

SEEING ENTROPY - The Incomplete Thermodynamics of the Maxwell Demon Bottle

M. V. SUSSMAN
Tufts University
Medford, Mass. 02155

MOST OF US have heard of Maxwell's Demon but entirely too few know of the Maxwell Demon Bottle [1, 2, 3]. This fact, although forgivable is entirely unfortunate because the bottle not only has sealed within it a Maxwell Demon—but also contains means for elucidating in a visible and even amusing fashion many of the fundamental concepts of thermodynamics.

What is the Maxwell Demon Bottle?

The Maxwell Demon Bottle (hereinafter called MDB) is the simple device shown in Figure 1 which consists of a long necked, sealed, flask containing a number (usually 10) of colored cork or rubber spheres. As usually constructed, 5 of these spheres are black and 5 are white. When the flask is held in a neck down position the spheres fall into the neck in a columnar array.

But what does this have to do with thermodynamics?

Well an important part of thermodynamics is concerned with the fact that Nature is not at all even-handed in the direction in which she moves phenomena. For example, apples fall from branches above to the ground below, and never move spontaneously in the opposite direction. A steel ingot always cools when taken from the furnace. Humpty-Dumpty never gets put together again. We grow older.

Nature's processes abound in irreversibility. They move easily in one direction and not at all or only with great effort in the reverse.

Why? Why, for example, is it easier to move from cream and coffee—separate entities, to—cream and coffee—mixture, than in the reverse direction? The MDB evolved from an old demonstration aimed at answering this very question; a demonstration to illustrate that *statistics*, rather than design, lies behind the irreversibility of a mixing process [4].

The demonstration is shown in Figure 1, 2 and 3. Starting with the flask in the neck down position and an array of 5 black spheres surmounted by 5 white spheres, the flask is tilted to allow the neck contents to run into the body. When the flask is reinverted the spheres again run into the neck but usually not in the initial array. The reason is, of course, that there are 252 possible arrays or permutations of 5 black and 5 white balls ($10!/5!5!$) and once the initial

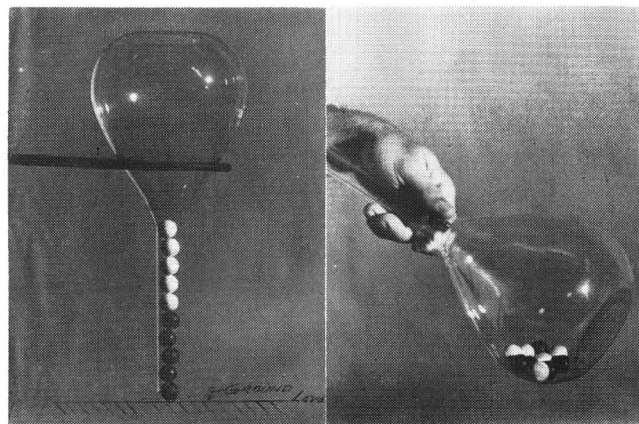


Figure 1. The "Maxwell Demon Bottle" in its initial state.

Figure 2. Tilting the bottle disarranges the initial state . . .

array is destroyed, the likelihood of reforming that array is very small.

In mixing solutions we are intermingling not 10 particles but numbers of particles in the order of 10^{23} . Therefore, the number of possible permutations of these particles is astronomically large and the likelihood of regaining the original array by chance is so small as to be considered impossible.

Thus, it is seen that mixing processes owe their irreversibility to statistics—that is, to the negligible likelihood that a unique array of an astronomically large number of particles, once disturbed, can be regained by chance selection—rather than to forces or to design.



Dr. M. V. Sussman is peripathetic professor of chemical engineering at Tufts University and the man who first bottled the Maxwell Demon (U.S. Patent No. 3,289,321). He is also the author of that recently published (Addison Wesley) thermodynamics textbook, with a perpetual motion machine (courtesy of M. Escher) on its dust jacket. Among other singular activities are works on continuous gas chromatography and "muscle" turbines. His thermodynamic ruminations have appeared in this journal previously ("Approaches to Statistical Thermodynamics," Chem. Eng. Ed., p. 113-118, Summer 1968).

