FLOW MODELING AND PARAMETER ESTIMATION USING RADIOTRACERS

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A common method of formulating dynamic models is to introduce a process upset (such as a tracer injection) and to measure the subsequent response. There are several important restrictions on the use of this method: the upset must be measurable without being dramatic enough to cause system failure, the production of off-specification product, or a threat to the safety of people and/or equipment; in addition, the experimenter must have access to the process stream, either through a control valve or tracer injection site, and he must have the capability of monitoring the process response. The latter requirement can be fulfilled in many industrial systems only if a radioactive isotope is used as a tracer.

In the experiment to be described, a radiotracer impulse is injected into a turbulent stream flowing in a straight pipe, and the count rate at two downstream points is measured as a function of time. The results are used to determine dispersion model parameters (mean residence time and effective axial dispersion coefficient) by the method of moments. The estimated values of the dispersion coefficient are then compared with values predicted by a published dimensionless correlation of Peclet number vs. Reynolds number.

The educational objectives of the experiment are to expose the students to the concepts of process modeling and experimental determination of model parameters, to lay a basis for subsequent presentation of material on process dynamics and the design and scaleup of nonideal process vessels, and to introduce the topics of tracer technology, radiosotopes handling and radiation detection, subjects not usually encountered in the undergraduate chemical engineering curriculum.

The background material presented to the class covers fully the problems associated with radioactive materials, and discusses alternative tracers such as dyes and electrolytes. It also briefly reviews problems which have been successfully attacked using radioisotopes (many of which are presented in the work by Gardner and Ely (1967)), including the determination of flow rates and flow channel volumes in process units, blood vessels and rivers and streams, formulation of models for industrial processes and biological systems, measurements of diffusion coefficients and kinetic rate constants, measurements of mixing efficiencies in stirred tanks, determination of the existence of channelling or bypassing and stagnant regions in process units, and measurements of currents and dispersion patterns in receiving waters. We believe that this list, as extensive as it is, represents only a fraction of the potential applications of radioisotopes to the solution of problems with which chemical engineers are likely to be confronted, and that having once been introduced to radioisotopes in their education, engineers will be more likely to think of them when dealing with such problems.

EXPERIMENTAL

A schematic of the experimental system is shown in Figure 1. Water is fed to the system from a constant head tank through a rotameter into a long ¾ inch NPD galvanized pipe, and from the pipe into a storage tank.

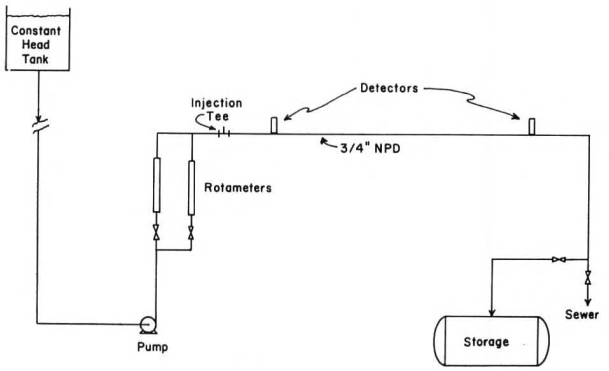
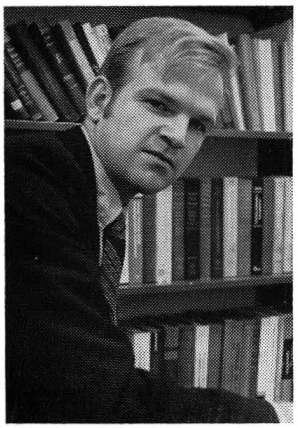
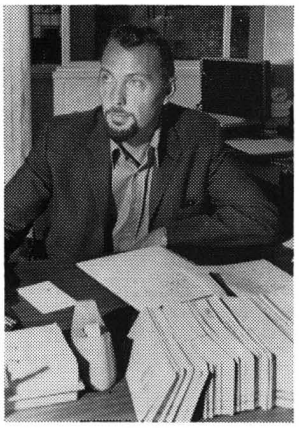
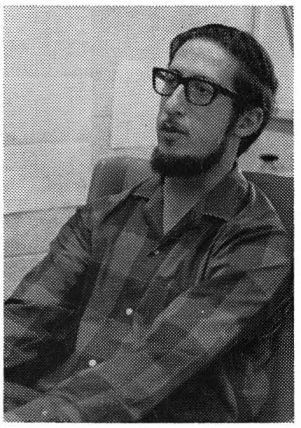


Fig. 1. Schematic of Experiment.

