WHAT IS PROBLEM SOLVING?

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PROBLEM SOLVING MAY be defined as an activity whereby a best value is determined for an unknown, subject to a specific set of conditions. In this paper we provide an overview of the four component parts of the activity of problem solving (see Figure 1). These are the types of problems to be solved, the necessary prerequisites, the strategies, the heuristics or hints, and the elements used in applying the strategies.

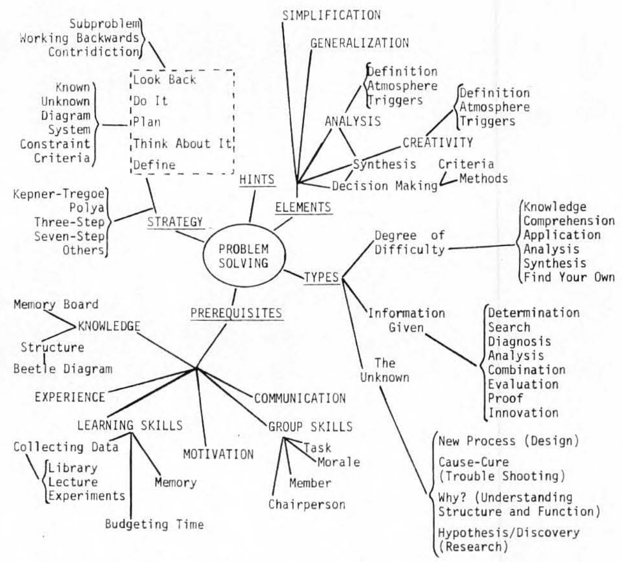

FIGURE 1. An Analysis of Problem Solving

PROBLEM TYPES

IF WE ARE GOING to solve problems, then there must be some problems to solve. We can classify the problems according to three different methods: what the unknown is, the difficulty, and what information is initially given. One basis for classification is "What the unknown is." For this basis there are four types of problems. **In the design type of problem**, the unknown is a new process, procedure or idea, e.g., design a plant to produce 30,000 tons/annum of fertilizer. The result will be

- **A sequence of equipment (with size, construction details specified) and interconnecting piping and conveyors that . . .**
- **Yields the specification quality and amount of product . . .**
- **Subject to the local site constraints . . .**
- **And satisfying the criteria of financial, technical, ethical, environmental and safety criteria. Some example texts that emphasize this type of problem include: Rudd, Powers and Siirola (1972); Dixon (1966); Buhl (1960); Sherwood (1963) and Bodman (1968).**

For the trouble shooting, diagnostic, clinical or critical instant problem the unknown is a cause of trouble that needs to be corrected and, in the long term, prevented from occurring again. e.g. "The product is black; it should be white. Get it fixed." There are very few examples published centering around this type of problem. Those that are available include AIChE "Engineering Case Problems" (1967), Doig (1977), and Kepner and Tregoe (1965). One discipline which is dominated by critical instant problems is the medical profession. Some elaboration on how to solve this type of problem in this context is given by Barrows and Tamblyn (1976) and Barrows and Bennett (1972).

In the understanding, anticipating or simulation type of problem, the unknown is why an existing process works. From this knowledge one can anticipate future bottlenecks or difficulties. Some work has been done on this type of problem with the resources being King (1967), Woods (1969) and Crowe et al (1971).

The final type of problem in **this classification has an hypothesis** as the unknown. Often this activity is called discovery or research and development. The text by Wilson (1952) is an illustration of this type of problem.

Another classification of the problem type would be according to how **difficult the problem is.** This classification is challenging to apply because "difficulty" is only seen in the eyes of the problem solver and not the poser of the problem. Nevertheless one could hypothesize that depending upon what the problem solver is asked to do, one could then hopefully identify degrees of difficulties that both the poser of the problem and the solver agree upon. From this standpoint, we use Bloom's (1956) Taxonomy as the over-riding principle to classify this type of problem. This classification is:

TYPE 1 — COMPREHENSION: Given a familiar situation (i.e., a law, principle, subject area under discussion), recall information and use it to solve a recognizable problem.

EXAMPLE: Given the statement, all the pertinent data and the necessary theory. Given Newton's second law, calculate the acceleration of a 360 g mass subjected to a force of 2 Newtons. Neglect all friction.

TYPE 2 — APPLICATION: Given a new situation (i.e., where the law, principle, subject area under discussion are not identified), identify memorized knowledge that applies and solve the problem as though it were a Type 1 problem.

EXAMPLE: Given the statement, all the pertinent data but not the necessary theory. Calculate the acceleration of a 360 g mass subjected to a force of 2 Newtons. Neglect all friction. (This example may be too easy because most readers will immediately recognize that we should use Newton's second law. However, this illustrates the difference.)