MORE THERMODYNAMICS: ASSIGNING AN ENERGY TO THE MDB

But the MDB contains more thermodynamics than this. For example, an *energy* can be assigned to the MDB by the following stratagem. Assume that the black spheres each have unit mass, while the white spheres are merely massless spacers. If the bottle is supported in a neck down position as in Figure 3 with the top of the neck resting at table level, we can assign an energy value to each ball position corresponding to its elevation above the ground or reference level. Thus, we will assign an energy of zero units to the lowest position in the neck of the bottle and

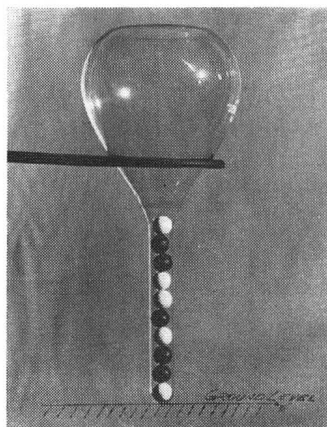


Figure 3. Re-inverting the bottle usually does not restore it.

energies of 1, 2, 3, 4, etc., energy units to each subsequent level. For example, the energy of the array in Figure 1 is

$$E_{k1} = 0+1+2+3+4 = 10 \text{ energy units}$$

whereas the energy of the array in Figure 3 is

$$E_{k3} = 0+4+5+7+8 = 24 \text{ energy units,}$$

and in general

$$E_k = \sum_{i=1}^M n_i \epsilon_i \quad (1)$$

where n_i is the number of black balls in the "i"th level and may be either 0 or 1, and ϵ_i is the energy magnitude of the "i"th level, which may have any integral value between 0 and M, the maximum energy level that can be reached. Now engaging in a flight of fancy nomenclature, let us call each array a "state." We have already pointed out that with 5 white and 5 black balls the system can have 252 "states." The states can have energies ranging from 10 units (state 0,1,2,3,4) to 35 units (state 5,6,7,8,9). The states are specified by the 5 ϵ_i 's of the black balls. One would therefore *expect* that on the average the energy of the MDB will be

$$\langle E \rangle = \sum_{k=1}^{252} P_k E_k \quad (2)$$

where P_k is the probability of state "k", or the chances of getting a particular permutation of black and white balls;

E_k is the energy magnitude of that state as given by equation (1); and

$\langle E \rangle$ is the expected energy, or the probable average of future energies, of the MDB, taking into consideration all possible permutations or "states."

If we now apply elementary calculus to Equation (2) we find that the differential of the expected energy of the MDB equals

$$d\langle E \rangle = \sum_k \epsilon_k dp_k + \sum_k p_k d\epsilon_k \quad (3)$$

which says that there are 2 ways of changing the energy $\langle E \rangle$:

(I) by changing the p_i 's while holding ϵ_k 's constant: and

(II) by changing the ϵ_k 's at constant P_k 's

Equation (3) is intriguingly similar to the equation in classical thermodynamics for the differential of energy in a simple system:

$$dU = d^J_{REV} - d^d_{REV} \quad (4)$$

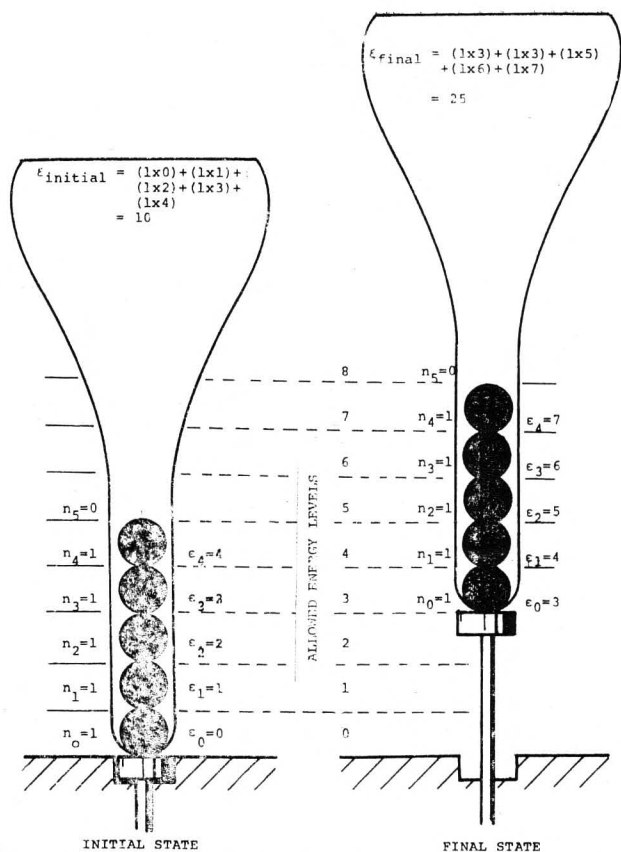


Figure 4. The energy of the "Bottle" may be increased by a Reversible Work Effect.

which also says that the energy can be varied in two ways:

- (I) by a heat effect, dQ_{REV} ; and
- (II) by a work effect, $-dW_{REV}$.