Tracer is injected into the pipe through a regular pipe tee fitted with a rubber septum. Detectors are mounted at either end of a 20 foot straight section of the pipe downstream of the tracer injection point. The storage tank is used to allow the radioactivity to decay to a negligible level before the effluent is sewered.







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Robin P. Gardner received his BChE and MS from North Carolina State University and his PhD from Pennsylvania State University. From 1961-63 he was Scientist in Charge of Short Courses on Radioisotope Applications in Industry at the Oak Ridge Institute of Nuclear Studies and later was Assistant Director of the Measurement and Controls Lab at the Research Triangle Institute. He was an IAEA Expert at the Institute of Radioactive Research in Belo Horizonte, Brazil and currently is Professor of Nuclear and Chemical Engineering at NCSU. (center)

Richard M. Felder did his undergraduate work at the City College of New York and obtained his PhD from Princeton. He spent a year at AERE Harwell, England on a NATO Postdoctoral Fellowship, two years at Brookhaven National Laboratory, and came to NCSU in 1969. Recently he has become involved with photochemical reactor analysis, radioisotope applications, and applications of engineering technology to medical and environmental problems. He has served as a consultant to the government of Brazil on industrial application of radioisotopes. (right)

Manganese—56, in the form of an aqueous $Mn(NO_3)_2$ solution, was selected as the tracer because it is a gamma ray emitter and hence can be detected through metal walls, and because it has a half-life of only 2.6 hours, which causes it to decay to negligible levels in less than 24 hours. Geiger-Mueller tubes were the detectors; their outputs were fed to a Nuclear Chicago rate meter coupled to a Leeds and Northrup strip chart recorder.

Both single and two-point detection methods were demonstrated in the experiments, using detectors placed at distances of 6 and 26 feet from the point of injection. While it is desirable that the detectors have approximately the same sensitivity, deviations from this condition are not serious because of the method of data analysis. Typical output from a run using both detectors is shown in Figure 2, which plots the counting rates recorded by the first and second detector vs. the time from injection.

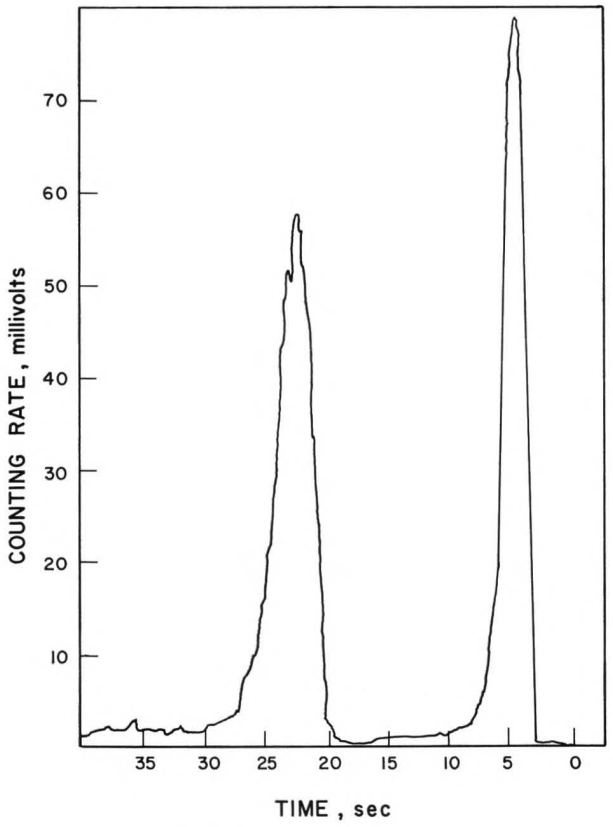

Fig. 2. Typical Analog Output.

