ChE laboratory

A Demonstration Experiment In NON-NEWTONIAN FLOW

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INTRODUCTION

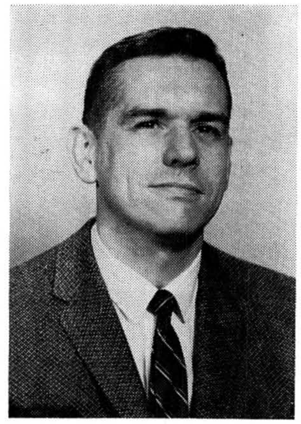
ALTHOUGH NON-NEWTONIAN FLOW is typical for all solutions and melts of high polymers at high shear rates, it is not an easy phenomenon to demonstrate to a large number of people at once. Other aspects of polymer solution behavior can be ilustrated by using extreme examples. Melt elasticity is shown by "Silly Putty" which is readily available, and, in fact, already familiar to most students. Flow birefringence and creep recovery also can be shown,[1]. The elasticity of dilute solutions of poly(ethylene oxide) giving rise to an "uphill" flow of a liquid has been packaged as a demonstration experiment (Edmund Scientific Co.). Drag reduction in turbulent flow also is amenable to demonstration. However the non-linear dependence of stress on rate-of-shear (or a viscosity which decreases on increasing stress or rate of shear) usually involves using a rotational apparatus with transducers connected to gauges or recorders so that the results can be seen by a large group at once.

There is one inexpensive type of capillary flow instrument which presents a range of stresses in a single experiment. The variable-head viscometer[2-4] consists essentially of a cylindrical reservoir connected to a capillary tube. When the liquid flows, the reservoir empties in such a way that the logarithm of the height in the reservoir (above the discharge enn of the tube) decays linearly with time for a Newtonian liquid. Although this can be used as the basis for a lecture demonstration, there are inherent advantages to using a triangular reservoir connected to a tube as in Fig. 1.

DEMONSTRATION

TO RUN THE experiment one fills the reservoir first with a Newtonian fluid selected to give a flow time of several minutes. Once the clamp is opened, members of the class can call out a signal at equal intervals of time, say 20 seconds. One of the students can mark the liquid level on the front of the reservoir with a crayon on each signal. If the reservoir is about 100 cm high, the experiment is clearly visible to a class of over 100. Next, while the reservoir is being refilled with a second (non-Newtonian) fluid, the points from the first run can be plotted on a paper graph next to the reservoir (as in Fig. 1). It is found that the points lie on a straight line.

The same routine is followed for the second fluid. It should be found that this time the reservoir does not empty linearly with time. With proper selection, a non-Newtonian fluid can be made to **cross over** the line for the Newtonian fluid.



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EXPLANATION

FOR A FLUID flowing through a capillary tube it can be shown[3] that the shear stress and rate of shear (both referred to the wall) are:

Shear stress,

$$\tau_w = \frac{\rho h g_c D}{4L} \qquad (1)$$

Rate of shear,

$$\dot{\gamma}_w = \left(\frac{1+3n}{4n}\right)\left(\frac{32Q}{\pi D^3}\right) \qquad (2)$$

where ρ is the fluid density, gm/cm³; h is height of fiuid in reservoir above the efflux point of the tube, cm; g_c is 981 dynes/gm; D is tube diameter, cm; L is tube length, cm; Q is rate of flow through tube, cm³/sec; and n is $\ln\tau_w/d\ln\gamma w$ (n = 1 for a Newtonian fluid).