On the strength of the analogy between equations (3) and (4) we will call

$$\sum_x \epsilon_x dp_x \equiv dq_{REV} \quad (5)$$

or the bottle analog of heat

$$\sum_x p_x d\epsilon_x \equiv -dw_{REV} \quad (6)$$

or the bottle analog of work and,

$\langle E \rangle \equiv U$ or the bottle analog of *internal energy*.

(WORK AND HEAT EFFECT)

A WORK EFFECT OCCURS when the p_x are held constant and the ϵ_x are varied. This is easily accomplished by changing the elevation of the bottle. If the bottle is raised, for example as in Fig. 4, the magnitude of each of the ϵ_x 's is increased without disturbing the n_x 's. Hence, the ϵ_x are changed without changing the p_x . In short, work is performed on the MDB system by a change of an external parameter, the elevation,

much like work is done on a gas by changing another external parameter, the volume.

Continuing the analogy, a heat effect, $\sum_x \epsilon_x dp_x$, is accomplished by changing the probability of a given state of black balls without changing the elevation. This may be done by adding or removing white balls from the bottle thereby changing the number of permutations and therefore the probability of a given permutation or state.

A profound difference between reversible work and heat becomes apparent when equal amounts of these forms of energy are added to a MDB. For example, the 2 black ball one white ball MDB of Fig. 5a has an expected energy of 2 units, (all arrangements are equally likely), which can be increased to 4 units either by raising the entire system one diameter (adding 2 units of work energy, 5b) or by adding 2 white balls (adding 2 units of heat energy, 5c).

Now although the 5b and 5c systems have the same store of expected energy, they differ in the *availability of that energy for doing work*. The 5b system can release 2 units of its energy as work by simply dropping to the 5a position, whereas the 5c system *cannot drop* and therefore *cannot* release energy as work (unless perhaps means are found to drain off excess white balls without reducing $\langle E \rangle$).

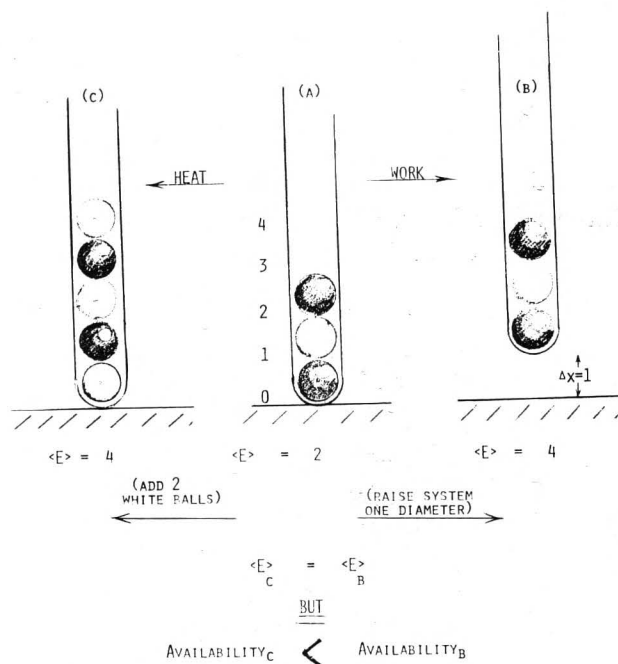


Figure 5. Changes produced by equal heat and work effects: Although (B) and (C) have equal interval energies, (B) can release work, whereas, (C) cannot, therefore (B) has more Available Energy.

The generalization of this observation is that if two systems store the same amount of energy, the more disordered of the two can do less work than the less disordered. I know no simpler illustration of this fundamental thermodynamic principle.

"SEEING" ENTROPY

LET US SUPPOSE that as suggested above, white balls, the massless spacers, can in some manner be added to the bottle at will, and starting from a system containing only 5 black balls, examine the effects of successive additions of white balls.

Nature's processes abound in irreversibility . . . they move easily in one direction and not at all or only with great effort in reverse.

With only 5 black balls (Figure 6a) the system has an energy of 10 units (0+1+2+3+4). On addition of 1 white ball (6b), the system may have any one of the 6 energies between 10 and 15 as the white ball occupies any of the 6 possible positions between ϵ^0 and ϵ^5 . For example, Figure 6b shows the system when it has an energy of 12 units. With 2 white balls added (6c), the energy of the system can take on values from 10 to 20 units and the balls can form:

$$\frac{7!}{5! 2!} = 21 \text{ states or permutations.}$$

Three white balls, (6d), make possible energies between 10 and 25 and allow:

$$\frac{7!}{5! 3!} = 56 \text{ states}$$

and as previously noted, 5 white balls allow energy states ranging from 10 to 35 units and permit 252 states.

