ChE classroom

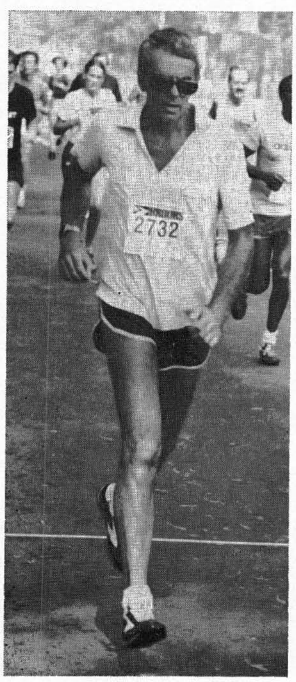
THERMODYNAMICS OF RUNNING

ALAN L. MYERS
University of Pennsylvania
Philadelphia, Pennsylvania 19104

TEXTBOOKS ON THERMODYNAMICS for chemical engineers contain numerous examples of the application of the first and second laws. These include chemical reaction, refrigeration, liquefaction, compression of gases, various power cycles (Brayton, Otto, Rankine), pipe flow and throttling. Recently chemical engineers have become interested in bioengineering. Therefore it is useful to add to this list an application to a biological system. Several problems suitable for classroom discussion are provided.

DIMENSIONAL ANALYSIS OF RUNNING

PHYSIOLOGICAL WORK PERFORMED during aerobic running is an excellent example of a biological process which can be analyzed thermodynamically in the same manner as other power cycles. In running, each step or stride constitutes a cycle.



Alan L. Myers did his undergraduate work at the University of Cincinnati after three years in the U.S. Navy during the Korean War. He obtained his Ph.D. at the University of California at Berkeley in 1964 and then joined the faculty at the University of Pennsylvania. His research specialties lie in thermodynamics and statistical mechanics, particularly adsorption and electrolyte solutions. Other interests include Russian language and literature, and also long-distance running. Alan runs several miles each day and has competed in many races, including the 1981 New York City Marathon.

The average velocity v of running, which is assumed to continue for several minutes or more until a steady state is achieved, is given by:

$$v = L\nu \qquad (1)$$

where L is the length of a single stride and ν is the frequency with which the feet strike the ground. This frequency is analogous to the rpm of the drive shaft of a mechanical engine. The mechanical power of running is:

$$P_{mech} = W_{int}\nu \qquad (2)$$

W_{int} is the internal work done by the muscles of the body during a single stride or cycle.

Both L and ν are functions of the velocity and other variables such as the mass (M) and height (H) of the runner. A biomechanical model of running sufficiently realistic for the numerical calculation of work would be very complicated, but some of the important dimensionless groups are identified as:

$$L^* = \frac{L}{H} = \frac{\text{length of stride}}{\text{height}} = \text{dimensionless stride}$$

$$v^* = \frac{v}{\sqrt{gH}} = \frac{\text{inertial force}}{\text{gravitational force}} = \text{dimensionless velocity}$$

$$W^* = \frac{W_{int}}{MgL} = \frac{\text{work/(unit distance) (unit mass)}}{\text{acceleration of gravity}} = \text{dimensionless work}$$

The relation between stride (L^*) and velocity (v^*) was established by experiments on a track. Most of the subjects were sophomore students in chemical engineering at the University of Pennsylvania, but the points plotted on Fig. 1 include measurements for children and middle-aged adults, males and females, champion athletes and non-athletes with builds ranging from slim to overweight. For each point, subjects ran one lap (400 m) at a steady pace measured by a stopwatch and L was determined by counting steps.

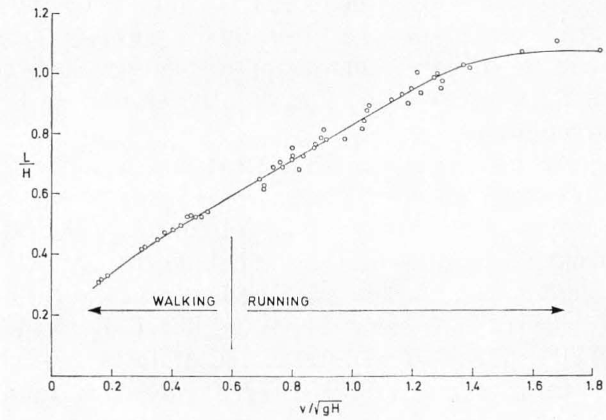


FIGURE 1. Length of stride (dimensionless) as a function of velocity (dimensionless).

