximation
7. Perfect Fluid Theory
8. Boundary Layer Theory
9. Turbulence
10. One-Dimensional Compressible Flows

ChE news

The following item on CACHE was submitted by Professor Warren D. Seider, University of Pennsylvania, Philadelphia, Pennsylvania 19104.

The CACHE (Computer Aids for Chemical Engineering Education) Committee has been organized to coordinate the development of computing systems for use in chemical engineering education. The committee includes twenty educators from sixteen universities. The principal goal of the CACHE Committee is to accelerate the integration of computing into the chemical engineering curriculum by sustained inter-university cooperation in the preparation of curriculum and course outlines and in the specification and creation of computing systems.

The CACHE Committee's curriculum subcommittee has organized a session for the AICHE Annual Meeting in Washington, D.C., entitled "Computers in Chemical Engineering Education." The session will emphasize topics relating to short and long range plans for the integration of computers into the curriculum. Ten members of the CACHE Committee will participate in the panel discussion after brief presentations.