In general n_w white balls added to a 5 black ball system will make possible energies ranging from 10 to $10+5n_w$ units with

$$\frac{(5+n_w)!}{5! n_w!} \text{ possible states.}$$

The introduction of white balls has two effects. It makes it possible for individual black balls to climb higher into the neck of the flask to higher energy levels making possible increased

values of ϵ_g ; and in addition it has the curious effect of introducing *indeterminacy* into the system, because each white ball increases the number of ways black balls and individual energy levels may be permuted.

The number of possible permutations of black balls in accessible energy levels is an intriguing characteristic of the MDB which we shall make the basis of a property here christened "Bottle Entropy," and defined as:

$$S' = k \ln w \quad (7)$$

Thus, S' is proportional, to the natural logarithm of the number of permutations, w (k is a constant). Clearly S' increases as we add white balls and increase the possible states of the system, and S' goes to 0 as the number of white balls goes to 0 and the permutations drop to unity.

EXTENSIVE NATURE OF "S"

LET US AGAIN CONSIDER an MDB containing only two black balls and one white ball. This system can exist in $3!/2!$ or 3 states, and if all these states have equal probability then,

$$S' = k \ln 3$$

The three states are shown as columns of B's and O's in Table 1a.

If we were now to place a second identical MDB alongside the first, that is, if we were to *double* our system, the entropy of the *doubled* system consisting of MDB_1 and MDB_2 would be

$$S_{1+2} = k \ln w_{1+2}$$

where w_{1+2} is the number of states available when 2 MDB's are considered simultaneously.

Now if bottle 1 were in state (0, 1) (Table 1b), bottle 2 could be in any of its three possible states. Similarly, if 1 were in state (0, 2) bottle 2 again could be in any of its three possible states. The magnitude of w_{1+2} is therefore found by combining every state of one bottle with all possible states of the other bottle. Or

$$w_{1+2} = w_1 \cdot w_2 = 3 \times 3 = 9$$

and $S_{1+2} = k \ln w_1 \cdot w_2$

$$= k \ln w_1 + k \ln w_2$$

or $S_{1+2} = S_1 + S_2 \quad (8)$

and $S \text{ (of } N \text{ identical MDB's)} = N(S_1) \quad (9)$

or S depends on the number of bottles comprising the system and therefore is an *extensive* property.

CONDITION (a)
THE POSSIBLE STATES OF A 2-BLACK, 1-WHITE BALL MDB

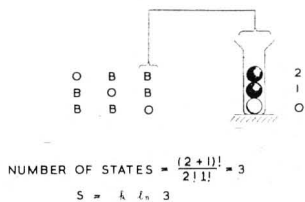


Table 1 (a)

CONDITION (b)
THE POSSIBLE STATES OF TWO MDB'S

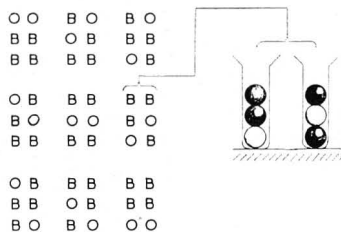


Table 1 (b)

CONDITION (c)
STATES OF TWO MDB'S HAVING DISTINGUISHABLE PARTICLES WHICH MIX

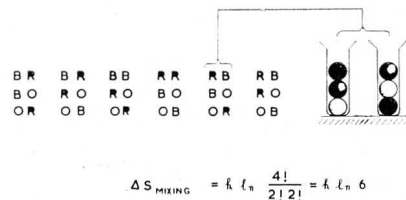


Table 1 (c)

THE ENTROPY OF MIXING

If, by some unspecified diffusion process, the black balls of bottle one can *interchange* positions with those of bottle 2, the entropy effect is that described in the previous paragraph because the black balls are all indistinguishable. If, however, the particles of bottle one are distinguishable from those in bottle 2, for example if 2 has red spheres and 1 has black spheres, (or if 2 has heavier spheres than 1), then there is a large and additional entropy increase because a large number of new 2-color states are created by the diffusional mixing. (Table 1c).

