

1976 Award Lecture

THE ROLE OF WAVES IN TWO PHASE FLOW: SOME NEW UNDERSTANDINGS

This paper is the ASEE-CED Award Lecture which was presented by this year's lecturer, Abraham E. Dukler, at the 1976 ASEE meeting at University of Tennessee, Knoxville, Tennessee.

Dr. Dukler graduated from Yale in 1945 and joined the Rohm and Haas Company in Philadelphia as a development engineer. In 1948 he entered the graduate program at the University of Delaware from which he received the M.S. and Ph.D. degrees. He was employed by Shell Oil Company as a research engineer in 1950 leaving there in 1952 to help start the new Department of Chemical Engineering at the University of Houston. He served in the ranks of assistant to full professor and chairman of the department. In the fall of 1976 he assumed the position of Dean of Engineering at Houston.

Dukler's research studies have centered on the flow mechanics and transport processes associated with gas-liquid systems. This work has included modelling various aspects of this complex phenomena combined with development of new measuring methods and extensive experimental measurements. A central theme in this work has been the generation and validation of models which are based on physically realistic approximations to the flow but which are simple enough to be of use to the industrial designer of two phase flow process equipment. The research has been supported by over \$1.5 million of grants from various federal agencies and industry and the results have appeared in over 45 papers.

At Yale Dr. Dukler was an Alfred Noyes Scholar and at Delaware he held the Shell and Research Corporation Fellowships. In 1967 he received a National Science Foundation Senior Fellowship Award and in 1970 the AIChE Alpha Chi Sigma Award in Chemical Engineering Research. He is a consultant to a number of companies as well as Federal and State agencies and was one of the organizers of the AIChE Design Institute for Multiphase Processing.



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INTRODUCTION

RESEARCH IN GAS-LIQUID flows has flourished over the past 35 years. During that time over 7,000 papers and reports have appeared! It would be satisfying to be able to report that as a result of all this work there exists even a phenomenological understanding of just one aspect of gas-liquid flow (say, momentum transfer) equivalent to Prandtl's mixing length theory for turbulent single phase flow. But such understanding is not yet at hand, despite the fact that Prandtl had only 1/50th this number of papers on which to draw. Two factors have contributed to this plethora of publication.

1. Two phase flow exists in a wide variety of practical operating situations of importance to government and industry programs.¹ These groups have provided financial support for research with the objective of obtaining information to insure safer operation of processes or more economical and reliable designs. In the absence of generalizing principles, a great many studies have been carried out on narrow parts of the problem which would

not have been necessary had the principles been understood.

- R. The field is rich in challenging fundamental problems, the solutions to which could find rapid application to practice.

This is, of course, an ideal situation for nurturing engineering research: the identification of need for improved designs, the definition of good fundamental research problems embedded in the design tasks and the availability of funding.

Why is it then that under this fruitful environment even first order generalizations have been so elusive? In gas-liquid flow the manner in which the two phases distribute in the pipe changes with flow rates, fluid properties, pipe size and inclinations. Thus, there are a variety of complicated boundary conditions associated with the location of the interface, its shape and motion. These boundary conditions vary with the operating conditions mentioned above, cannot be independently specified and are, in general, not known. As a demonstration of this problem, refer to Figure 1 which shows the "flow regimes" observed for air-water flowing in a 2-inch horizontal pipe. At low liquid and gas rates the liquid flows as a stratified layer with a smooth interface. An approach to modelling transport with this phase distribution is easy to visualize once we know that stratified configuration exists (Taitel & Dukler, 1976A). At a higher gas rate the liquid is stratified but the interface is wavy. The boundary conditions which are controlled by wave shape and motion are not understood. At still higher gas rates the liquid wraps around the fall and flows as an eccentric annulus with the gas flowing in the core. Again the interface is wavy and in this case droplets flow with the gas. Defining the boundary conditions requires (1) knowing when annular flow exists (2) given that it does exist, determining how the liquid flow distributes between drops and film and (3) finding the shape and motion of the wavy interface. At different liquid and gas rates slug flow is observed

**Chemical and Petroleum Processing: Reboilers, condensers, gas-liquid flow reactors, trickle bed contactors, absorbers.*

Production of Oil, Gas and Geothermal Steam: Gas lifts, offshore pipelines and gathering systems, geothermal wells.

