SOLUTION OF DIFFERENTIAL EQUATION MODELS BY POLYNOMIAL APPROXIMATION

By John Villadsen and Michael L. Michelsen
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This book is a welcome addition to the engineering literature. It introduces polynomial approximation for the solutions of differential equations with a demonstration of its performance in various problems leaning mostly on reaction engineering.

Chapter 1 lays down the scope of applications through a coverage of mathematical models commonly encountered in chemical engineering; more specifically, reaction engineering, separation processes, and polymer processing are some of the examples cited.

Chapter 2 contains an exposition of the *modus operandi* of polynomial approximation. The accent is on the approximation of the expansion coefficients while the choice of polynomials is essentially confined to those of the Jacobi class. Some useful computational schemes are introduced in Chapter 3 for efficient calculation of the polynomials.

Linear problems are the subject of Chapter 4 in which boundary and initial value problems have been treated. Nonlinear problems are dealt with in Chapter 5. The emphasis is on the nonisothermal catalyst problem (understandably so, since it has been actively investigated).

Chapter 6 is devoted entirely to a treatment of the one-point collocation method and its accomplishments (generally in reaction engineering) in spite of its startling simplicity.

In Chapter 7 demonstrations are made of the usefulness of the "global spline collocation" method in solving a variety of boundary value problems, especially entry length problems.

The discussion of coupled ordinary differential equations occupies Chapter 8. The treatment of stiff equations and parametric sensitivity of solutions deserves special mention.

The final chapter is concerned with the role of collocation (and spline collocation) methods in selected research problems. The low Peclet number Graetz problem, the asymptotic stability problem of a catalyst particle and fixed bed reactor dynamics are featured in this chapter. The Graetz problem at low Peclet numbers appears to have been treated well for the first time. In regard to this problem the criticism of Fourier series solution is somewhat inappropriate but excusable since it is based on past work, much of which has been plagued with errors. One also gets a good account of the catalyst stability problem for Lewis numbers different from unity.

The book makes comfortable reading for those with mathematical background normally available to graduate students in their first year of graduate school and qualifies for a supplementary text in a follow-up course on approximate methods; supplementary because of the constraint on *polynomial* approximations.

A feature that perhaps deserved some further attention in this book is the *convergence* of approximation methods. There are proofs available from functional analysis of the convergence of such methods to certain classes of operator equations. (The authors are not unaware of the role of functional analysis since they briefly allude to it in Chapter 5). While it would be a heavy undertaking to use the language of functional analysis in this book, it might have been possible to classify those equations for which convergence proofs are available along with rates of convergence). The restriction to differential equations becomes somewhat unnecessary especially because the use of approximate methods in other equations does not involve any special change of technique. Furthermore, while differential equations are indeed natural to chemical engineering models, integral equations of the Fredholm and Volterra types, and integro-differential equations (such as in population balances) occur with sufficient frequency to merit consideration. The omission of integral equations is not a special feature of this book but an unfortunate fact of the chemical engineering literature. Many realistic and important boundary value problems are best approached via integral equations.

Notwithstanding the foregoing criticism, this book is an important contribution to the chemical engineering profession because it brings together a class of approximation techniques that have been immensely valuable in the solution of a wide variety of engineering problems. □