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ChE book reviews

Mathematics Applied to Deterministic Problems in the Natural Sciences

By C. C. Lin and L. A. Segel
MacMillan, New York, 1974

Reviewed by R. Aris, University of Minnesota

It is generally agreed that it is all very well to teach students the methods of solving different types of equation, but that it is much more difficult—yet even more essential—to teach them how to develop a mathematical model, to assure themselves of its reasonableness and to get as much insight into it as possible before trying to compute a solution of its equations. Of the available books on the methods of applied mathematics the one under review is unique in addressing the more difficult task without neglecting the easier. As an introduction to the craft, as well as to the skills, of the applied mathematician it is as distinguished and valuable as the distinction and accomplishments of its authors would, in any case, lead us to expect.

Like that well-known province of the Roman empire, "liber est omnis divsa in tres partes." The first part introduces the scope and range of applied mathematics in a dramatic fashion by outlining the way in which the physical descriptions of galactic structure and the chemostactic behavior of slime mold amoebae lead to challenging mathematical problems. Deterministic systems and the generation of ordinary differential equations and random processes and their connection with partial differential equations are the subjects of the next two chapters and Fourier analysis, illustrated by problems of heat conduction, that of the two that follow. These five chapters provide a survey of the interaction of mathematics with physical phenomena for, besides those mentioned above, planetary orbits, the pendulum, Brownian motion, coagulation, twisted beams and DNA molecules all come into the picture. Nor are the mathematical notions confined to the pedestrian, for the authors do not hesitate to take up Poincaré's perturbation theory of periodic orbits and allude to the Gibbs phenomenon.

The second part of the book would be called, in the argot of our day, "very unique" for here some of the fundamental modes of applied mathematical thinking are explained in detail. Indeed the detail is often "painful,' not in the contemporary connotation but in the older and more honorable sence of 'painstaking.' I know of no other place where the beginner and proficient alike will find a systematic account of the way in which the model and its equations should be handled and of how intelligent simplifications, dimensional analysis and the understanding of scale can be used to bring the problem into its most responsive form. This is the foreplay of mathematical analysis and, as might be anticipated by analogy, calls for sensitivity and intellectual tact. These techniques are admirably illustrated by formulating and solving a problem in osmotically driven flow. But the second part also contains a chapter on regular perturbations and an introduction to singular perturbation theory that is a model of all that an elementary exposition of a deep subject should be. Chapter 10 takes this exposition further by treating Michaelis-Mention (or Langmuir-Hinshelwood-Hougen-Watson, not to mention Briggs and Haldane) kinetics in detail. Finally the phase plane, multiple scale expansions and linearization are introduced in connection with the simple pendulum.

After the virtuosity of this second movement, the third movement (just to mix my metaphors absolutely and uniformly) is relatively traditional. It provides an introduction to the theory of continuous fields and their associated partial differential equations, considering first the elastic vibrations of a bar, then continuum mechanics and inviscid flow and, finally, potential theory with an acoustical example. In a second volume we are promised more continuum mechanics, with Cartesian tensors viscous flow and elasticity, dispersive wave theory and variational methods.

In short it is a book that can be recommended without reservation both for its style and content. It can be recommended for chemical engineering courses; for the chemical engineering student—the least parochial of the engineering family—will welcome the catholicity of example which the chemical engineering teacher will find much instruction in working up further examples of his own.

An Introduction to Nonlinear Continuum Thermodynamics

by Gianni Astarita
Societa Editrice di Chimica, 133 pp.

Reviewed by Martin Feinberg,
University of Rochester

Since the turn of the century anyone who has set pen to paper in an attempt to advance thermodynamics has come under attack from one quarter or another, and the only thing upon which we all agree is that Gibbs was a very smart fellow. So, not knowing what to make of the battles raging around us, we opt for neutrality: we confine our teaching to the substance and style of 19th century thermodynamics. Although this course of action has served us reasonably well and, incidently, lends the subject an undeniable charm, at some point we must ask if such a state of affairs is to prevail forever.

It might be argued that, before we commit our classrooms to anything new, we ought to sit back and wait until the relative merits of various 20th century theories are settled by experiment. Well, I don't think things work that way for a subject as broad in scope as thermodynamics. What happens, I suppose, is that a theory is offered by Professors A,B, and C, is learned and found compelling by Professors X,Y, and Z, who in turn teach the theory to students, write textbooks, try their best to make converts of colleagues, and so on. If the theory has appeal and/or the political climate of the day is favorable, it penetrates into and diffuses through the mainstream of scientific thinking and ultimately flourishes if, in a sense difficult to make precise, that theory is "successful" in applications.

In 1963 Bernard Coleman and Walter Noll published a paper[1] which articulated a simple, yet powerful, line of thermodynamic reasoning based upon the Clausius-Duhem inequality and, by way of example, demonstrated how that line renders results for familiar classes of materials—e.g., linearly viscous fluids which are Fourier heat conductors. In 1964 Coleman published a remarkable paper[2] in which he used the methods proposed a year earlier to deduce results for materials with fading memory (e.g. polymer solutions and melts). Since then the theory has been explored and used extensively by others (notably Gurtin) in an explosion of papers generally endowed with high technical excellence. One can never be certain of these things, but I believe the body of theory precipitated by the Coleman-Noll paper of 1963 will, in fact, be "successful" and will come to play a permanent role in the way chemical engineers think about thermodynamics.

If this is so, then Professor Astarita's monograph must certainly be regarded as an important step in the coming assimilation process. Although it is not the first volume which describes modern methods based upon the Clausius-Duhem inequality, it is, I believe, the first written by a chemical engineer with other chemical engineers predominantly in mind. As such, the book is likely to be a critical, if not decisive, factor in the manner with which our colleagues and students respond to the theory, at least in the immediate future.

Before I discuss the monograph in detail, let me state my own biases plainly. I admire the work Astarita admires, probably for the same reasons. Plausible premises are stated clearly at the outset (to be accepted or rejected as one sees fit) and conclusions are drawn from these using standards of logic, rigor, and linguistic precision normally insisted upon in all other areas of science we deem

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