Also, the viscosity (poise) is defined as

$$\eta = \tau_w/\dot{\gamma}_w \qquad (3)$$

In the present apparatus, Q can be related to the change of h with time by

$$Q = -A\ dh/dt \qquad (4)$$

where A is the cross-sectional area of the reservoir in cm² at a height of h. The reservoir is positioned so that the apex of the triangle is at the same height as the effluent point of the horizontal tube. This,

$$A = \left(\frac{BW}{H}\right) h \qquad (5)$$

where B, W, and H are dimensions (in cm) of the reservoir in Fig. 1. Therefore, we have

$$\tau_w = \eta\dot{\gamma}_w \qquad (6)$$

$$\frac{\rho h g D}{4L} = \eta\left(\frac{1 + 3n}{4n}\right)\left(\frac{32}{\pi D^3}\right)\left[-\left(\frac{BW}{H}\right) h \frac{dh}{dt}\right] \quad (7)$$

For a Newtonian liquid all the terms are constant except h, which cancels out, and n=1, which is the advantage of the triangular reservoir. Now we have:

$$-\frac{dh}{dt} = \left(\frac{\pi g}{128}\right)\left(\frac{\rho}{\eta}\right)\left(\frac{D^4}{L}\right)\left(\frac{H}{BW}\right) \quad (8)$$

The last three terms on the right are separate functions of the fluid, the tube, and the reservoir. Of course, when dh/dt is constant, h decays linearly with time.

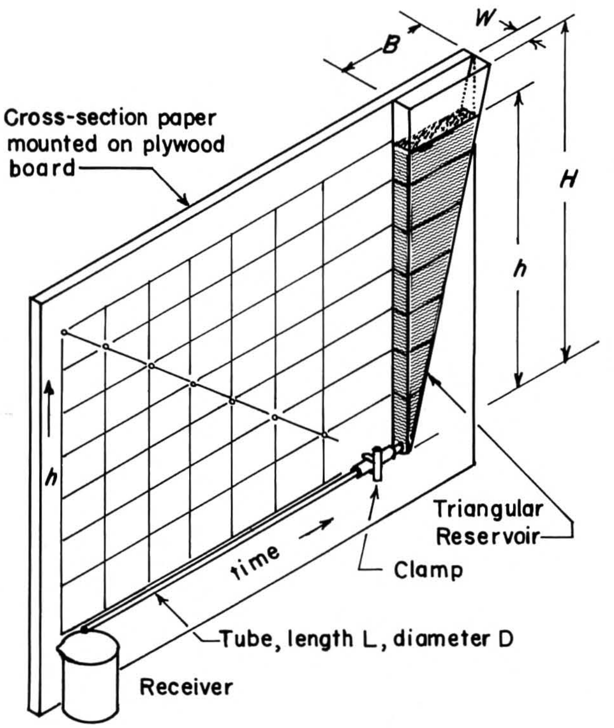

Figure 1. Variable-head viscometer with triangular reservoir.

The analysis of the non-linear plot for the non-Newtonian fluid can be carried out several ways. The easiest is to define an "apparent kinematic viscosity" as

$$(\eta/\rho)_a = \left(\frac{1+3n}{4n}\right)\left(\frac{\tau_w}{\dot{\gamma}_w}\right) \quad (9)$$

Then

$$\left(\frac{\eta}{\rho}\right)_a = \left(-\frac{dt}{dh}\right)\left(\frac{\pi g}{128}\right)\left(\frac{D^4}{L}\right)\left(\frac{H}{BW}\right) \quad (10)$$

The apparent viscosity is inversely proportional to the slope of the h, t plot, Most polymer solutions are "pseudoplastic", that is, the viscosity decreases with increasing shear stress (which is readily calculated from equation 1).

MATERIALS AND APPARATUS DIMENSIONS

THE SUCCESS OF a lecture demonstration is always affected directly by the choice of materials and conditions. A reasonable model made by joining a glass tube to an acrylic reservoir by rubber tubing has the dimensions: B = 20 cm; W = 1 cm; H = 100 cm; L = 21.5 cm; D = 0.40 cm.

