ChE classroom

THE ANALOGY BETWEEN FLUID FLOW AND ELECTRIC CIRCUITRY

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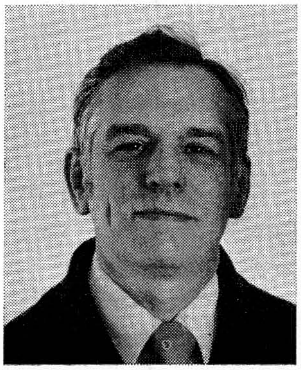
THE BEHAVIOR OF fluids in pipe networks resembles that of currents in electric circuits. Of course, in the area of Process Control, extensive use has been made of the parallels between control systems and electric circuits. Also, the analogy between thermal conductivity and electric conductivity is often invoked to introduce or reinforce concepts of conductive heat transfer. However, none of the texts usually used by chemical engineers appears to have used electric circuitry as a tool for teaching fluid flow.

Most college students encounter fluid flow for the first time as sophomores or juniors, long after they have been introduced to Ohm's and Kirchhoff's Laws. In fact, many have had a multiple exposure to the concepts of electrical circuitry in high school and in freshman college physics.

The analogy is most useful in dealing with pipe networks with laminar flow, but it has some advantage even in turbulent flow. A factor favoring the use of the analogy is the growing adoption of SI units which make the parallel between mechanical and electric systems more obvious. One common misunderstanding which the analogy helps to clear up comes from the usual form of the mechanical energy balance for a flowing fluid. The friction term in energy per unit of flowing mass often becomes identified by students as a resistance whereas it is, in fact, in the nature of a potential. Perhaps because engineers often express the friction term as "head" in feet or meters (where force and mass units have been cancelled out), the image of a barrier or resistance seems to occur naturally.

In Table 1, the identification of kg in a mass flow system is made with coulombs in the electric circuit. When the familiar volts, ohms, and amperes are expressed as joules, coulombs, and seconds, the analogy becomes more apparent. The usual mechanical energy balance for a fluid flow process is:



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$$\frac{\Delta V^2}{2g_c} + \frac{\Delta p}{\rho} + \frac{g\Delta z}{g_c} + h_f + \frac{W_t}{\eta_t} - W_p\eta_p = 0 \qquad (1)$$

where the last two terms represent contributions of turbines and pumps. The term h_f represents frictional dissipation of energy. In the absence of pumps or turbines, and with negligible changes in kinetic energy:

$$-h_f = \frac{\Delta p}{\rho} + \frac{g\Delta z}{g_c} \qquad (2)$$

The identification of h_f as a potential rather than as a resistance should be obvious from equation 2, but, as previously noted, the common units may confuse some students.

BRANCHED FLOW

IN THE ELEMENTARY CASE (Table 2) of parallel resistances, almost every student has been told

TABLE I
The Fluid Flow—Electric Current Analogy

	FLUID FLOW	ELECTRIC CURRENT	EQUATION
Physical system:	Pipe: $\leftarrow \Delta p, \Delta z, h_f \rightarrow$ $\leftarrow L,D \rightarrow$	Resistor: $\leftarrow E \rightarrow$ $\leftarrow L,D \rightarrow$	
Flowing unit:	mass, kilogram	electricity, C	
Flow rate:	$\dot{m}$, kg/s	I, C/s (A)	
Potential:	h_f, J/kg	E, J/C (V = W/A)	
Resistivity: (Laminar flow)	$(32\mu)/(D^2\rho^2 g_c), \frac{\text{J-m-s}}{\text{kg}^2}$	$\rho', \frac{\text{J-m-s}}{\text{C}^2}$ (ohm•m)	(1-1)
Resistance:	$R_f = (32)\left[\frac{4L}{\pi D^2}\right]/(D^2\rho^2 g_c), \frac{\text{J-s}}{\text{kg}^2}$	$R = \rho' L/A = \rho'\left[\frac{4L}{\pi D^2}\right], \frac{\text{J-s}}{\text{C}^2}$ (ohm)	(1-2)
Ohm's Law:	$h_f = R_f(\dot{m})$	$E = R(I)$	(1-3)
Power:	$P = h_f(\dot{m}) = R_f(\dot{m})^2$	$P = E(I) = R(I)^2$	(1-4)

that an equivalent resistance, R_e, is given by the reciprocal of the sum of the reciprocals of the individual resistances R_1 and R_2. It is a simple consequence of Kirchhoff's laws. The potential E across each resistance is the same, but the total current, I, is given by the sum of the individual currents I_1 and I_2. Thus,

$$E = R_e I = R_1 I_1 = R_2 I_2 \text{ and } I = I_1 + I_2 \quad (3)$$

Rearrangement gives

$$\frac{I}{R_1} = \frac{I_1}{R_e} \text{ and } \frac{I}{R_2} = \frac{I_2}{R_e} \quad (4)$$

Combining equations 4 and 3 to eliminate currents gives

$$(R_e)^{-1} = (R_1)^{-1} + (R_2)^{-1} \quad (5a)$$

The general case for n resistances in parallel is

$$(R_e)^{-1} = \sum_n (R_i)^{-1} \quad (5b)$$

The power (energy/time) to cause the flow is given by the product of total flow and the friction term (equation 1-4, Table 1).

EXAMPLE OF BRANCHED, LAMINAR FLOW

STATEMENT: A stream of 18 m^3/hr is split into three pipes, A, B, and C, with diameters of 20, 30, and 40 mm respectively and lengths of 50 m each. What power is dissipated as friction?

