ChE problems for teachers

Analog Simulation of the Dispersion of Atmospheric Pollutants

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PROBLEM

The problem is to devise an analog program for simulation of the dispersion of atmospheric pollutants. Beginning with a commonly accepted dispersion model, which is analytical in nature, show the steps involved in the analog simulation of the vertical concentration profile at successive points downwind of a line source. This problem can be used as a very effective demonstration or as an introductory problem at various stages of a course on analog simulation.

SOLUTION

The most commonly used models of atmospheric dispersion from continuous sources are the Gaussian Plume Models (D. B. Turner, Workbook of Atmospheric Dispersion Estimates, U. S. Department of HEW, 1967). For an infinite line source such as might be used to simulate automotive emissions on a freeway, the model has the following analytical form

$$\chi = \frac{q}{\sqrt{2\pi}\,\sin\phi\,\sigma_z u}\left\{\exp\left[-\frac{(z-H)^2}{2\sigma_z^2}\right] + \exp\left[-\frac{(z+H)^2}{2\sigma_z^2}\right]\right\} \quad (1)$$

where

- H is the effective height at which the emissions occur (meters).
- z is the vertical distance above the ground (meters).
- u is the wind speed (meters/sec).
- Φ is the angle between the source line and the wind direction (limited to $\Phi \geq 45°$)
- q is the rate of emission per unit length (grams/(meter sec)).
- χ is the concentration of the pollutant (grams/cubic meter).
- σ_z is called the vertical dispersion coefficient (meters).

σ_z has been empirically related to the distance, x meters, from the line source to the point of interest and to the stability condition of the atmosphere. Suggested correlations are given by Turner. As approximations to these correlations:

$$\sigma_z \simeq 0.023\ x \quad (2)$$

when the turbulent nature of the atmosphere is minimal to

$$\sigma_z \simeq 0.14\ x \quad (3)$$

when the atmosphere is very turbulent.

Models very similar in nature are also presented by Turner for point and instantaneous source emissions.

Analog Simulations

Using the model described above, two situations will now be considered for analog simulation. The first is the simulation of the vertical concentration profile at a fixed distance downwind assuming, q, Φ, σ_z, u, and H are fixed. The second is the simulation of the vertical profile in an automated manner as the distance downwind increases linearly.

• Vertical Profile at Fixed Downwind Distance

To develop a model suitable for analog simulation, the basic dispersion model is first written as

$$\chi = \frac{q}{\sqrt{2\pi}\,\sin\phi\,\sigma_z u}(\alpha + \beta) \quad (4)$$

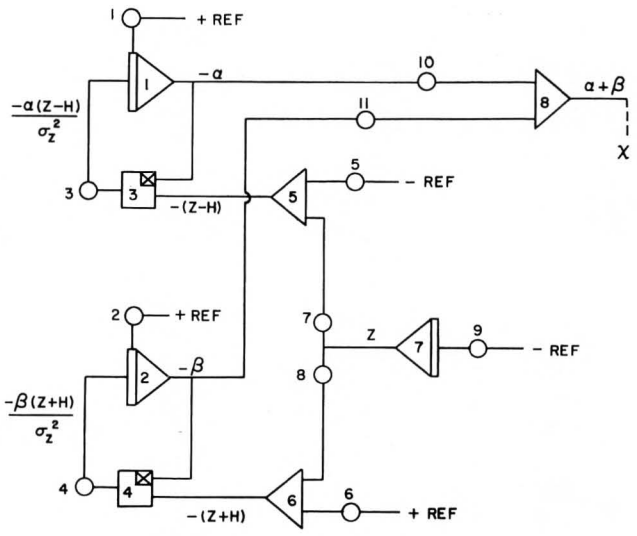
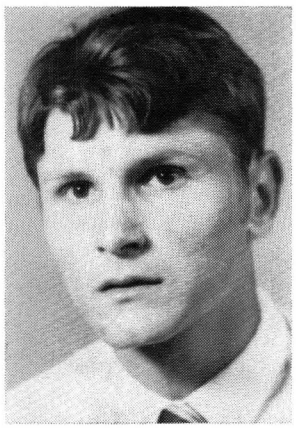
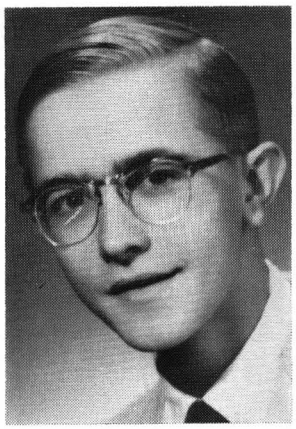


