TEACHING OPTIMIZATION: THE BEST OF ALL POSSIBLE APPROACHES*

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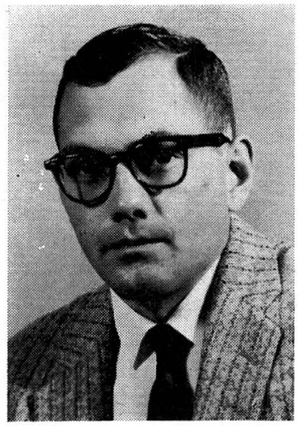
A distant cousin to modern day optimization was the philosophy of optimism articulated about 1700 by Leibniz, the German pholisopher and mathematician. From his experience and insights, Leibniz deduced that this is the "best of all possible worlds." With such a historical precedent the author, with tongue-in-cheek, is cavalier enough to propose the "best of all possible approaches" to teaching optimization. However, before discussing the teaching of optimization, it is appropriate to consider the question, "Why teach optimization at all?"

WHY TEACH OPTIMIZATION ?

Engineers, and most everyone for that matter, are concerned about doing the "best" they can. Optimization concepts and formal optimization procedures provide a framework and a means by which one can be more systematic about doing the best possible job. Using optimization methods, the engineer may be able to design a more profitable process or to reduce the costs of operating an existing system. Aside from these obvious benefits, there are other significant advantages that can accrue from teaching optimization. First, the fact that a criterion of goodness must be established in order to carry out an optimization, forces identification of a single goal. This goal is usually economic in nature and thus can serve to develop the student's ability to identify and concentrate upon economically significant portions of the system. Second, because the process or system to be optimized must be described quantitatively, the student must draw upon his previously acquired engineering know-how. As part of the problem formulating activity, the economic trade-offs and optimizable variables must be determined. The net result is a sharpening of the student's technological and economic insights. Third, upon completion of an optimized design, the student can more fully appreciate the rules of thumb that are used for quick estimates. For example, after optimizing an extraction system design, the rule of thumb that the optimum solvent to feed ratio is usually in the range of 0.5-1.0 will ring true to the student. Finally, there are certain useful generalities that the student encounters in studying optimization. Such adages as "in engineering economics you don't get something for nothing," "look before you leap," and "it is often better to plan as you go rather than make complete plans ahead of time," are vivified in the study and application of optimization concepts.



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PERSPECTIVE AND BACKGROUND

The focus of this paper is upon teaching optimization at the introductory level. The twofold objective of such teaching is to lay the

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foundations for more advanced studies in optimization theory as well as to equip the students with some tools and ideas that will be directly applicable in their engineering careers.

Because the generalizations and suggestions presented in this paper are deeply rooted in the author's experience in teaching optimization to chemical engineers in a continuing education program,* this experience will be described briefly. The extrapolation of this experience plus that gained in teaching optimization material to both graduate students and undergraduates in chemical engineering at the University of Idaho provide the background for the teaching guidelines and subject matter coverage proposed by this paper.

EXPERIENCE WITH TIME-SHARING

The idea of using time-shared computer terminals as a tool to teach optimization occurred to the author during his Ford Foundation Residency* and began to take shape at the University of Michigan Computer Workshop** in April 1967. An optimization course actually using time-shared computer terminals was presented to thirty-six‡ of Union Carbide's chemical engineers in May 1967. Three computer terminals, tied to the GE-265 system, were installed in the classroom. The course content is shown in Table I. During the eight, 3½-hour classes, approximately two-thirds of the time was spent in work sessions. In these work sessions, teams of three used the remote terminals while the others in the class worked on problems which were suited to hand calculations. Typically an optimization method was first introduced in a lecture session, then some simple hand calculations provided familiarization with the basic concepts. Thirdly, the participants would solve a "realistic" chemical engineering problem using the remote terminal to do the calculations of the objective function but doing the optimization logic off-line. This allowed the use of interesting, relevant problems but avoided tedious hand calculations. Finally, the participants set-up and used a fully automated version of the particular optimization method programmed for the time-sharing system. This general approach was adapted as necessary to the different optimization methods covered. As a secondary benefit to this course, the participants were thoroughly familiarized with the time-sharing computer system without the usual fanfare attendant to courses in computer programming and usage.