From equation 8 we learn that

$$-(dh/dt) = \left(\frac{\pi 981}{128}\right)\left[\frac{(0.40)^4}{21.5}\right]\left(\frac{100}{20}\right)\left(\frac{\rho}{\eta}\right) = 0.143\ (\rho/\eta) \quad (11)$$

If a reasonable flow time for h going from 90 down to 20 cm is two minutes, then

$$(\eta/\rho) = 0.143\ (120/70) = 0.245 \text{ stoke}$$

or about 25 times the viscosity of water. The shear stress is given in dyne/cm^2 by equation 1 when ρ is in gm/cm^3 and h is in cm:

$$\tau_w = \left(\frac{981}{4}\right)\left(\frac{0.4}{21.5}\right)\rho h = 4.56\ \rho h \quad (12)$$

Figure 2 shows a non-Newtonian polyacrylamide solution (0.94 wt.%) crossing over the straight line for a poly(vinyl alcohol) (4.0 wt.%) solution. The viscosity of the Newtonian solution is 0.56 stoke since -dh/dt = 0.254 cm/sec. When the slope of the other curve is plotted against h (Figure 3) a straight line results. This means that the fluid can be represented by the "power-law" model in this range of stresses.

$$\tau_w = K\dot{\gamma}_w^n \quad (13)$$

From equations 10 and 11 we can derive

$$(\eta/\rho)_a = 0.143\ (-dh/dt) \quad (14)$$

This, with equations 12 and 13 (and with ρ = 1.0 gm/cm^3) can be rearranged to give

$$(-dh/dt) = 0.01\pi\ (4.56/K)^{1/n}(4n/3n+1)h^{(1-n)/n} \quad (15)$$

From the slope in Figure 3, n = 0.59, and from the intercept, K = 4.73 dyne, sec.$^{0.59}$/cm^2. This is in good agreement with the behavior of this same polymer solution in rotational viscometers[5].

Most water-soluble polymers that have an intrinsic viscosity less than two will give Newtonian solutions under the conditions of this experiment. In addition to poly(vinyl alcohol), some other materials are hydroxyethyl cellulose, dextran, poly(vinyl pyrrolidone), polyacrylamide, glycerol, and the lower glycols. On the other

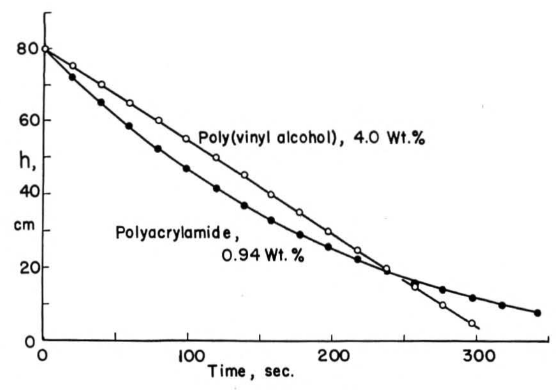

Figure 2. Head decays linearly with time for the Newtonian solution but not for the pseudoplastic solution. Both aqueous solutions were run at 25°C with L = 21.5 cm, D = 0.40 cm, and (BW/H) = 0.20.

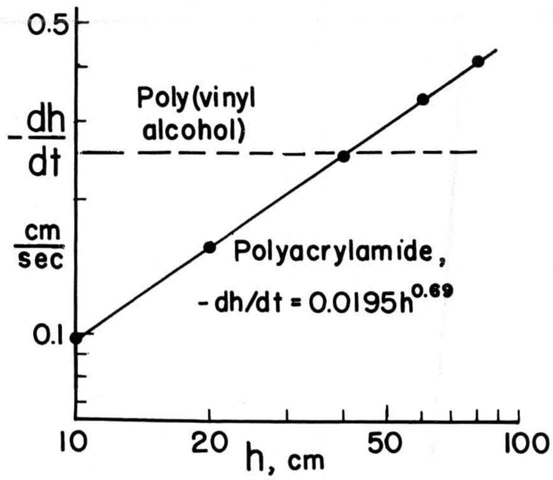

Figure 3. The logarithm of the slope for the non-Newtonian solution taken from Figure 2 increases linearly with log h corresponding to a power-law model.

hand, a non-linearity for the non-Newtonian material demands a very high molecular weight. Poly(ethylene oxide) is available which performs very well (Polyox FRA, Union Carbide Corp.). The polyacrylamide used here was made by placing a beaker containing a mixture of 50 gm acrylamide, 50 gm water, and 0.001 gm riboflavine-5'-phosphate sodium on the light table of an overhead projector for 10 minutes[6]. The resulting polymer gel dissolved in sufficient water to make a 1 wt.% solution while being gently rocked for one week.