TYPE 3 — ANALYSIS: Identify relationships, omissions, parts, pieces; recognize unstated assumptions; distinguish fact from opinion, conclusions from evidence; recognize what particulars are relevant to the validation of a judgment; detect fallacies in logic, missing information, incorrectly defined problems. Translate a real world problem into a mathematical model problem. Then, when the analysis is complete, this type becomes a Type 2 problem.

EXAMPLE: Given the statement, some data but not all the pertinent data and not the necessary theory. Calculate the acceleration of a Speedster Automobile if it has an 80 hp motor.

In the understanding, anticipation or simulation type of problem, the unknown is why an existing process works.

TYPE 4: SYNTHESIS: Creation of alternatives to satisfy a given criterion. The integration of the parts to form a whole. Once this is completed this becomes a Type 3 problem.

EXAMPLE: Given the statement, but data and theory are missing. Design a comfortable car that will have an acceleration of 10 ft/sec^2.

TYPE 5 — FIND YOUR OWN: Analysis of a situation to identify a worthy or worthwhile problem. Once this is completed this becomes a Type 4 problem.

EXAMPLE: Given a situation but no statement, data or theory. Given today's transportation problems, what problem, if any, exists?

A third classification, devised by Fuller (1974) is based on what principal parts of information are given initially to the problem solver. He classifies the given information differently from that based on Bloom's taxonomy. On this basis he identifies eight different classes of problems.

NECESSARY PREREQUISITES

NOW THAT WE HAVE looked at the types of problems that one is trying to solve, we might look at what sort of prerequisite information is useful in solving problems. Here there are a variety of sub-categories.

A first prerequisite might be that the solver has the necessary knowledge, attitudes and manual skills. These prerequisite skills are vital; indeed some believe that one's inability to solve problems occurs because the problem solver just does not "understand the basic knowledge." We believe that there is much more to solving problems than basic knowledge, as this article attests, but at the same time we acknowledge the need for a sound understanding of the knowledge as a prerequisite. Some activities that have helped us to internalize basic knowledge have been:

- **To study how our memory works and try to use those ideas to improve our memory. Buzan (1975) has summarized this conveniently.**
- **To try various methods to discover the structure in a subject. We have tried different classification techniques—traditional point outline and the beetle diagram approach of Buzan.**
- **To try to identify the minimum number of basic laws and the relationships between those laws. Some of this we displayed on a 'memory board' illustrated in Figure 3 of Woods et al (1975). Brown (1977) requests that his students prepare a separate, personal memory board book.**

- **To use Larkin's (1975) checklist method to help us develop details about the major ideas.**
- **To try to identify cognitive learning preferences within ourselves. Most material presented in a lecture will have the cognitive preference of the instructor. A personal awareness will help us identify why we might be having difficulty understanding a given instructor or a given subject. Once this is identified, we can then seek or try to develop on our own the necessary resources, information and viewpoint. Although there are rather detailed methods available, as described, for example, by Hill (1969) and Hoogasian (1971), we only identified preferences for verbal, mathematical or pictorial viewpoints. Details of this are available elsewhere (see Moore et al (1977), Woods et al (1977a), Woods et al (1977b)).**

Next we need some experience which helps us to develop judgment. This is different from the cognitive skills in that this is a memorization of

A sixth prerequisite, required if group problem solving is used, is "the ability to work effectively in a group."

'how big is big' and 'how small is small.' Most experience factors come from the numerical data given in the various problems that we solve, the numerical answers we calculate and any data that we have to look up. To assist us, we prepare an experience board or summary sheet periodically throughout each year. We know that we need these numbers to allow us to make judgments and to solve approximations to the real problem during the "Think About It" stage.

The third prerequisite could be 'learning skills.' This is a deceptively simple idea; we need skill at planning time, obtaining information and organizing that information into an internalized structure of knowledge. Yet, we are beginning to suspect that how we store the knowledge and experience is a key factor in how well we can later retrieve the information to solve problems. A major portion of the freshman year was spent trying to improve learning skills. One skill that we find particularly challenging is how to take good lecture notes. We tend to copy only that material which is written on the board.

For example, one lecturer in freshman Chemistry said "Consider now how we calculate the bond energies. Such energies are very useful and important in Chemistry. We focus first on the enthalpy changes that occur when the bonds in a molecule are broken. For example, consider the following reaction: gaseous hydrogen chloride decomposes into gaseous hydrogen and gaseous chlorine." This instructor's board work was as follows.

"Bond Energies
Enthalpy changes
$HCl\ (g) \rightarrow H(g) + Cl\ (g)$**"**

All of us wrote down only what was on the board; several of us even forgot to note the gaseous condition for the components in the reaction. However, from an analysis such as this we could adjust to improve our skill at taking lecture notes.