*Union Carbide Corporation, South Charleston, West Virginia, June 1967-September 1968.

**National Science Foundation Workshop on Computers in Engineering Design Education.

‡Two sections of eighteen each.

TABLE I — CONTENTS OF PILOT COURSE TAUGHT AT UNION CARBIDE CORPORATION

1. Introduction to Optimization Concepts and Attitudes
2. One-Dimensional Optimization; Golden Section Search
3. Multi-dimensional Search Techniques (including Lagrange multiplier and penalty function approaches to constrained problems).
   A. Nongradient Methods—("Pattern" Search)
   B. Gradient Methods—(Steep Ascent)
   C. Stochastic Optimization—(Simplex EVOP)
4. Introduction to Linear Programming
5. Exploiting System Structure—Introduction to Dynamic Programming

TEACHING GUIDELINES

From the above described experience and that gained at the University of Idaho without a time-sharing system, the author proposes the following general guidelines for teaching introductory material in optimization. Suggested subject matter coverage follows in the next section.

I. **Optimization should be taught in the context of applications.** For the engineer, optimization is somewhat like economics and computers in that its utility lies in applying it to real problems. Thus a substantial number of the problems used in teaching optimization need to be of engineering significance so that the student sees optimization in its proper context. Use of pithy problems also provides the student with a situation in which he learns about the technical and economic aspects of engineering as well as learning about optimization. Some typical problems for the chemical engineering student include;

1. Optimization of chemical processes or systems. The Denbigh reaction problem[2] and extraction system described by Treybal[9] are examples that can be adapted for use in teaching optimization.
2. Free energy minimization in complex chemical equilibrium problems[11].
3. Nonlinear parameter estimation by minimizing sums of squares. For example, estimation of kinetic parameters from experimental data[6]
4. Root finding for nonlinear equations and

solution of implicit functions. Solving for specific volume from virial-type equations of state can be attacked as an optimization problem. In fact, any trial and error calculation can be cast as an optimization problem.

II. **Optimization material should be integrated into the curriculum through courses rather than being taught as a separate course.** This method of presentation emphasizes that optimization is part of a bigger picture and not an end in itself. Most all engineering curricula have an introductory course in the freshman or sophomore year in which a one-dimensional and a simple multidimensional direct search optimization method could be introduced. The students could actually write computer programs for these methods. Whether they write their own or become familiar with an existing program, the optimization programs should be readily available to them for use in other courses.

The senior year process analysis and process design courses are where the other optimization concepts and methods are introduced. Optimization and engineering economics should be developed in parallel and applied to engineering examples. In chemical engineering, the recent textbook by Rudd and Watson[10] is particularly good in this regard. Finally, when the students are ready to attack a substantial design problem, they have some optimization know-how at their disposal.

III. **Emphasis should be placed on "learning by doing" through a lecture-laboratory sequence.** At the introductory level, students seem to grasp the concepts and methodology of optimization more quickly by actually trying to put them to use than by listening to formal lectures. It is in the laboratory situation where the initial insights and understanding develop most quickly. Some formal presentation of material is necessary but should be held to a minimum.

At this point, time-shared computing comes on the scene. The combination of significant engineering problems in a "learn by doing" or laboratory context is made possible by remote computer terminals used in a conversational mode. The computer does the calculations that would be too lengthy to be done by hand or simulates experimental or process equipment including random error. This frees the student to concentrate on optimization logic and the engineering significance of the results. The laboratory actually involves three-way interaction among the instructor, students and computer. The instructor must be available for questions and discussion as well as to provide general guidance.

Figure 1 is a block diagram depicting a general sequence for presenting optimization methods. All the steps in this sequence need not receive equal emphasis. In fact, steps should be omited entirely in some situations. For example, in introducing linear programming it is not desirable to do the optimization logic off-line but rather to use an existing program after the student appreciates the essentials of the underlying theory.

At present the primary disadvantage to the extensive use of a time-sharing system as described above is cost. Although it is sound pedagogy to have students conversing with the computer, it is also expensive. Fortunately time-sharing costs are on the downtrend and the future looks promising in this regard[3].

A look to the future also sees the scheme shown in Figure 1 being used in a programmed learning environment. The lecture material could be on video tape and the time-sharing system itself could conduct the laboratory sessions.