Nuclear Power: Emergency core cooling systems for pressurized water (PWR) and boiling water (BWR) reactors.

Aerospace: Film cooling of high performance jet engines.

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(high velocity liquid slugs flow down the pipe followed by a wavy stratified liquid film over which gas flows). At high liquid rates the gas distributes in a continuous liquid phase. The velocity at which these transitions take place will differ with pipe size, inclination and fluid properties. Certain additional flow regimes are observed for vertical upflow or downflow not seen in the horizontal configuration. It thus becomes clear that a root cause for the difficulties in modelling two phase flows is the problem of predicting the phase distributions or the location of liquid and gas interfaces, given all the operating conditions. It is only very recently that this has been accomplished for horizontal pipes (Taitel & Dukler, 1976B) and completely satisfactory models for vertical upward flow are yet to appear.

WAVES ON THE INTERFACE AND THEIR EFFECT

OVER A WIDE RANGE of flow rate space the liquid-gas interface is continuous and is

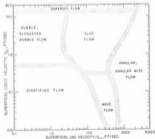


FIGURE 1. Flow Regime Transitions Air-Water in a Horizontal 1" i. d. Pipe.

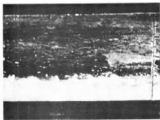


FIGURE 2. Stratified Flow with a Wavy Interface.

covered with waves. For example, Figure 2 shows a stratified wavy liquid with concurrent air flow; Figure 3 is liquid falling as a thin film down a vertical plate in the absence of gas flow; Figure 4 displays an axial view of upward annular flow of steam and water. In all cases the interface is shown to be covered with a well developed wave structure. Now the question arises naturally: What is the role of the waves in the transport of heat, mass or momentum in the gas and liquid phases? If the effects can be shown to be negligible then, aside from theoretical interest in the wavy motion, one can proceed to model these transfer processes ignoring the presence of the waves. However, evidence is to the contrary:

Data for pressure drop in the gas phase during concurrent downward isothermal flow of water in a film with steam in the core appear in Figure 5 (Dukler & Elliott, 1965). The flow

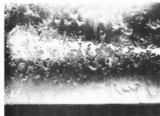


FIGURE 3. Waves on a Falling Liquid Film.

is in a 2-inch i.d. by 16-foot long smooth copper tube under vacuum. The vacuum level at the exit was held at 25 inches of mercury. This flow simulates conditions in a falling film evaporator of a type which has been investigated for water desalination. The broken, lowest, curve represents data obtained in the absence of water flow. The large increases in pressure drop (thus, in rate of radial momentum transfer) in the gas phase is apparent as water in the form of a wall film is added. For example, at a steam rate of 100 lb/hr the pressure drop doubles when 600 lb/hr of liquid flows on the wall. This is a very low liquid rate indeed for a 2-inch pipe and measurements show that the area occupied for liquid flow is negligible. So, the enhanced momentum transfer results not from the increased velocity of the gas in the presence of liquid, but is connected in some way to the presence of this mobile interface.

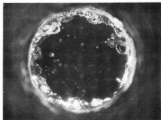


FIGURE 4. Waves on a Rising Liquid Film.

Figure 6 shows some classical data for mass transfer in the liquid phase (Emmert & Pigford, 1954). Experiments were made in situations where the resistance to mass transfer in the gas was negligible. Note that the ordinate varies *inversely* as the mass transfer coefficient. Experiments were conducted with falling liquid films in the presence of natural waves of the type shown above and also with these waves suppressed using surface active agents. It is evident that the rate of mass transfer in the liquid phase is enhanced by a factor of 2-3 in the presence of waves.