Data: $\mu = 0.10$ Pa-s, i.e. (1 poise), $\rho = 1$ Mg/m^3, i.e. (1 g/cm^3)

Calculations:

$\dot{m} = 5.0$ kg/sec

Resistances calculated from equation 1-2 (Table 1):

$R_{fa} = 12.7$ kJ•s/kg^2

$R_{fb} = 2.51$ kJ•s/kg^2

$R_{fc} = 0.794$ kJ•s/kg^2

Equivalent resistance, R_{fe}, from equation 2-2 (Table 2):

$R_{fe} = 0.576$ kJ•s/kg^2

Potential, h_f, from equation 1-3 (Table 1):

$h_f = 0.576 \times 5.0 = 2.88$ kJ•/kg

Power, p, from equation 1-4 (Table 1):

$P = 2.88 \times 5.0 = 14.4$ kW

Individual streams calculated from

$\dot{m}_i = h_f/(R_{fi})$:

$\dot{m}_1 = 0.23$ kg/s,

$\dot{m}_2 = 1.15$ kg/s,

$\dot{m}_3 = 3.63$ kg/s

Individual Reynolds numbers from equation 3-2 (Table 3):

$(N_{re})_1 = 146$, $(N_{re})_2 = 488$, $(N_{re})_3 = 1155$

TABLE 2
Branched Flows

GENERAL CASE AT A JUNCTION:	FLUID FLOW	ELECTRIC CURRENT	EQ.
	$\dot{m} = \Sigma m_i$	$I = \Sigma I_i$	(2-1)
Equivalent Resistance:	$\frac{1}{R_{equiv}} = \Sigma \frac{1}{R_i}$		(2-2)

TURBULENT FLOW

THE RESISTIVITY IN turbulent flow varies with the flow rate (Table 3). In laminar flow, the resistivity is a function only of μ, ρ, and D. In turbulent flow, the friction factor f decreases in non-linear fashion as the Reynolds number increases. For smooth pipes at N_{re} above about $5x10^4$ the behavior is approximated by equation 3-2 (Table 3). A modified resistance M_f can be defined (Table 3) so as to be independent of flow rate. The analogy with equations 3, 4, and 5 can be extended to give:

$$h_f = M_{fe}(\dot{m})^{1.8} = M_{f1}(\dot{m}_1)^{1.8} = M_{f2}(\dot{m}_2)^{1.8} \quad (6)$$

for two pipes with modified resistances M_{f1} and M_{f2} and individual flows of m_1 and m_2, respectively. The result for the general case where i = 1, 2, etc. is:

$$(M_{fe})^{-0.556} = \Sigma(M_{fi})^{-0.556} \quad (7)$$

AN EXAMPLE OF BRANCHED, TURBULENT FLOW

STATEMENT: Same conditions as in previous, laminar case except that μ = 1.0 mPa•s (that is, 1.0 centipoise)

Calculations:

Modified resistances calculated from equation 3-3 (Table 3):

TABLE 3
Turbulent Flow

Resistance: $$R_f = h_f/\dot{m} = \frac{32 f \dot{m} L}{g_c \pi^2 D^5 \rho^2} \quad (3-1)$$

If $f = 0.046(N_{re})^{-0.2}$ and $N_{re} = (4\dot{m})/(\pi D \mu)$ (3-2)

Then $$R_f = \frac{0.1421(\mu)^{0.2} L}{g_c (D)^{4.8} \rho^2}(\dot{m})^{0.8} = M_f(\dot{m})^{0.8} \quad (3-3)$$

M_{fa} = 255.0 (J/kg) $(s/kg)^{1.8}$

M_{fb} = 36.4 (J/kg) $(s/kg)^{1.8}$

M_{fc} = 9.16 (J/kg) $(s/kg)^{1.8}$

Equivalent modified resistance from eq. 7:

M_{fe} = (same units)

Potential, h_f, from eq. 3-1, 3-3 (Table 3):

$h_f = 3.84 \times (5.0)^{1.8} = 69.6$ J/kg

Power, P, from equation 1-4 (Table 1):

P = 69.6 x 5.0 = 348 W

Individual streams calculated from

$\dot{m}_i = (h_f/M_{fi})^{0.556}$:

$\dot{m}_1 = 0.49$ kg/s

$\dot{m}_2 = 1.43$ kg/s

$\dot{m}_3 = 3.09$ kg/s

Individual Reynolds numbers from equation 3-2 (Table 3):

CONCLUSIONS

THE EMPHASIS HERE has been on the use of electric circuits as analogs in teaching concepts of pipe flow in networks. Extensive computer programs have evolved for handling complex circuits. These can be adapted for fluid-handling systems, also. □

GLOSSARY:

D, Diameter, m
E, Electric potential, volt (= J/C)
g, Acceleration due to gravity, 9.81 m/s^2
g_c, Proportionality constant, 1.00, dimensionless in SI system
h_f, Energy loss in pipe flow, J/kg
I, Electric current, ampere (C/s)
$\dot{m}$, Mass flow rate, kg/s
M_f, Modified resistance term, equat. 3-3, Table 3, (J/kg)$(s/kg)^{1.8}$
N_{re}, Reynolds number
Δp, Pressure drop, Pa
P, Power, W
R, Electric resistance, ohm (= $(J\text{-}s)/C^2$)
R_f, Fluid flow resistance, $(J\text{-}s)/kg^2$
$\overline{V}$, Fluid velocity (average), m/s
W_p, Energy supplied to system by pumps, J/kg
W_t, Energy taken from system by turbines, J/kg
Δz, Change in elevation, m.
η_p, Pump efficiency
η_t, Turbine efficiency
ρ, Density, kg/m^3
ρ', Electric resistivity, ohm-m
μ, Viscosity, Pa-s