The total states in a system consisting of a 2 black sphere—one white sphere MDB, and a 2 red sphere—one white sphere MDB, is all the states of a two-bottle system, multiplied by the number of ways the 2 red and 2 black spheres can be interchanged or permuted in each of these double-bottle states. Now, each of the double bottle states can be permuted. $(2+2)!/2!2! = 6$ ways (Table 1 c, d)

Therefore $w_{r+b} = w_r \cdot w_b \cdot \frac{(n_r + n_b)!}{n_r!n_b!} = 3 \times 3 \times 6 = 54$ (10)

Consequently $S_{r+b} = k \ln w_r + k \ln w_b + k \ln w_{\text{mixing}}$

OR $S_{r+b} = S_r + S_b + \Delta S_{\text{mixing}}$ (11)

where $\Delta S_{\text{mixing}} = k \ln \frac{(n_r+n_b)!}{n_r!n_b!}$ (12)

When we deal with very large numbers of particles as for example in the preparation of solutions of liquids or gases, Stirling's Approximation for $\ln(n!)$ may be employed

$\ln(n!) = n \ln(n) - n$ (13)

in Equation (12) which transforms it to

$\Delta S_{\text{mixing}} = -nk \sum_i x_i \ln x_i$ (14)

where x_i is the fraction of particles that are of the "i" kind, and n is the total number of particles.

If in addition k is taken to be the Boltzmann constant or, R/\bar{N} , the gas constant divided by Avagardo's number, then nk becomes NR , the total number of moles multiplied by the gas constant. Equation (14) is then the classical expression for the entropy of mixing ideal solutions:

$\Delta S_{\text{mixing}} = -NR \sum_i x_i \ln x_i$ (15)

TABLE 1(D) ENTROPY AND ENERGY OF MIXING

POSSIBLE STATES AND ENERGIES OF TWO MDB'S
THE COLORED PARTICLES OF WHICH ARE DISTINGUISHABLE AND INTERCHANGE POSITIONS

e_i	00 00 00 00 00 00	00 00 00 00 00 00	00 00 00 00 00 00	00 00 00 00 00 00	00 00 00 00 00 00	00 00 00 00 00 00	00 00 00 00 00 00	00 00 00 00 00 00	00 00 00 00 00 00	E_i
2	00 00 00 00 00 00	00 00 00 00 00 00	00 00 00 00 00 00	00 00 00 00 00 00	00 00 00 00 00 00	00 00 00 00 00 00	00 00 00 00 00 00	00 00 00 00 00 00	00 00 00 00 00 00	E_2
1	00 00 00 00 00 00	00 00 00 00 00 00	00 00 00 00 00 00	00 00 00 00 00 00	00 00 00 00 00 00	00 00 00 00 00 00	00 00 00 00 00 00	00 00 00 00 00 00	00 00 00 00 00 00	
0	00 00 00 00 00 00	00 00 00 00 00 00	00 00 00 00 00 00	00 00 00 00 00 00	00 00 00 00 00 00	00 00 00 00 00 00	00 00 00 00 00 00	00 00 00 00 00 00	00 00 00 00 00 00	
4	4 4 2 6 4 4	5 5 3 9 7 7	6 8 6 10 8 10							E_2
2	00 00 00 00 00 00	00 00 00 00 00 00	00 00 00 00 00 00	00 00 00 00 00 00	00 00 00 00 00 00	00 00 00 00 00 00	00 00 00 00 00 00	00 00 00 00 00 00	00 00 00 00 00 00	
1	00 00 00 00 00 00	00 00 00 00 00 00	00 00 00 00 00 00	00 00 00 00 00 00	00 00 00 00 00 00	00 00 00 00 00 00	00 00 00 00 00 00	00 00 00 00 00 00	00 00 00 00 00 00	
0	00 00 00 00 00 00	00 00 00 00 00 00	00 00 00 00 00 00	00 00 00 00 00 00	00 00 00 00 00 00	00 00 00 00 00 00	00 00 00 00 00 00	00 00 00 00 00 00	00 00 00 00 00 00	
7	7 7 3 9 5 5	8 8 4 12 8 8	9 11 7 13 9 11							E_1
2	00 00 00 00 00 00	00 00 00 00 00 00	00 00 00 00 00 00	00 00 00 00 00 00	00 00 00 00 00 00	00 00 00 00 00 00	00 00 00 00 00 00	00 00 00 00 00 00	00 00 00 00 00 00	
1	00 00 00 00 00 00	00 00 00 00 00 00	00 00 00 00 00 00	00 00 00 00 00 00	00 00 00 00 00 00	00 00 00 00 00 00	00 00 00 00 00 00	00 00 00 00 00 00	00 00 00 00 00 00	
0	00 00 00 00 00 00	00 00 00 00 00 00	00 00 00 00 00 00	00 00 00 00 00 00	00 00 00 00 00 00	00 00 00 00 00 00	00 00 00 00 00 00	00 00 00 00 00 00	00 00 00 00 00 00	
10	8 6 10 8 6	11 9 7 13 11 9	12 12 10 14 12 12							E_2

