The object of this column is to enhance our readers' collection of interesting and novel problems in Chemical Engineering. Problems of the type that can be used to motivate the student by presenting a particular principle in class or in a new light or that can be assigned as a novel home problem are requested as well as those that are more traditional in nature that elucidate difficult concepts. Please submit them to Professor H. Scot Fogler, ChE Department, University of Michigan, Ann Arbor, MI 48109.

THE DOLPHIN PROBLEM

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WHALES, DOLPHINS AND PORPOISES are able to maintain surprisingly high body temperatures even though they are immersed continuously in cold, cold water. As can be seen from Figure 1, the extremities of these animals (tails, fins, flukes) have a large surface to volume ratio, and a large portion of the heat loss occurs there. Now an ordinary engineering junior designing a dolphin from first principles would probably view the heat loss from a flipper somewhat as shown in Figure 2.

Let us suppose that blood at 40°C enters the flipper at 0.3 kg/s, feeds the flipper, is cooled somewhat, and then returns to the main part of the body. The dolphin swims in 4°C water, the overall heat transfer coefficient is 100 cal/s·m²·K and the area of the flipper is 3 m².

a) At what temperature does the blood reenter the main part of the body of the dolphin?

Frankly, an ordinary engineer (which you obviously are not) would design a lousy dolphin. Let's try to do better; in fact, let us try to learn from nature. Let us see if we can reduce some of the undesirable heat loss by transferring heat

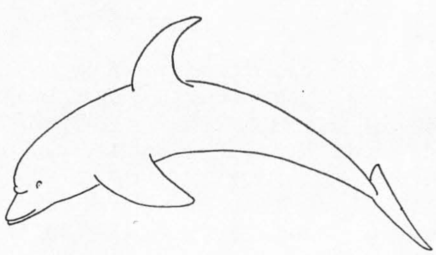


FIGURE 1

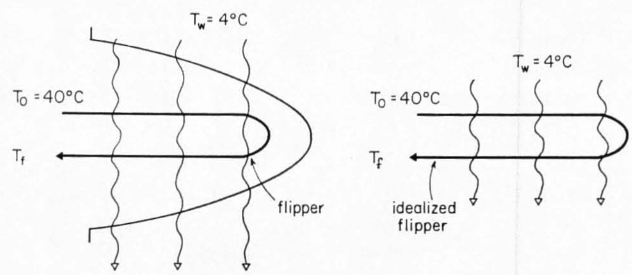


FIGURE 2

from the outgoing warm arterial blood to the cooled venous blood. Such a scheme is idealized as shown in Figure 3. Assume for this internal exchanger B that

$$A_B = 2 \text{ m}^2$$

and

$$U_B = 150 \text{ cal/s·m}^2\text{·K}$$

b) With this extra exchanger find T_3, the temperature of blood returning to the main part of the body; and, in addition, the fraction of original heat loss which is saved. Approximate the properties of blood by water.

NOTE: Heat conservation of this sort, by having arteries and veins closely paralleling each other, in counterflow, is one of nature's clever tricks.

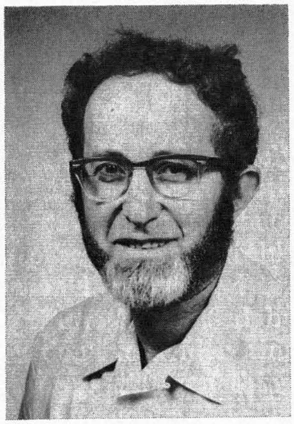
SOLUTION

a) Flipper alone

First, a plot of temperature of blood and water on a q vs T diagram (see Figure 4) gives straight lines meaning that the log mean ΔT is the proper driving force for this process. A heat balance then gives

(heat lost by blood) = (heat transfer rate)

In symbols



Octave Levenspiel, professor at OSU, is primarily interested in problems of chemical reactors. He has written a text on this subject, and has won the ASEE Lectureship Award for his early visions in this field. His weakness for scientific curiosities has led to flirtations with 4-colorologers, 2nd law repealers, Fibonacciics, boomerologists, topolographers, and other such. He is also 1975 president of the Northwest Neothermo Society.