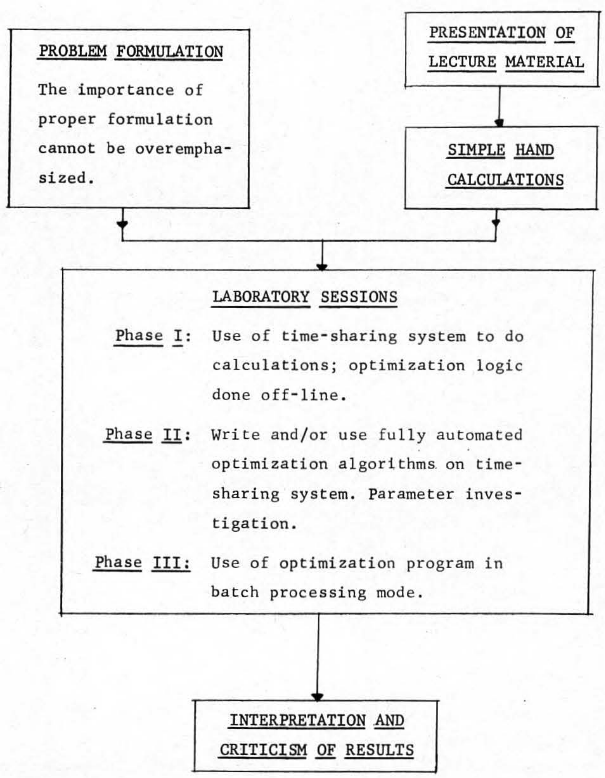

Figure 1. General Lecture-Laboratory Sequence for Teaching Optimization.

SUBJECT MATTER COVERAGE

Although the optimization literature is voluminous and generally sophisticated, there are many optimization techniques that are both easy to understand and of practical importance. The specific topics listed below are of this type. They also include a broad spectrum of concepts. These topics are effectively presented using the lecture-laboratory sequence shown in Figure 1 with modifications and adaptions as appropriate.

I. Direct Search Optimization Methods.
   A. One-Dimensional: Golden Section Search[13].
      Simple but very efficient. Serves as a good starting point for introducing students to optimization concepts.
   B. Multi-Dimensional: Pattern Search[5].
      Alarmingly simple but very useful. Exposure to the concepts of multi-dimensional geometry broadens the student's horizon.
   C. Constraints: Penalty Functions[1].
      Simple and convey to the student the idea of constrained optimization. Used in combination with Pattern Search. Penalty functions have something of an *ad hoc* nature and can handle only a limited number of constraints.
   D. Stochastic Optimization: Simplex EVOP[7].
      The Spendley, Hext, Himsworth method is easy to use and very powerful.
      The time-shared terminal can simulate an actual piece of equipment or process, including random error, by superimposing normally distributed, random values[8] on the computed values for the criterion function. Since the student does not know the underlying computational routine, the results obtained from the remote terminal for a given set of independent variables are in fact experimental data. This type of experimental data can be obtained quickly and easily. Therefore students can use the remote terminal as if it were on actual process and search for the optimum operating conditions using the EVOP technique.
II. Indirect Optimization Methods.
   A. Linear Programming[4].
      Without a doubt, linear programming is the most widely used optimization technique. Linear programming on the time-shared terminals allows the student to experiment with shadow prices and sensitivity analysis.
   B. Geometric Programming[14].
      A novel approach to optimization and should be included to broaden the student's base. Problems with one or two degrees of difficulty can be solved using Golden Section Search or Pattern Search.
III. Optimization Methods that Exploit System Structrue: Dynamic Programming[12].
   Elementary dynamic programming concepts are easily grasped and form the basis for future learning in this area. For simple problems the time-shared terminal can be used to perform individual stage optimizations while the student applies the "Principle of Optimality" off-line.

SUMMARY

The key point in this paper is that availability of remote time-shared computer terminals opens a new dimension in teaching optimization at the introductory level. This new dimension is a laboratory in which the student can attempt to apply optimization techniques to stimulating engineering problems without long turn-around times or burdensome hand calculations.

The paper proposes that optimization material be integrated into the engineering curriculum rather than taught as a separate course. This emphasizes optimization as an engineering tool rather than an end in itself. The subject matter coverage suggested by the author combines simplicity, practicality, and breadth.

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