$S_{r+b} = k \ln 54 \times \frac{(2+2)!}{2!2!} = S_r + S_b + \Delta S_{\text{MIXING}}$; $\Delta S_{\text{MIXING}} = k \ln 6$

$\langle E \rangle_{r+b} = \frac{U}{r+b} = \frac{U}{r} + \frac{U}{b} = 6$ ENERGY UNITS ; $\langle \Delta S \rangle_{\text{MIXING}} = 0$

Table 1 (d)

ENERGY OF MIXING: IDEAL SOLUTIONS

IN CONTRAST to the entropy, the total energy does not change on mixing, if the mixing

occurs as in Table 1c and 1d. To illustrate this we assign a mass of one unit to the black balls and of three units to the red balls. The pure red ball MDB at ground level, therefore, may have energy states of 3, 6, and 9 energy units and an expected energy

$$\langle E \rangle_r = 6 \text{ energy units}$$

whereas the pure black ball MDB has an expected energy

$$\langle E \rangle_b = 2 \text{ energy units}$$

If there are no energy effects on mixing, it must follow that the energy of the mixed system

$$\langle E \rangle_{r+b} = \langle E \rangle_1 + \langle E \rangle_2 = 2 + 6 = 8 \text{ energy units}$$

The entropy function of classical thermodynamics is seen to be a monotonic measure of the number of ways that the particles that make up the system can be permuted in the energy levels available to that system.

That this is indeed the case may be seen by examining the energies of all the states in Table 1(d). The possible energy states of the mixed system range from a low of 2 to a high of 14e.u. (which is greater than the 4 to 12 range of 2 unmixed systems). Thus the average or expected energy is

$$\langle E \rangle = 8$$

$$\Delta U_{\text{mixing}} (= \Delta \langle E \rangle_{\text{mixing}}) = 0 \quad (16)$$

It also follows that, if there are no elevation changes during mixing

$$\Delta H_{\text{mixing}} (= \Delta U - \Delta(PX)) = 0 \quad (17)$$

Consequently the free energies of mixing

$$\Delta G_{\text{mixing}} = \Delta A_{\text{mixing}} = -T\Delta S_{\text{mixing}} = NRT \sum_i x_i \ln x_i \quad (18)$$

Equations (15), (16), (17) and (18) are the classical expressions for the isothermal mixing properties of ideal solutions. In fact, they define the "ideal solution."

BOTTLE AND CLASSICAL ENTROPY

A FITTING CLOSE to our entropic elucidation would be to relate "Bottle Entropy"

(Equation 7) to the classical entropy defined as

$$dS = \frac{dq_{\text{rev}}}{T} \quad (19)$$

A general expression of S' may be written as

$$S = -k \sum_{\lambda} p_{\lambda} \ln p_{\lambda} \quad (20)$$

which reduces to equation (7) when the p_i 's all have the same constant value

$$p_{\lambda} = \frac{1}{w} \quad (21)$$

This is always true when one permutation is as likely as another because

$$\sum_{\lambda} p_{\lambda} = 1 \quad (22)$$

Furthermore it can be shown that S is a maximum when

$$p_{\lambda} = \frac{e^{-E_{\lambda}/kT}}{Z} \quad (23)$$

where k is the same constant as in Equation (7) and T and Z depend only on $\langle E \rangle$ and not on any of the p_i .

Equation (23) reduces to Equation (21) when $T \rightarrow \infty$.

From Equation (20) it follows that:

$$dS' = -k \sum_{\lambda} (\ln p_{\lambda} + 1) dp_{\lambda} \quad (24)$$

and if $\ln p_i$ is expressed in terms of Eq. (23):

$$dS' = -k \sum_{\lambda} \left[- \left(\frac{E_{\lambda}}{kT} \right) - \ln Z + 1 \right] dp_{\lambda} \quad (25)$$

but on substituting Eq. (22) in (25):

$$dS' = \sum_{\lambda} \frac{E_{\lambda} dp_{\lambda}}{T} \quad (26)$$

which on using Eq. (5) becomes:

$$dS' = \frac{dq_{\text{rev}}}{T} \quad (27)$$

Thus, the function S defined by equation (20), which is related to the number of permutations of particles in accessible energy levels, or more generally to the probability of all such permutations, is seen to be the differential of the bottle analog of heat divided by T , which we recognize as the classical definition of the differential of *Entropy*.