The solid line drawn on Fig. 1 is the function $L^*(v^*)$. The average deviation between the points and the curve is 2%. This scatter is due to a combination of experimental error and the neglect of other variables which might affect the stride such as body proportion, obesity and training.

Fig. 1 indicates that the length of the stride L increases with velocity until it is equal to the height H of the runner. After that, it is necessary to increase the frequency ν to run faster as shown by Eqn. (1). More experimental points are needed to establish this apparent leveling-off in the length of the stride. However, the portion of the curve above $v^* = 1.4$ corresponds to sprint races less than one mile in distance and does not apply to the aerobic running under discussion. Fig. 1 also shows that the functions $L^*(v^*)$ for walking and running form a single, continuous curve, although this was not expected because the mechanics of walking and running are different.

For $v^* < 1.4$, the solid line of Fig. 1 is nearly linear:

$$L^* = (0.238) + (0.594)v^*$$

Example 1. Calculate the stride length and frequency for a person running at a speed of 15 km/hr. The mass of the person is 75 kg and the height is 1.8 m.

Using MKS units, the velocity is 4.17 m/s.

$$v^* = v/\sqrt{gH} = (4.17)/\sqrt{(9.8)(1.8)} = 0.99$$

$$L^* = 0.83 \text{ (from Fig. 1)}$$

$$L = (L^*)H = (0.83)(1.8) = 1.49 \text{ m}$$

$$\nu = v/L = (4.17)/(1.49) = 2.80 \text{ s}^{-1}$$

. . . the internal work done by the muscles is identified as the sum of three terms: work of overcoming gravity . . . work against inertial forces while accelerating, and work to overcome wind resistance . . .

WORK DONE BY MUSCLES DURING RUNNING

WORK PERFORMED BY THE muscles cannot be measured directly because it is done internally. An analogy with an automobile operated on level terrain is useful. For a car the mechanical work supplied by the engine is equal to the product of the torque transmitted to the drive shaft and its angular displacement. If the entire automobile is taken as the thermodynamic system, the work performed by the engine and transmitted to the wheels through the drive shaft is performed internally. This work is the sum of three terms: work against the drag force exerted by the wind, work to accelerate the car, and work performed to overcome rolling friction of the tires. The sum of these three terms is equal to the work done internally by the engine. Returning to the human body, work is performed internally by the muscles. Since the body is the thermodynamic system, the internal work done by the muscles (W_{int}) is identified as the sum of three terms: work of overcoming gravity (the body bobs up and down while running on level terrain), work against inertial forces while accelerating, and work to overcome wind resistance:

$$W_{int} = \Delta E_k + \Delta E_p + W_{wind} \tag{3}$$

Increases in kinetic (ΔE_k) and potential (ΔE_p) energy are periodic, occurring once each stride, whereas work against wind resistance (W_{wind}) is steady. For running on level terrain at steady state, the time-averaged change in kinetic and potential energy is zero. The periodic increases in kinetic and potential energy are supplied by muscle work. Each time the foot strikes the running surface, there is a force against the ground but no displacement if the surface is firm. However, the body decelerates while the foot is touching the ground, and muscle work must be performed to restore the lost kinetic energy. Also, the center of gravity of the body falls a few centimeters with each stride, and again muscle work is required to regain the lost potential energy.

The kinetic and potential energy terms in

Eqn. (3) are the periodic increases which occur with each stride. What happens to the corresponding decreases in kinetic and potential energy? Once more an analogy with a car is helpful. If the car accelerates from 50 to 55 mph, the additional work required by the engine is equal to the increase in kinetic energy of the car. If the car then brakes from 55 to 50 mph, the kinetic energy decrease is dissipated as heat by the brakes and there is no work done by the engine against inertial forces until the next acceleration. The human body maintains a constant average velocity but there are periodic accelerations and decelerations at the frequency with which the feet strike the ground. The center of gravity rises and falls at the same frequency. The periodic decreases in kinetic and potential energy are dissipated as heat to the surroundings.