Similar evidence is presented in Figure 7 (Kafesjian, et al, 1961) for the effect of waves on rate of transfer in the gas phase. Shown are the analyses of data from several sources for the

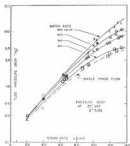


FIGURE 5. The Effect of Waves on Momentum Transfer in the Gas.

evaporation of water from falling films into flowing air. Of course, such a system exhibits no resistance to transfer in the liquid, and in that sense is the inverse of the situation displayed in Figure 6. At any fixed gas phase Reynolds number, increasing the liquid flow from a Reynolds number of 40 where the wave motion is weak to 800 where a well-developed wave structure exists results in a change in transfer rate in the gas phase of about 50-75%. Since changes in the velocity distribution in the liquid phase can have no effect, one must conclude that in some way the presence of a wavy interface is the cause of this

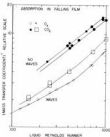


FIGURE 6. The Effect of Waves on Mass Transfer in the Liquid.

observed result. In the calculation of these transfer coefficients, the surface area of the undisturbed film was used so that additional transfer area associated with the waves could contribute to the apparent increase. But it seems to have been conclusively demonstrated (Poetalaki, 1964) that the increase in area is small compared to the increase in coefficient.

A more quantitative description of these waves can be found in Figure 8. These are time traces of the height of the interface for a thin water film falling down the inside of a 2-inch i.d. smooth vertical tube. The measurements were based on the use of changes in electrical conductance which take place as the film thickness varies between two closely spaced probes mounted flush with the wall (Chu & Dukler, 1974). The most

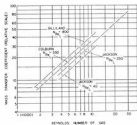


FIGURE 7. The Effect of Waves on Mass Transfer in the Gas.

striking observation is the fact that there appear to be two classes of waves; very large waves exist which are separated by a very thin substrate and which show randomness in separation time and amplitude. The substrate and the large waves are covered by a smaller, more regular wave structure. This existence of a two-wave structure suggests that characterizing each of these classes of waves could provide insights into the role each class plays in the processes of transport.

What are these characteristics? Analyses of the amplitude vs time traces from multiple locations in the direction of flow using techniques of random signal analysis provided the needed information of which the results of Figure 9 are representative. This profile shown is very much

out of proportion and it is of some use to see this in perspective. Scale the substrate with a thickness of one inch. The small waves display an amplitude of about 1/10 inch but are 2 feet long. The large waves project a distance of 6 inches from the substrate, each wave is 35 feet long and successive waves are separated by a distance of 70 feet! The properties of the two classes of waves are dramatically different in terms of amplitude to base thickness, celerities, wave lengths and all other characteristics shown in the accompanying table. It is also possible to con-

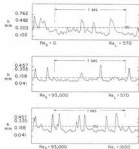


FIGURE 8. Time Traces of Film Thickness.

clude from this picture that the added transfer area contributed by the wavy surface is negligible because the surface slopes are so small. Considering the fact that two successive large waves 70 feet apart must communicate pressure and velocity information through a substrate only 1 inch thick strongly suggests that these "waves" are of isolated character. Perhaps they should be called liquid lumps rather than waves. Furthermore, these are mass-carrying or continuity waves. It is easy to observe that the flow rate leaving the bottom of the tube varies with time. Large instantaneous flows are seen which correspond to the passage of large lumps out of the tube.

In view of this picture, it is possible to deduce that transfer processes in the films must depend only on

- Substrate thickness
- Structure of the large waves
- Small wave structure

One concludes that transfer in the gas phase is controlled by the structure of the small waves . . . which suggests that the small waves act much as a roughness does to a moving gas phase in single phase flow.

Now it is of interest to explore what is known of each of these factors.

THE SUBSTRATE

WITH TIME TRACES of film height such as those of Figure 8 on magnetic tape, it is possible to develop software which provides a complete statistical description of each class of waves. For example, the probability density of the film thickness is shown in Figure 10 for a liquid Reynolds number of 4500 in the absence of gas flow. The circles represent the data. Decomposing the density into contributions due to the large and small waves gives the results shown (Chu & Dukler, 1974). Now it is possible to use the probability density of the substrate thickness to obtain other information of interest. For example, the variation of the mean thickness of the substrate, $\langle h_s \rangle$ (the first moment of the probability density) with gas and liquid rates appears in Figure 11. Since the waves on the substrate are of small amplitudes, as a first ap-

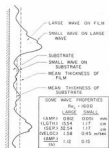


FIGURE 9. The Two Types of Waves.

proximation their presence can be ignored and from these mean thicknesses the flow rates in the substrate calculated. The surprising result appears in Figure 12. Note, for example, that when the total film Reynolds number is 1000 that in the Reynolds number in the substrate is only 70, or 7%! At a total liquid Reynolds number of 5000 the substrate flow, $Re_s = 400$.