COMPUTATIONAL PROCEDURES

Flow systems in which a measurable but finite amount of axial mixing occurs are commonly simulated by two-parameter models, such as axial dispersion and tanks-in-series models. The parameters of these models are simply related to moments of the system impulse response function,

and may be conveniently estimated from numerically calculated moments of experimental response curves. In contrast, parameter estimation techniques such as time-domain and frequency-domain regression require considerable computational effort, and are consequently less suitable for inclusion in a junior-level laboratory course.

The quantities needed to estimate the parameters of a two-parameter flow model by the method of moments are

$$\mu_i = \frac{\int_0^\infty t\, R_i(t)\,dt}{\int_0^\infty R_i(t)\,dt} \qquad i = 1,2 \qquad (1)$$

$$\sigma_i^2 = \frac{\int_0^\infty (t-\mu_i)^2 R_i(t)\,dt}{\int_0^\infty R_i(t)\,dt} \qquad i = 1,2 \qquad (2)$$

where $R_i(t)$ is the counting rate at the i^{th} detection station, and u_i and σ_i^2 are respectively the mean and variance of this function. The procedure is to choose a model, derive expressions for the model parameters in terms of u_i and σ_i^2, evaluate the latter quantities from the experimental impulse responses and Eqs. (1) and (2) by numerical quadrature, and use the resulting values to solve for the model parameters. In the experiments run so far, hand calculations of the moments from analog output were required; in the future, the output from the rate meter will be punched on paper tape, which will in turn be used as input to a Simpson's Rule quadrature subroutine.

The model chosen for illustrative purposes was the doubly-infinite axial dispersion model. The assumptions inherent in this model and solutions for the impulse response and its moments are outlined by Himmelblau and Bischoff (1968) and Levenspiel (1972). The relations used in the single-detector measurements are as follows:

$$\bar{t}_i = \frac{L_i}{v} = \mu_i \qquad i = 1,2 \qquad (3)$$

$$\frac{\sigma_i^2}{\mu_i^2} = \frac{2}{P_i} + \frac{8}{P_i^2} \quad \text{and} \quad P_i = vL_i/D \qquad (4,5)$$

where L_i is the distance from the injection point to the i^{th} detector, v is the mean flow velocity in the tube, and D is the effective axial dispersion coefficient. The procedure followed was to calculate u_i and σ^2 from the experimental R_i data using Eqs. (1) and (2), then to evaluate the velocity v from Eq. (3), the Peclet number P_i from Eq. (4), and finally D from Eq. (5). The two-detector method provides two advantages over the single detector method: the injection does not have to be a perfect impulse, and errors in synchronizing the time of injection with the recorder output are eliminated. The equations for the two detector method are

$$\frac{L_2-L_1}{v} = \mu_2-\mu_1 \qquad (6)$$

$$\frac{\sigma_2^2 - \sigma_1^2}{(\mu_2-\mu_1)^2} = \frac{2}{P} \qquad (7)$$

$$P = \frac{v(L_2-L_1)}{D} \qquad (8)$$

In this case the velocity v is calculated from the numerical moments using Eq. (6), the Peclet number P is calculated from Eq. (7), and D is calculated from Eq. (8).

Table 1 shows values of flow velocities determined with a rotameter and calculated as outlined above. It is interesting to compare the results obtained by the one- and two-point techniques: the average error is 16% for the single detector at 6 feet, 6% for the single detector at 26 feet, and 2% for the two detectors. Figure 3 in a plot of the effective axial dispersion coefficient vs. the Reynolds number, and shows that the data fall within the range predicted by a

Table 1—Flow Velocities

Run	u(from rotameter)	u_1	u_2	u_{12}
1	2.01ft/sec	1.77	1.88	1.93
2	1.53	1.07	1.37	1.49
3	1.0	0.938	0.945	0.99
4	1.0	—	1.03	—