Put another way; the entropy function of classical thermodynamics is seen to be a monotonic measure of the number of ways that the particles that make up the system can be permuted in the energy levels available to that system. It is a measure of the *freedom* enjoyed by the system's particles to move through the energy levels accessible to these particles. It also may be looked upon as a measure of the *uncertainty* associated with an assignment of the system to any one of its accessible states.

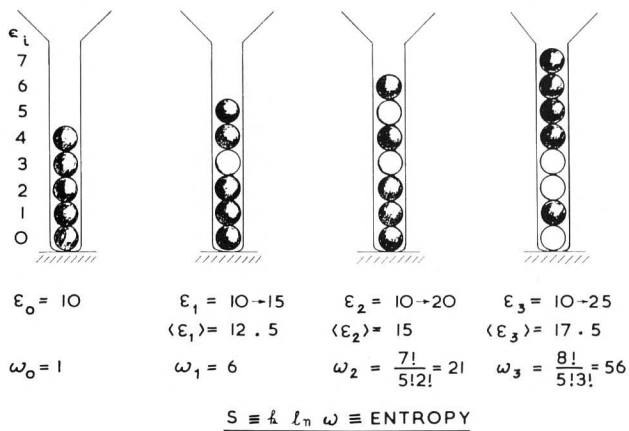


Figure 6. Seeing Entropy. Entropy is a measure of the permutability of a system. Adding white balls increases the number of energy levels accessible to the black balls and hence increases the permutations of black particles and energy levels.

RESPECTABLE DEGENERACY

WE CONTINUE OUR examination of the thermodynamics of the Maxwell Demon Bottle by looking more closely at the 252 allowed states of the 5 black, 5 white ball system. Each of these states can be described by a unique combination of 5 integers representing the 5 energy levels occupied by each of the black balls. The energy of any state "i" will be designated as ϵ_i and is given by the sum of these integers.

The individual quantum states (λ 's) are unique. The ϵ_λ , however, or not, and the number of l states that have the same ϵ_λ is called the degeneracy of energy state ϵ_λ , or w_{ϵ_λ} . Values of ϵ_λ , w_{ϵ_λ} and l for the 5 white—5 black system are shown in Table II. The degeneracy has a maximum value of 20 when ϵ_λ equals 22 or 23.

PARTITION FUNCTION

The constant Z in Equation (11) is called the *Partition Function* and can be determined by combining Equation (10) with (11), from which it follows that

$$\sum_{\lambda} p_{\lambda} = 1 = \sum_{\lambda} \frac{e^{-E_{\lambda}/kT}}{Z}$$

Therefore
$$Z = \sum_{\lambda=1}^{252} e^{-E_{\lambda}/kT} \quad (28)$$

The summation is over all possible *quantum* (l) states. Z may also be expressed as a summation over *energy*, (ϵ_i) states

$$Z = \sum_{\epsilon_i=10}^{35} w_{\epsilon_i} e^{-E_{\lambda}/kT} \quad (29)$$

in which the summation index ϵ_λ takes on all

integral values between 10 and 35 and each p_i term is multiplied by its degeneracy w_{ϵ_i} . Thus Z in Equation (27) has the same value as in Equation (26).

CONVENTIONAL THERMODYNAMIC SYSTEMS

WE HAVE TAKEN a simple set of black and white balls and ascribed to the set properties that we usually associate with thermodynamic systems. We have given the MDB an "energy" and have shown how this "energy" can be changed by "heat" and "work" effects. We have also shown that a system whose energy accumulation is the result of heat effects can do less work (has a lower availability) than a system of equal energy accumulated as the result of reversible work effects. In addition we have determined an "entropy" for the balls which is related to the number of ways the black balls may be arranged in the energy levels available to each

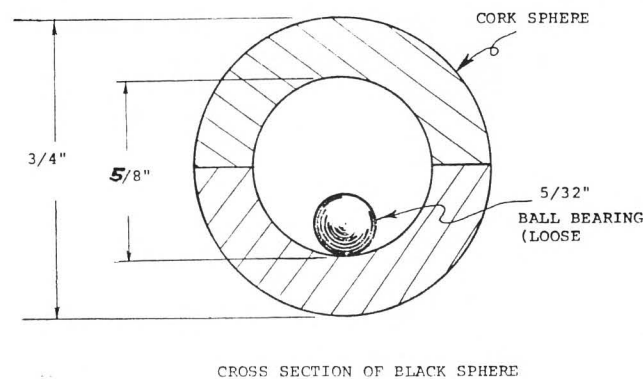


Figure 8. Cross-section of a black sphere.