This gives a rather different picture of flow and of the related process of mass transfer in a falling film. Figure 13 gives a revised view of mass transfer from the gas into the film. A large liquid lump, acting as a reservoir of fresh liquid, moves rapidly downward sweeping up liquid from

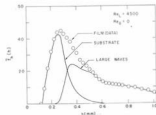


FIGURE 10. Contributions of the Two Types of Waves to the Probability Density of the Film Thickness.

the film in front and laying down a fresh film behind. The initial concentration distribution is uniform and the concentration distribution develops with distance along the film behind the lump. Thus, the rate of transfer is high just after the lump passes and decays with time. This continues until the next lump arrives and a fresh film is generated. Thus the process is one of renewal and unsteady mass transfer rather than the usual picture of steady state mass transfer.

The factors controlling transport in the liquid film consistent with this model must be

- Substrate thickness which determines the velocity and concentration distribution.
- Celerity of the large waves or lumps.
- Separation distance between these lumps.

A simple model for substrate thickness can be arrived at by the considerations shown in Figure 14. At the left is a sketch of the large lumps flowing down over the slow moving substrate at a celerity, C_s . As already shown, this

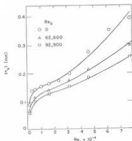


FIGURE 11. The Mean Film Thickness.

lump serves as a reservoir for the film so it should be roughly equivalent to the situation shown at the right. There we picture a plate drawn upward at the same velocity, C_s , from a reservoir of liquid. The thickness of the liquid film can be calculated from the theory of White and Tallmadge (1965) for laminar flow withdrawal and a comparison between values calculated this way. The measured substrate thickness is shown in Figure 15. Except at the higher substrate Reynolds numbers where the flow becomes turbulent, the agreement is quite satisfactory.

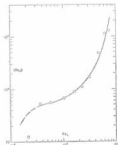


FIGURE 12. Comparison of Flow in the Substrate, $\langle Re_s \rangle$, with Total Liquid Flow, Re_L .

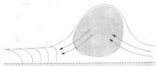


FIGURE 13. A Renewal Mechanism for Mass Transfer into the Film.

Thus, we conclude that the substrate thickness depends uniquely on the celerity of the large waves. Since as indicated above the mass transferred into the liquid film is determined by the substrate thickness as well as the celerity and separation distance of the large waves (the renewal rate) and since the substrate thickness depends only on the large wave celerity, it becomes apparent that the entire process of transfer in the liquid is controlled primarily by the large scale wave structure.

THE LARGE WAVES OR LUMPS

IT BECOMES OF interest to examine these large isolated lumps and to evaluate what can be predicted of their character. Figure 16 is an expanded view of a single lump with the measured time scale replaced by distance using the measured celerity. The amplitude scale is greatly expanded. Note that the slope seldom exceeds 5%. The amplitude is at least five times the substrate thickness, the characteristic base dimension is 150 times the amplitude and the front of the wave rises more steeply than does the back.

Procedures for modelling such waves frequently use Fourier series to describe the shape of the surface so it is of interest to examine the

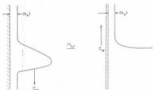


FIGURE 14. A Model for Determining Substrate Thickness, $\langle h_s \rangle$.

number of terms necessary to make a reasonable fit. The points in this figure result from the use of eight coefficients for each of the sine and cosine terms. Truncation after only 6 terms gives poor agreement.

Modelling of the wave motion has proceeded through a variety of approaches (Dukler, 1972). Attempts have been made to directly solve the equation of motion for the wavy film through use of small perturbation expansions for the stream function by making boundary layer type solutions of the equations and by the use of integral methods. In all of these approaches it is necessary at some stage to describe the shape of the surface

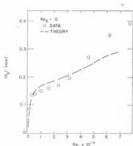


FIGURE 15. Substrate Thickness: Theory vs Experiment.

in the course of the mathematical procedure. This has been done in all cases using a Fourier series with unknown coefficients and the objective of the solution is to find these coefficients, thus making it possible to describe the wave amplitude and shape. Because of the complexity of the procedure, no one has successfully developed a method which includes more than three terms in the series.