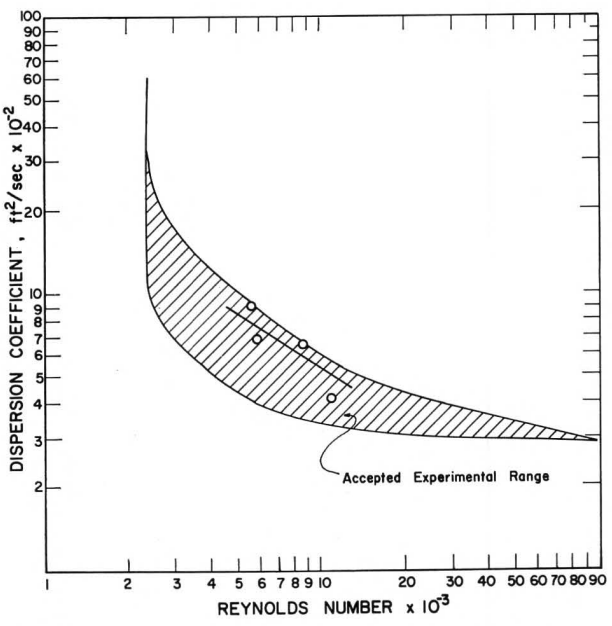

Fig. 3. Dispersion Coefficient as a Function of Reynolds Number.

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correlation reprinted by Himmelblau and Bischoff (1968). Since doing so would not serve any particular purpose in light of the educational objectives of the experiment, we have not attempted to confirm the negligibility of adsorption other than to note the absence of long tails on the impulse response curves.

DISCUSSION

The students are given two 3-hour laboratory periods to complete the experiment. In the first period they go through the entire experiment without actually using the radiotracer. Under this format they each get a chance to become familiar with the equipment and injection procedure without the worry of spills or other hazards. Also during the first 3-hour session they are instructed in fundamentals of radiation detection, particularly as related to Geiger-Mueller characteristics, and in radiation safety. In the second 3-hour period the actual runs are made, but the injections and tracer handling are not done by the students, who merely observe. The runs themselves are not particularly time-consuming, so that each student has sufficient time to carry out the necessary calculations and to seek individual instruction on any aspect of the experiment.

The participating students appeared to get a great deal out of the experiment; the only strong objection was to the necessity of calculating moments graphically from analog output, a requirement which will be eliminated in the future when digital output equipment becomes available. A potential problem with an experiment of this sort being given on a junior level is that the students may have to accept on faith the utility of much of what they are doing. This did not appear to be a matter of concern to the students, however, possibly due in part to the fact that they had been introduced to elementary concepts of modeling (and in particular to the dynamic response of a first-order process) in a previous course. The experiment also served to make related material subsequently encountered in senior courses on reactor design and process dynamics and control a great deal more meaningful to those who participated in it.

Once the basic procedure for a stimulus-response experiment of this type has been established, it is a relatively easy matter to study a variety of flow systems using the same technique. We are currently making provisions for the following experiments:

1. Flow and dispersion in packed columns.
2. Flow in obstructed tubes
3. Detection of stagnancy and channeling
4. Residence time distribution of stirred tanks in series
5. Variation of RTD with stirring rate in a single stirred tank—determination of mixing efficiency.
6. Impulse responses of laminar flow systems.

Our present plan is to include one of these experiments (probably the one on the packed column) in the laboratory course, either in addition to or instead of the empty tube experiment, and to use the others as senior projects.□

REFERENCES

Gardner, R. P. and R. L. Ely, Jr., "Radioisotope Measurement and Applications in Engineering," Reinhold Publishing Co., 1967.

Himmelblau, D. M. and K. B. Bischoff, "Process Analysis and Simulation," Chapter 4, John Wiley and Sons, Inc., 1968.

Levenspiel, O., "Chemical Reaction Engineering," Chapter 9, John Wiley and Sons, Inc., 1972.

LAB WORKSHOP (Continued from p. 125)

D. J. Graves discussed the use of audio-visual packages in the preparation phase of the laboratory experiment. The student prepared modules used to describe complex pieces of equipment, measurements points, flow paths, and complex procedures. At the University of Pennsylvania the audio-visual modules have reduced laboratory time and allowed students to continue to work on project experiments after previous groups have graduated. Two sample audio-visual modules were shown to illustrate their use with typical experiments.

The entire proceedings of the laboratory workshop are to be published and can be obtained by sending a check for $10.00 payable to The University of Michigan to Professor H. Scott Fogler, Department of Chemical Engineering, The University of Michigan, Ann Arbor, Michigan 48104. The price of the bound proceedings, which will be available after May 1, 1973, was established by the ASEE ChE Division in order to minimize any loss of funds from printing costs.