

FIGURE 2A : AT THE BEGINNING OF IRREVERSIBLE FREE EXPANSION (II' STEP).

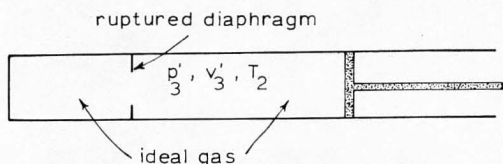


FIGURE 2B : AT THE CONCLUSION OF IRREVERSIBLE FREE EXPANSION (II' STEP).

Step II:

Heat absorbed, $Q_{II} = RT_2 \ln (v_3/v_2) = Q_2$
 Entropy change, $\Delta S_{II} = R \ln (v_3/v_2)$
 Work done, $W_{II} = Q_{II} = RT_2 \ln (v_3/v_2)$

Step II':

Heat absorbed, $Q_{II'} = 0$; Work done, $W_{II'} = 0$
 The entropy change can be calculated by assuming a reversible isothermal process between thermodynamic states 3 and 3'. $\Delta S_{II'} = R \ln (v_3'/v_3)$

Step III':

Heat absorbed, $Q_{III'} = 0$;
 Entropy change, $\Delta S_{III'} = 0$;
 Work done, $W_{III'} = -\Delta U_{III'} = C_v(T_2 - T_1)$

Step IV:

Heat absorbed, $Q_{IV} = RT_1 \ln v_1/v_4' = Q_1'$
 Entropy change, $\Delta S_{IV} = \ln v_1/v_4'$;
 Work done, $W_{IV} = Q_{IV} = RT_1 \ln v_1/v_4'$

For the entire cyclic process:

$$W = \sum W_i = Q_2 + Q_1' = RT_2 \ln(v_3/v_2) + RT_1 \ln(v_1/v_4') \quad (3)$$

and

$$\Delta S_{cycle} = R \ln (v_3/v_2) + R \ln (v_1/v_4') + R \ln (v_3'/v_3) \quad (4)$$

By considering the two reversible adiabatic steps, viz., I and III', we can show that

$$v_3'/v_2 = v_4'/v_1 \quad (5)$$

Upon substituting equation (5) into equation (4) we shall obtain:

$$\Delta S_{cycle} = R \ln 1 = 0 \quad (6)$$

By combining equations (3) and (5) we find:

$$W = [R \ln(v_3/v_2)] [T_2 - T_1] - RT_1 \ln (v_3'/v_3) \quad (7)$$

The efficiency of the modified cycle is given by:

$$\eta' = 1 - \frac{T_1}{T_2} - \frac{RT_1 \ln (v_3'/v_3)}{RT_2 \ln (v_3/v_2)} \quad (8)$$

Since v_3' is greater than either v_3 or v_2 , the third term on the right side is a negative quantity. Hence it follows that η' is less than Carnot efficiency η . The latter is defined as follows:

$$\eta = 1 - \frac{T_1}{T_2} \quad (9)$$

The diminution in the efficiency of the modified cycle is directly linked to the presence of the irreversible step II',

The greater the departure of v_3' from v_3 the larger the decrease in the efficiency.

In order to determine the entropy change of the universe, due to the modified cyclic process, we need to calculate the entropy changes of the surroundings. The hot reservoir "lost" an amount of thermal energy equal to $-Q_2$. The cold reservoir "gained" an amount of heat which is equal to $-Q_1'$.

$$\text{Entropy change of hot reservoir} = -Q_2/T_2 = R \ln (v_3/v_2) \quad (10)$$

$$\text{Entropy change of cold reservoir} = -Q_1'/T_1 = -R \ln (v_1/v_4') \quad (11)$$

Summation yields:

$$\Delta S_{surroundings} = -R \ln (v_3/v_2) - R \ln (v_1/v_4') \quad (12)$$

Upon combining equation (12) with equation (5), we shall find that:

$$\Delta S_{surroundings} = R \ln (v_3'/v_3) \quad (13)$$

Since $v_3' > v_3$, this quantity is positive. The entropy change of the universe is given by:

$$\Delta S_{universe} = 0 + R \ln (v_3'/v_3) \quad (14)$$

CONCLUSIONS

In conclusion it may be said that the presence of an irreversible step in an otherwise reversible cyclic process will cause a diminution of the efficiency of a heat engine operating on that cycle; furthermore, there will be an augmentation of the entropy of the universe each time the cycle has been completed.

Thus, unlike the strict Carnot cycle, the modified cycle causes an increase in the entropy of the universe.

REFERENCES

1. Darken, L. S. and R. W. Gurry, "Physical Chemistry of Metals," McGraw-Hill Book Company, Inc., New York, 1953.
2. Sears, F. W. and G. L. Salinger, "Thermodynamics, Kinetic Theory and Statistical Thermodynamics," 3rd edn., Addison-Wesley Publishing Company, Reading, Mass., 1975.

ChE conferences

The Second International Symposium on Innovative Numerical Analysis in Applied Engineering Science will be held at the Ecole Polytechnique in Montreal, Canada, from June 16-20, 1980. Over 100 papers, covering a range of disciplines from solid mechanics (elasticity, fracture, visco-elasticity, elasto-plasticity) and structure to fluid mechanics (aerodynamics, free surface flow, acoustics) and fluid-structure interaction to diffusion, electromagnetic and biological problems, have been accepted. An equally wide range of numerical techniques (finite differences, finite elements, boundary integral equations, etc.) will be represented, with all papers published in proceedings to be available at the meeting. Keynote addresses are scheduled to be presented by O. Zienkiewicz of the U.K., J. Hess, G. Fix of the U.S.A., J. Nedelec of France and M. Fortin of Canada. Further information may be obtained from A.A. Lakis, Mechanical Engineering Dept., Ecole Polytechnique de Montreal, C.P. 6079, Station A., Montreal, Quebec, Canada H3C 3A7.