A comparison of some of these attempts to model the waves with data (Chu & Dukler, 1975) appear in Figures 17-19. The comparison is made both for the large and small waves and it is seen that no theory adequately describes either wave type. The source of the difficulty rests in the attempt to use Fourier series to fit the waves. Figure 16 shows that 8 terms in the series are required. However, the theories developed to date become impossible complex when over 3 terms are used.

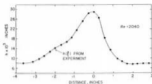


FIGURE 16. Fitting a Large Wave with a Fourier Series.

An alternative procedure is to use an orthogonal series which can fit the shape of the wave reasonably well using only a few terms. A Gram-Charlier series accomplishes this end. Its use for this purpose has recently been discussed (Chu, 1973) and further work is now in progress.

However, one can deduce some characteristics of these large waves by the use of integral balances. Consider a control volume consisting of the front of a large wave shown in Figure 20 which flows down a vertical surface in the absence of interfacial shear. The dimensions, h , h_{sub} and h_{max} , as well as the celerity are all measurable as discussed above. Permit the coordinate system to translate downward at the velocity C . The integral momentum and continuity equations are then

$$\sigma D[\rho(V_{sub}-C)h_s](V_{sub}-C) - \sigma D[\rho(V_{max}-C)h_{max}](V_{max}-C) - F_s + F_b = 0$$

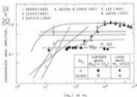


FIGURE 17. Comparison of Data with Various Theories for Wave Amplitude:

$\langle A \rangle$, $\langle A_s \rangle$ = Amplitude of Large, Small Waves
 $\langle h \rangle$, $\langle h_s \rangle$ = Thickness of Film, Substrate

$$[V_{sub}-C]h_s = (V_{max}-C)$$

Combining, these two equations yield a relationship for C .

$$C = V_{sub} + \left[\frac{g h_s \left(1 + \frac{h_{max}}{h_s} - \frac{\tau_w}{\rho g h_s} \right)^{1/2}}{1 - \frac{h_s}{h_{max}}} \right]^{1/2}$$

All the quantities in the square bracket are known except the wall shear, τ_w . In the absence of information we consider two possibilities: (1) $\tau_w = \rho g h_s$, and (2) $\tau_w = 0$. The first assumes that just after a wave overruns the substrate the velocity gradients near the wall are unchanged from those in the substrate and the second implies

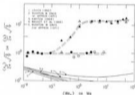


FIGURE 18. Comparison of Data with Various Theories for Wave Length:

λ , λ_s = length of Large, Small Waves.
 We , We_s = Weber Number of Film, Substrate

a mixing vortex and separation at the wall in the front of the wave. A comparison of the values calculated from theory using these two assumptions and the data appear in Figure 21. The reasonably good agreement with assumption (2) is clear and the flow pattern one could deduce is shown in Figure 22. Note how well this agrees with the earlier speculation on the renewal mechanism for mass transfer in the liquid film.

WAVES AND GAS TRANSPORT

THE LARGE WAVES control the process of transfer in the liquid as has been demonstrated above. Now it is of interest to determine the role they play in causing the observed increase in gas phase transport shown in Figure 5

Since the mass transferred into the liquid film is determined by the substrate thickness as well as the celerity and separation distance of the large waves and since the substrate thickness depends only on the large wave celerity, it becomes apparent that the entire process of transfer in the liquid is controlled primarily by the large scale wave structure.

structure of the small waves. This result seems to suggest that the small waves act much as a roughness does to a moving gas phase in single phase flow. However, the roughness is of a unique shape and the usual relative roughness correlations cannot be expected to apply.

With this understanding the approach to modelling the enhanced gas phase transfer seems clear. It is necessary first to model the size and shape of these small waves more successfully than has been done in the past. These small waves are also skewed so that a Fourier series is not a convenient tool to describe their shape. Perhaps a Gram-Charlier series can be used. Then the form drag associated with these special shapes must be determined, probably through the use of drag coefficients. It will then be possible to estimate the velocity distributions in the turbulent gas flow and to calculate the pressure gradients associated with those distributions. This work is now underway.