black ball. We could also have derived the conventional relationships for the thermodynamic properties in terms of the partition function of the bottle. We are able to do all this because our system, like all physical systems, may be described in terms of discrete particles having discrete energy levels; and also because we have defined entropy by Equation (20). An implicit and essential part of this definition is that the p_i in equation (20) have that set of values which *maximizes* S . These p_i are evaluated by standard mathematical maximization techniques (2, 5) that take into account such constraints as Equation (22), which says that the set of states must be exhaustive; and Equation (2), which says that the energy magnitudes of all the accessible quantum states is known as is the average or expected energy of the system. The maximization

E_x	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	
V_{E_x}	1	1	2	3	5	7	9	11	14	16	18	19	20	20	19	16	14	11	9	7	5	3	2	1	1		
	01234	01235	01236	01237	01238	01239	01240	01241	01242	01243	01244	01245	01246	01247	01248	01249	01250	01251	01252	01253	01254	01255	01256	01257	01258	01259	01260

Table II

Quantum States, Energy States and Degeneracy of the 5-Black, 5-White Sphere MAXWELL DEMON BOTTLE

procedure yields the set of p_i 's given by Equation (23), wherein the probability of a quantum state is a function of its energy magnitude (except at infinite temperature).

Equation (20) provides a formalism for assigning p_i 's in an objective fashion. We, therefore, need not hypothesize ergodic behavior and have no difficulty establishing the thermodynamics of a five, or, for that matter, a single particle system.

It is interesting to note that the MDB particles behave as "Fermions" that is, like electrons, in that two particles can not occupy the same quantum level. The diameter of the neck of the bottle imposes a "Pauli Exclusion Principle" on the system.

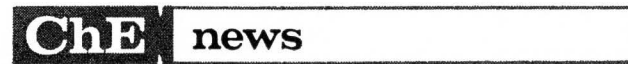
AND IN CONCLUSION: THE MAXWELL DEMON

THE READER MAY have wondered how does one get the bottle back to the state of Figure (1) in preparation for a repeat demonstration. One could, of course, make repeated trials and trust to chance that the initial state would be restored. But how much nicer it would be to call upon Maxwell's Demon to perform the separation for us. It was alleged at the beginning of this article that the Demon is sealed within the bottle. To demonstrate that this is no empty allegation, and to invoke the Demon, one grasps the flask at the base of the neck in a neck-up position as in Figure 2 and swirls its contents using a circular wrist motion. Then the flask is turned to the neckdown position while the swirling is continued so that the spheres are held in the body of the flask by centrifugal force. On reducing the rate of swirling, the spheres rolling inside the flask body follow a spiral path into the neck with the black spheres falling into the neck before the white ones. The "Demon," or whatever other name one may wish to give this gentle deception, lies in the black spheres which are constructed so

as to lose momentum rapidly. A "demon" design that has worked satisfactorily is shown in Figure 8. Commercially bottled demons are available, (3). □

REFERENCES

1. Sussman, M. V., J. Chem. Educ. 40, 49 (1963).
2. Sussman, M. V., "Elementary General Thermodynamics" p. 233 Addison Wesley, 1972.
3. Sargent, Welch Scientific Co. Cat. No. 1710C 7300 N. Linder Avenue, Skokie, Illinois 60076.
4. Urbain, G. and Boll, M., "La Science" p. 372-3, Librairie Larousse, Paris (1934).
5. Tribus, M., "Thermostatistics and Thermodynamics" Van Nostrand, 1961.



ENVIRONMENTAL CONFERENCE AT WORLD'S FAIR

Washington State University will sponsor a three-day conference, "Environment and the Economy: Exploring the Tradeoffs," in Spokane September 5-7 in cooperation with the world environmental fair, EXPO '74.

Program chairman for the conference is Dr. Joseph Brink, chairman of the WSU Chemical Engineering Department. The meeting will be managed by the Engineering Extension Service.

According to Brink, the agenda will focus on the interface between the environment and the economy, the benefits and limits of growth, land use, costs of pollution control, the political framework as it relates to energy, resources, and recycling.

Among the featured speakers will be Donald P. Hodel, Bonneville Power administrator; Dr. Eric Farber, solar energy researcher, University of Florida; Brock Evans, Sierra Club, Washington D.C.; Dr. John McKetta, chemical engineer, University of Texas; and Dr. Roger Cortesi, director of the Washington Environmental Research Center of the Environmental Protection Agency. □