Fig. 1. Vertical Profile Generation





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where

$$\alpha = \exp\left[-\frac{(z-H)^2}{2\sigma_z^2}\right]; \quad \beta = \exp\left[-\frac{(z+H)^2}{2\sigma_z^2}\right] \qquad (5,6)$$

The variables α and β are then differentiated with respect to z to yield the following differential equations for the purposes of analog solution.

$$\frac{d\alpha}{dz} = -\frac{(z-H)}{\sigma_z^2}\,\alpha \qquad (7)$$

$$\frac{d\beta}{dz} = -\frac{(z+H)}{\sigma_z^2}\,\beta \qquad (8)$$

By reference to Equations (5) and (6), the appropriate initial conditions are found to be

$$\alpha\,(z=0) = \exp\left[-\frac{H^2}{2\sigma_z^2}\right] = \beta(z=0) \qquad (9)$$

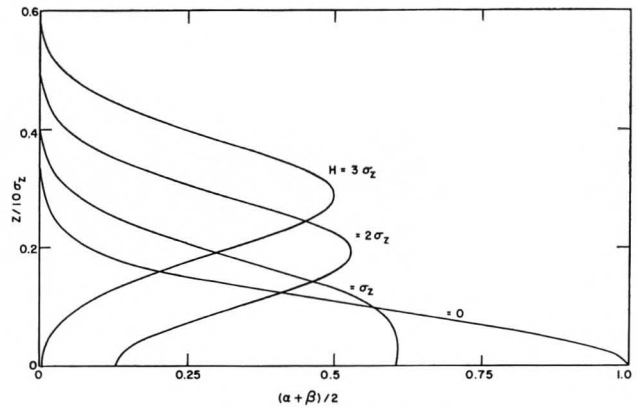

Fig. 2. Vertical Profiles for Various H.

Using Equations (7) and (8), the analog diagram presented in Figure 1 can be used to compute χ as a function of z for fixed q, Φ, u, σ_z, and H. Table 1 below details the variables and parameters of the circuit before and after scaling. For convenience in this table a superscript * is used to indicate the maximum magnitude and B is used to represent the time scaling factor.

TABLE 1. ANALOG VARIABLES AND PARAMETERS

	Before Scaling	After Scaling
Amp 1	$-\alpha$	$-\alpha/\alpha^*$
Amp 2	$-\beta$	$-\beta/\beta^*$
Amp 3	$-\alpha(z-H)$	$-\alpha(z-H)/(\alpha^*(z-H)^*)$
Amp 4	$-\beta(z+H)$	$-\beta(z+H)/(\beta^*(z+H)^*)$
Amp 5	$-(z-H)$	$-(z-H)/(z-H)^*$
Amp 6	$-(z+H)$	$-(z+H)/(z+H)^*$
Amp 7	z	z/z^*
Amp 8	$\alpha+\beta$	$(\alpha+\beta)/(\alpha+\beta)^*$
Pot 1	$\alpha(0)$	$\alpha(0)/\alpha^*$
Pot 2	$\beta(0)$	$\beta(0)/\beta^*$
Pot 3	$1/\sigma_z^2$	$(z-H)^*/(\sigma_z^2\ B)$
Pot 4	$1/\sigma_z^2$	$(z+H)^*/(\sigma_z^2\ B)$
Pot 5	H	$H/(z-H)^*$
Pot 6	H	$H/(z+H)^*$
Pot 7	1	$z^*/(z-H)^*$
Pot 8	1	$z^*/(z+H)^*$
Pot 9	1	$1/(z^*B)$
Pot 10	1	$\alpha^*/(\alpha+\beta)^*$
Pot 11	1	$\beta^*/(\alpha+\beta)^*$

Using the values given in Table 2, the sample results given in Figure 2 were obtained by varying H from 0 to 3 σ_z.

TABLE 2 . EXAMPLE NUMERICAL PARAMETERS

Parameter	Value
α^*	1
β^*	1
z^*	$10\,\sigma_z$
$(z-H)^*$	$10\,\sigma_z$
$(z+H)^*$	$20\,\sigma_z$
$(\alpha+\beta)^*$	2
σ_z	10

As indicated in Figure 2, at low volues of H the maximum concentration occurs at ground level. In this context, it is worth noting that the model assumes complete reflection of the pollutant at the ground. At higher values of H, the maximum concentration occurs at H, the profile becomes more symmetric, and the maximum value of $(\alpha+\beta)$ approaches $1/\sigma_z$ as α be-

comes increasing dominant in the sum. It may be further observed by reference to Equations 7, 8, and 9 that z and H may be related to σ_z as suggested in Figure 2 in order to obtain a general solution valid for σ_z.