To help us organize our time we made a detailed analysis of the total time available and the demands on that time. Then we prepared our own daily time schedules and long range calendars. We attempted to use Critical Path Methods and Gantt charts to help give us a sense of accomplishment. However, we found this latter method too complicated. The methods described under basic knowledge were also used to improve our learning skills. A required sophomore course on communication skills helped us to gain confidence in locating information in the library. To help us highlight our experience, and what we have learned about problem solving we try to complete a standard form at the completion of each problem. This form asks us to do three things:

- **Dream up a sample problem that you think you could solve based on the knowledge tested in this problem.**
- **What did you learn about problem solving from doing this problem?**
- **Experience factors. Record the experience factors you used or calculated in this problem.**

A fourth prerequisite is desire or motivation to solve the problem. Usually the requirement of a mark for an assignment is sufficient motivation to most students. However, the greatest hurdle is to get started on a "difficult" problem. Sometimes the hurdle is so great that we rationalize postponement. We have found that familiarity with a strategy for solving problems is sufficient to overcome this hurdle. For more details, see the student comments at the end of each problem solving presentation day as summarized in Woods (1977b) and our other papers, especially Moore et al (1977). When we have solved the problem, we must be able to communicate our results. A sixth prerequisite, required if group problem solving is used, is 'the ability to work effectively in a group.' This means skills as facilitators and a willingness for all in the group to develop task and group building attributes.

THE STRATEGIES

HOW DOES ONE SOLVE PROBLEMS? Some type of organized strategy, approach, or set of tactics is needed. Many strategies have been suggested.

One that we have become more familiar with is a five step strategy: define, think about it, plan, do it and look back. Other types include a four step, three step, and a seven step strategy. Many of these are reviewed by Woods (1977a). Others include Jones's (1974) three-six step strategy with matrix moves. These are:

1. **Analysis**
   1. **Brief issued**
   2. **Design situation explored**
   3. **Problem structure perceived and transformed**
   4. **Boundaries located sub-solutions described and conflicts identified**
2. **Synthesis**
   5. **Sub-solutions combined into alternative designs**
3. **Evaluation**
   6. **Alternative designs evaluated and final design selected.**

An intriguing feature of Jones' approach is that he allows one to move easily among the steps by

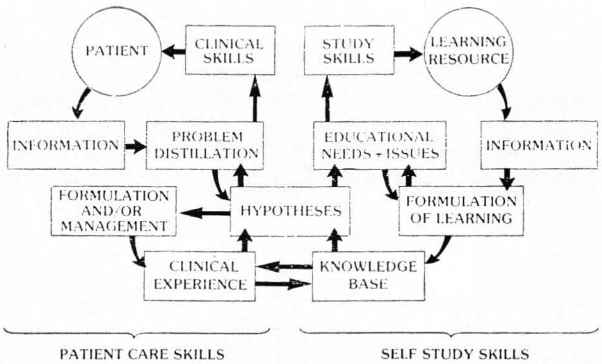

FIGURE 3. Strategy for Developing Patient Care and Self Study Skills

identifying techniques that can be used to go between any of the steps. Kepner and Tregoe (1965) (1973) have developed four sequential strategies to be applied to solving problems in industrial operations. These strategies are

- **Situation Analysis (SA) which is an attempt to break down a complex problem so as to decide which of the three following strategies should apply.**
- **Problem Analysis (PA) which is essentially to determine the cause.**
- **Decision Analysis (DA) which is to decide how to correct the cause.**
- **Potential Problem Analysis (PAA) which is how to prevent the cause, identified in step 2 (PA), from reoccurring.**

Barrows and Tamblyn (1977) have developed two interlinking strategies that solve clinical problems and develop self study skills. This is illustrated in Figure 3. Doig (1974) developed a detailed elaboration on how to apply Polya's four step approach to the selection and design of engineering equipment. An analysis of various strategies has been given by Sears (1977).

A fourth aspect to problem solving is what is used when we apply a strategy.

ELEMENTS

A FOURTH ASPECT to problem solving is what is used when we apply a strategy. In other words, there are certain skills we use over and over again when we go through the various steps in the strategy. These skills we call elements. There are four major elements: analysis, synthesis, generation and decision making. Some authors use these terms synonymously with "problem-solving." For example, Riggs (1968) uses the term decision-making to mean our word problem solving. Hence, to avoid ambiguity we define each of these terms in the present context.