$$\dot{m}C_p\,(T_o - T_f) = UA\frac{(T_o - T_w) - (T_f - T_w)}{ln\,\dfrac{T_o - T_w}{T_f - T_w}} \qquad \text{(i)}$$

Rearranging gives

$$T_f = T_w + (T_o - T_w)\,\exp\!-\!\left(\frac{UA}{\dot{m}C_p}\right)$$

and on replacing values

$$T_f = 4 + 36e^{-1} = 17°C \qquad \text{(a)}$$

b) Flipper Plus Exchanger

Here we must make heat balances about both units to solve for the unknown temperatures T_1, T_2 and T_3. So for the internal exchanger B we have

$$\begin{pmatrix}\text{heat lost}\\\text{by hot}\\\text{blood}\end{pmatrix} = \begin{pmatrix}\text{heat}\\\text{gained by}\\\text{cold blood}\end{pmatrix} = \begin{pmatrix}\text{heat}\\\text{transfer}\\\text{rate}\end{pmatrix}$$

In symbols

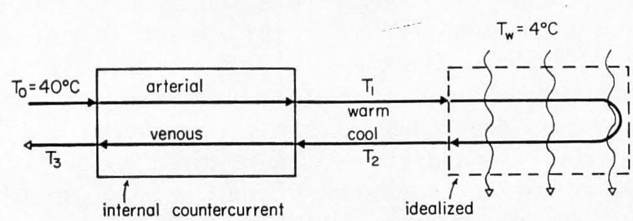


FIGURE 3

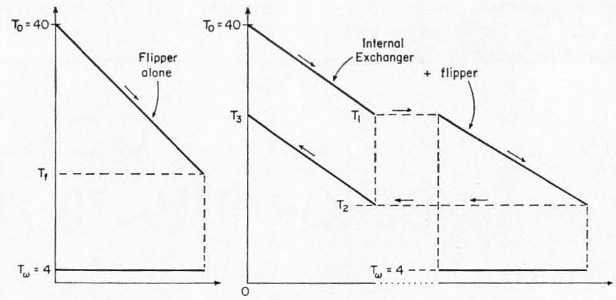


FIGURE 4

$$\underset{\textbf{I}}{\dot{m}C_p\,(T_o - T_1)} = \underset{\textbf{II}}{\dot{m}C_p\,(T_3 - T_2)} = \underset{\textbf{III}}{[UA\,\Delta T_{lm}]_B} \qquad \text{(ii)}$$

For the flipper alone, i.e. exchanger A, we obtain, as with Eq. (i)

$$\dot{m}C_p\,(T_1 - T_2) = UA\frac{(T_1 - T_w) - (T_2 - T_w)}{ln\,\dfrac{T_1 - T_w}{T_2 - T_w}} \qquad \text{(iii)}$$

Solving (ii) and (iii) simultaneously gives the unknown temperatures T_1, T_2 and T_3. All else is known. The first step in this solution is to combine I and II to give

$$40 - T_3 = T_1 - T_2 = \Delta T$$

This expression shows that the driving force is the same at both ends of the internal exchanger B. Consequently we should use the arithmetic ΔT, not the log mean ΔT in Eq. (ii). This fact is shown in the q vs T diagram of Fig. 4. The rest is straightforward, giving

$$\text{For Eq. (i)}: 40 - T_1 = T_3 - T_2 = T_1 - T_2$$

$$\text{For Eq. (ii)}: \; ln\,\frac{T_1 - 4}{T_2 - 4} = 1$$

From which

$$T_1 = 26°C, \qquad T_2 = 12°C, \qquad T_3 = 26°C \qquad \text{(b)}$$

This modification represents a heat savings of

$$\frac{26 - 17}{40 - 17} = 39\% \qquad \text{(b)}$$

EDITOR'S NOTE: Professor Levenspiel's problem statement was published in the 1981 fall issue of CEE. At that time we issued an invitation to our student readers to submit solutions. We congratulate Mike Glass, Washington Univ. (St. Louis) who has submitted the first correct solution and in so doing has won a subscription to CEE.