• Spread of Profile in Downwind Direction

To simulate the dispersion as a function of the downwind distance, it is necessary to change σ_z as suggested by the approximations in Equations 2 and 3. If an analog capable of two speed integration, possessing a moderate amount of logic, is available, the simulation of this downwind dispersion can be automated to yield an effective demonstration. In essence, the automation procedure is to use a slowly continually operating analog circuit to generate σ_z, $\sigma(0)$, and $\beta(0)$ for use in the more rapidly repetitive operating circuit given in Figure 1. Pots 1 and 2 are, of course, eliminated and Pots 3 and 4 must be replaced by dividers to handle a varying σ_z. In addition, a final divider must be used to compute the $(\alpha + \beta)/\sigma_z$ terms in x as indicated by

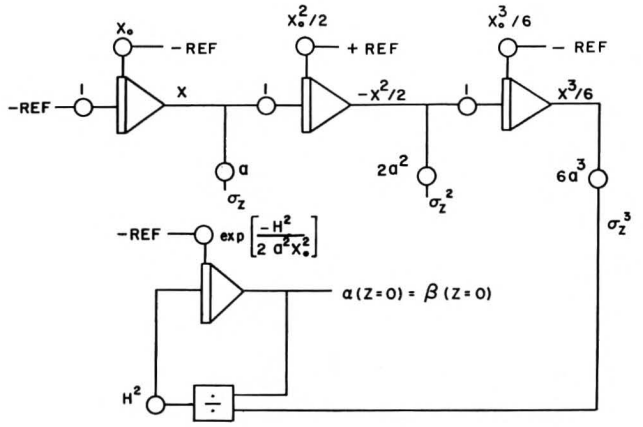

Fig. 3. Generation of Parameters

Equation 4.

Assuming $\sigma_z = ax$ where a is a constant and noting that

$$\alpha(z=0) = \beta(z=0) = \exp\left[-\frac{H^2}{2\sigma_z^2}\right],$$

either x or σ_z can be used as the independent variable on the slowly operating circuit. As indicated in Figure 3, σ_z and σ_z^2 are obtained by integration and $\alpha(z = 0) = \beta(z = 0)$ are determined using

$$\frac{d\,\alpha(z=0)}{d\,\sigma_z} = \frac{H^2}{\sigma_z^3}\,\alpha(z=0) \qquad (10)$$

Initial conditions on Equation (10) are chosen at the lowest non-zero value of x to be simulated. Note that x = 0 can not be assumed because it is a singular point in the model. Again referring to Figure 3, the generated values of σ_z, σ_z^2, $a(z = 0)$, and $\beta(z = 0)$ are fed to the repetitive operating circuit which computes χ. If the secondary circuit operates very slowly, σ_z and σ_z^2 will change neglible during any one repetitive run and thus can be used directly in the primary circuit. If the secondary circuit is fairly rapid, a pair of track and store units can be used to interface the two circuits and maintain σ_z and and σ_z^2 constant during each repetitive run. By plotting χ + x versus z the dispersion of the pollutant as a function of the downwind direction can be automatically obtained. Figure 4 presents several example profiles obtained in this manner. χ is chosen to have a maximum value of 0.1 at a distance of 100 meters downwind. The resulting dispersion downwind at 200, 300, and 400 meters is given.

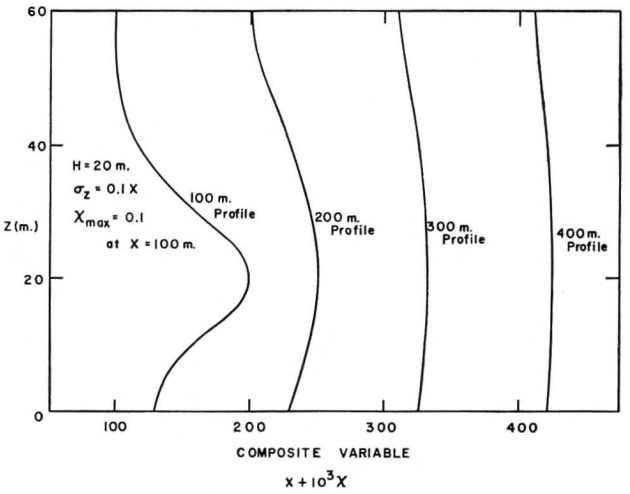

Fig. 4. Dispersion Downwind of Source.

CONCLUSIONS

The analog simulation of the dispersion of atmospheric pollutants has been briefly examined in this paper. It has been shown that the problem can be used as an effective demonstration in order to examine the essential features of atmospheric dispersion. In addition, the problem presents several more difficult scaling problems for the student. The analytical nature of the solution provides, however, a ready check of the simulation and thus the simulation can prove to be an effective student problem at several stages of an analog course. □