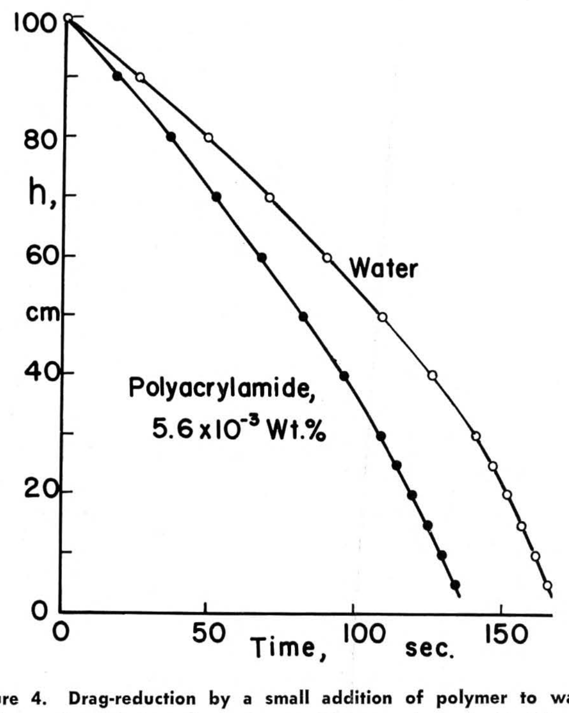

Figure 4. Drag-reduction by a small addition of polymer to water is shown in turbulent flow N_{re} = 2100 at about h = 20). For this experiment at 25°C, L = 91.5 cm, D = 0.30 cm, and (BW/H) = 0.20.

OTHER EXPERIMENTS

A QUALITATIVE DEMONSTRATION uses two setups in tandem so that both solutions can be run at once. The effect of having the Newtonian fluid flow more slowly than the other at first, but then catching up and passing up the non-Newtonian fluid in tortoise-like fashion illustrates the difference in behavior even without recording the time dependence of head.

The same apparatus can be used for an experiment in turbulent flow. The common parameters are the friction factor, f, and Reynolds number, N_{re}.

$$f = (hDg_c)/(2L\bar{u}^2) \qquad (16)$$

$$N_{re} = D\bar{u}\rho/\eta \qquad (17)$$

The average velocity, ü, is given by

$$\bar{u} = (BW/H)(4/\pi)(h/D^2)(-dh/dt) \qquad (18)$$

Since f changes slowly with N_{re}, the qualitative prediction of equations 16 and 18 is

$$-dh/dt \propto (1/h)^{1/2} \qquad (19)$$

The increase in slope with decreasing h is borne out by experiment (Figure 4) down to the point where laminar flow sets in (N_{re}= 2100).

A small amount (56 parts per million) of the poly (acrylamide) will make the reservoir empty even faster due to the well-known phenomenon of drag-reduction[7,8]. From raw data of head versus time, the student should be able to construct a friction factor, Reynolds number plot. In turbulent flow experiments, end effects must be taken into account to achieve agreement with literature values of friction factors.

In all experiments with this apparatus, temperature control is difficult so that there is an inherent limit to the accuracy obtainable. Two other complications are the drainage error and surface tension. The first is aggravated by fast flows with viscous fluids. The second can be compensated for by first dipping the flow tube in the liquid and then aligning the apex of the triangular reservoir with the meniscus of the liquid in the tube rather than with the effluent point.

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