A SUMMARY

THE DIFFICULTY of modelling gas liquid flow results from the fact that the distribution of the two phases in the pipe are usually unknown and vary with flow rates, fluid properties, line size and inclination. The location, shape and motion of these interfaces determine the boundary conditions in the mathematical models.

Over much of the flow rate space the phases



FIGURE 21. Calculated vs Measured Celerity of the Large Waves for Using Two Models for Wall Shear.

distribute in such a way that axially continuous interfaces exist and these are usually covered with waves. It is demonstrated that the waves cause substantial increases in transport of mass, energy and momentum in both the gas and liquid phases.

The interface is covered by two type of waves. Large isolated waves or lumps of fluid carrying most of the mass of the system are separated by long substrates. A small wave structure exists on these substrates and on top of the large waves. The process of transfer in the liquid phase is shown to be controlled by the large wave structure while the enhanced transfer in the gas is due to the small waves.

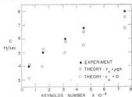


FIGURE 22. Flow Pattern Deduced from the Results of Figure 21.

Existing models for predicting this wave structure are shown to be inadequate to describe the waves observed in experiment and the reason for the failure of all the existing theories is shown to be the attempts to use Fourier series to describe the shape of the wave. An alternate approach is suggested.

Possible directions for predicting the relation between small wave structure and gas phase transport are discussed. □

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- No reduction in load is granted for advisement of college organizations.
- Most schools do not grant release time for new course and laboratory development.
- About one half the schools do not give release time for advisement of senior projects, M.S. and Doctoral Students. About 70% of these, however, are on a 6-9 hour teaching load rather than a 12 hour base load.
- Differential weighting of graduate versus undergraduate courses is not the rule in most schools. It can be included in an adequate overall reduction in load for the graduate program. In schools with graduate programs, the base load is 6-9 hours rather than 12 hours and a reduction has already been considered.
- No reduction in load is granted for college committee work since most replies seem to feel that committee assignments even out.
- An average correlation of FTE Faculty (F) and FTE Students (S) is

$$F = \frac{8}{4.641 + 0.00139 S}$$

TABLE 6. Auxiliary Departmental Personnel, (Mid America State Universities Association Peters, Eng. Ed. 61, No. 7, 840-843, 1971)

TYPE OF PERSONNEL	FTE FACULTY REQUIRED PER FTE OF EACH TYPE OF PERSONNEL
Teaching Assistants	2
Secretarial Assistance	
Lower Division	10
Upper Division	6
Master's Program	2
Doctoral Program	2
Recommended	4
Technician Assistance	
Lower Division	10
Upper Division	6
Master's Program	2
Doctoral Program	2
Recommended	4

ChE books received

GLOSSARY OF CHEMICAL TERMS

C. A. Hampel and G. G. Hawley

Van Nostrand Reinhold, 1976, 281 pp., \$14.95.

This glossary is a reference for students of chemistry and chemical engineering and professionals in other sciences who need basic definitions of chemical technology. It contains 2,000 entries including terms used in the several subdivisions of chemistry and chemical engineering and those in common usage in the chemical industries.

BOOK REVIEW

PETROLEUM AND THE CONTINENTAL SHELF OF NORTH WEST EUROPE—Volume 2 Environmental Protection

Edited by H. A. Cole,

Halsted Press, 1975, 126 pages.

Reviewed by James D. Wall, HYDROCARBON PROCESSING, Houston, Texas

This work is a compilation of articles and floor discussion from a meeting involving geologists associated with the North Sea. Thirteen articles discuss definition of the pollution problem in producing oil offshore, the general effects of oil pollution on elements of the environment and isolated requirements for control involving political and monitoring considerations.

The work is disappointing for those familiar with the oil industry and the environment. It suffers from lack of depth in review for those familiar with the subjects. Particularly does it suffer from lack of significant association to the problems in the North Sea. Most of the work could have been written for any offshore operation or any oil-water situation.

For those unfamiliar with oil production or environmental protection, the work does give a review of part of the data such that an opinion could be developed relative to the significance of the problems encountered. □

DUKLER: Role of Waves

Continued from page 117.

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