Analysis: The act of dividing a whole into parts so that there is a meaningful relationship among the parts ("resolution into simple elements" Oxford) (synonyms: critical study, critique, logical study)

Our attitude should be critical, looking for subtle differences, careful scrutiny of something.
Major tools used during an analysis: logic
Procedure:

1 Identify the system to be analyzed.
2 Identify the objective of the analysis—what meaningful relationship are we seeking and how do we know when we achieve this?
3 Identify and compare the elements in the system to identify the basis for dividing the system into parts. (May need some creativity to do this)
4 Divide the system into parts.
5 Look back. (May need some creativity to do this)

Example: Which of the following numbers do not belong in this in this set? 3 5 7 9 11

Step 1. System of numbers.
Step 2. Meaningful—most numbers in this set have a common property; at least one does not contain that property.
Step 3. In looking at the set we can identify the following attributes:

3 — one digit, odd number, arabic, typed, integer, rational, prime, open, curved lines.
5 — one digit, odd number, arabic, typed, integer, rational, prime, open, combined straight and curved lines.
7 — one digit, odd number, arabic, typed, integer, rational, prime, open, straight lines.
9 — one digit, odd number, arabic, typed, integer, rational, not a prime number, closed, curved lines.
11 — two digit, odd number, arabic, typed, integer,

Heuristics or hints are general suggestions that lead to a successful solution to a problem or successful completion of one step in the problem solution.

rational, prime, open, straight lines.

Hence, all have common properties except 9 which is a non-prime number and 11 which has two digits instead of one.

Step 4. There are two answers. 11 does not belong on the basis of digits. 9 does not belong on the basis of prime numbers.

Step 5. Both answers seem reasonable.

Synthesis: The act of putting pieces together to form a whole in some unique way to satisfy a purpose (combination of elements, putting together) (synonyms: creation, creativity, design)

Creativity: the act of generating ideas. Synthesis is a combination of creativity and analysis. Creativity is used to get different ideas about how things might be put together; analysis is used to determine which is a unique idea satisfying the purpose. Hence, since analysis was discussed above, we emphasize the creative aspects of synthesis here.

Our attitude: should be imaginative, free from constraints, day dreaming, off-in-left-field.

Major tools used during synthesis: imagination

Procedure:

1 Identify the system to be studied.
2 Identify the objective of the synthesis—what unique way?
3 Create ideas via brainstorming, attribute listing, and triggers.
4 Evaluate the ideas by analysis.
5 Decide which ones meet the criteria.
6 Look back.

Hence, since synthesis is a combination of analysis, creativity and decision-making only creativity is the additional element needed.

Decision-making. Decision-making is the process whereby one of many possible actions, ideas, or objects is chosen as being the best. The process includes calculating the criteria, comparing and selecting the optimum to yield the "best." Although good decision-making requires that a good choice is made of the criteria, we do not include the activity of choosing the criteria as being a part of the decision-making process.

This activity is preceeded by analysis (to define the problem and choice and method of measurement of "best"), creativity (to generate ideas), and analysis of these ideas to develop alternatives that are feasible. Then, for each feasible idea, the criteria are calculated and compared to select the idea with the optimum value.

Generalization: Generalization is the process by which a relationship is identified that is possessed by a number of things. It is deriving and deducing from particulars. In this process we identify a broad overall character without being limited, modified or checked by narrow, precise considerations. Furthermore, in this process we isolate a common attribute and ignore other attributes (Synonyms: abstraction, reverse of analysis)

Procedure:

1. Identify the purpose or focus for the generalization.
2. Identify all the attributes of the given entity.
3. Based on the purpose of the generalization, identify which attributes can be ignored and which attributes relate to the purpose and therefore cannot be ignored.
4. Create lists of other entities with similar attributes. The lists should pertain to the purpose of the generalization.
5. Identify a name of the class of entities that share the same attributes. Note that in this process some differences between entities will be ignored.
6. Repeat the process based on the class of entities.

Thus, the four skills or elements we use when we apply a problem-solving strategy are analysis, creativity, decision making and generalization. Heuristics or hints are general suggestions that lead to a successful solution to a problem or successful completion of one step in the problem solution. The applicability and usefulness of such suggestions cannot be proved precisely.

For example, "check the units," or "neglect small terms" or "use crude approximations" are some heuristics. Some that we used are described elsewhere (32) (33). Other examples, are given by Aris (34), Newell and Simon (35), Rubinstein (36) and Polya (37). Simplification is the process of reducing the complexity and the number of complications.

SUMMARY

PROBLEM SOLVING IS a general activity made up of four components. First, we can classify the type of problems we have to solve. Three different classifications were reviewed. Next, we can identify six prerequisites: knowledge, experience, learning skills, motivation, communication and group skills. Some details were given of how we have tried to develop these prerequisite skills.

A wide variety of strategies for solving problems have been presented in the literature. We have highlighted some of these and emphasized similar features. Four elements are used in applying the strategy: analysis, creativity, decision-making and generalization. These elements were defined and discussed briefly. □

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