Physiological work performed during aerobic running is an excellent example of a biological process which can be analyzed thermodynamically in the same manner as other power cycles.

FIRST LAW OF THERMODYNAMICS

THE FIRST LAW OF thermodynamics for aerobic running at steady state on level ground is:

$$\Delta H = Q - W_{wind} \qquad (4)$$

The control volume for the first law is the body as a whole and the only work term is against wind resistance. W_{wind} is the work done by the body against the constant drag force imposed by the wind, which is self-generated by the motion of the body even in still air. It is assumed that the true wind velocity is zero, so that the wind velocity relative to the runner is equal to his average velocity v (see Appendix). The muscle work is performed internally and therefore does not appear in Eqn. (4). The periodic changes in kinetic and potential energy vanish for running on level terrain at steady state.

ΔH is the exothermic heat of reaction for the combustion of foodstuffs stored in the body. For example, for glucose:

$$C_6H_{12}O_6 + 6O_2 = 6CO_2 + 6H_2O \qquad (5)$$

the heat of combustion is:

$$\Delta H_c = -\Delta H = 670 \text{ kcal/mole} = 3.7 \text{ kcal/g}$$

For running on a treadmill there is no wind resistance and the work term in Eqn. (4) is zero. In this case, the value of ΔH is equal to the heat transferred from the body to the surroundings (Q is negative). Thus all of the energy derived from food is eventually dissipated as heat to the surroundings.

The heat of combustion is calculated indirectly from the measured oxygen consumption. A respiration calorimeter is used to measure the rate of oxygen uptake by the lungs. According to the stoichiometry of Eqn. (5), the heat of combustion of glucose is 5 calories per milliliter of oxygen (STP). The heat of combustion of fatty acids is 4.5 calories per milliliter of oxygen. An intermediate value of 4.8 calories per milliliter is used by physiologists [10] to relate oxygen consumption to the heat of combustion of foodstuffs.

The energy generated by the combustion of carbohydrates and fats is dissipated as heat from the body to the surroundings by several mechanisms: conduction and convection, radiation, evaporation of water from the skin, and respiration. The relative importance of these different modes of heat transfer depends upon a number of factors such as amount of clothing, the temperature difference between the body and the surroundings, the humidity, etc. Nevertheless the total heat loss is given by the first law, Eqn. (4).

Example 2. In a five-minute treadmill test, a person with a mass of 65 kg consumes 14.5 liters of oxygen (STP) while running at maximum speed. Estimate the heat loss assuming steady state.

From Eqn. (4):

$$-Q = -\Delta H = \left(\frac{4.8 \text{ cal}}{\text{ml}}\right)(14{,}500 \text{ ml})\left(\frac{\text{kcal}}{10^3 \text{ cal}}\right)$$
$$= 69.6 \text{ kcal}$$

The specific maximum rate of oxygen consumption is a measure of running ability because it is proportional to the power-to-mass ratio of the person. Values range from 30 ml/kg-min for below-average capability, to 60 for athletes and as high as 70 ml/kg-min for marathon runners [4]. For this example,

$$\text{Max. oxygen capacity} = \frac{(14{,}500 \text{ ml})}{(65 \text{ kg})\ (5 \text{ min})}$$
$$= 44.6 \text{ ml/kg-min}$$

This is a typical value for an average person of age 25.

SECOND LAW OF THERMODYNAMICS

THE PROCESS OF RUNNING is essentially isothermal. The muscle cells are the engines that transform chemical energy into mechanical energy. The second law of thermodynamics states that the maximum work which can be derived from the oxidative reaction by the muscles is given by the decrease in Gibbs free energy:

$$W_{int} = -\Delta G = -(\Delta H - T\Delta S)$$

The maximum work is for a reversible process, so the actual irreversible work W_{int} performed by the muscles must be less than $(-\Delta G)$. The thermodynamic efficiency of running is:

$$\epsilon = \frac{W_{int}}{(-\Delta G)}$$

and the second law requires that $\epsilon<1$. Although ΔS is large for Eqn. (5), the product $T\Delta S$ is much smaller than ΔH and in practice the efficiency is defined by:

$$\epsilon = \frac{W_{int}}{\Delta H_c} \quad (6)$$

This efficiency can be determined by independent measurements of mechanical work and heat of combustion. The overall efficiency defined this way is 29% [10]. It is the product of two values of efficiency, one for the synthesis of ATP (60%) [8] and another for the performance of positive muscle work by contraction associated with hydrolysis of ATP (49%) [10]. This overall efficiency of 29% for converting chemical energy into mechanical work is comparable to the efficiency for producing electricity in commercial power plants by combustion of fossil fuels (30 to 40%).

DIMENSIONLESS WORK OF RUNNING

SINCE THE DIMENSIONLESS stride L^* is a function of dimensionless velocity v^*, there should be a relation between the dimensionless work and v^*. The positive internal work per stride is given by Eqn. (3). The instantaneous kinetic energy of the body is:

$$E_k = \frac{M}{2}v^2$$

The differential of E_k is:

$$dE_k = Mvdv$$

The process of running is essentially isothermal. The muscle cells are the engines that transform chemical energy into mechanical energy.

The increase in kinetic energy for each stride can be estimated from the increase in velocity:

$$\Delta E_k = Mv\Delta v \quad (7)$$

This can be measured with an accelerometer, which is used to find the impulse imparted to it by the foot and thus the increase in momentum $M\Delta v$. The increase in potential energy with each stride is:

$$\Delta E_p = Mg\Delta z \quad (8)$$

where Δz is the increase in elevation of the body's center of gravity above the running surface, which can be measured using high-speed photography. For the special case of running on a treadmill, the wind resistance is zero and substitution of Eqns. (7) and (8) into (3) yields the muscle work performed by the body per stride:

$$W_{int} = Mv\Delta v + Mg\Delta z \quad (9)$$

In non-dimensional form, Eqn. (9) becomes:

$$W^* = \frac{W_{int}}{MgL} = \frac{\nu\Delta v}{g} + \frac{\Delta z}{L} \quad (10)$$

Example 3. The person described in Example 1 ran on a treadmill at a steady speed of 15 km/hr. The stride length was 1.49 meters and the frequency was 2.80 strides per second. Measurements with an accelerometer and high-speed photography indicated that the increase in elevation of the center of gravity of the body was Δz = 6.6 cm, and the increase in velocity with each stride was Δv = 0.79 km/hr. What is the mechanical power expended and what is the dimensionless work of running at this velocity?

Using MKS units, the velocity is 4.17 m/s and Δv = 0.219 m/s.

$$W_{int} = Mv\Delta v + Mg\Delta z = (75)(4.17)(0.219) + (75)(9.8)(0.066) = 117 \text{ J/stride}$$

$$P = W_{int}\nu = (117)(2.80) = 328 \text{ watts} = \text{mechanical power}$$

$$W^* = \frac{\nu\Delta v}{g} + \frac{\Delta z}{L} = \frac{(2.80)(0.219)}{(9.8)} + \frac{(0.066)}{(1.49)} = 0.107$$

MECHANICAL POWER AND ENERGY EXPENDITURE

THE DIMENSIONLESS WORK OF running ($W^* = 0.107$) calculated in Example 3 is for a particular velocity. Since L^* is a function of v^*, it was anticipated that W^* would also be a function of v^*. Surprisingly, experiments have shown [2, 5] that the kinetic energy term in Eqn. (10) increases with velocity and the potential energy term decreases with velocity such that the sum of both terms is nearly constant. Thus it is a good approximation to assume that the dimensionless mechanical work of running is a constant, independent of velocity:

$$W^* = 0.107 = \text{constant} \qquad (11)$$

Substituting Eqn. (11) into (2):

$$P_{mech} = W_{int}\,\nu = W^*Mgv \qquad (12)$$

According to Eqn. (6), the rate of energy consumption derived from oxidation of carbohydrates and fats is:

$$\frac{dE}{dt} = \frac{W^*Mgv}{\epsilon} \qquad (13)$$

where $W^* = 0.107$ and $\epsilon = 0.29$. Eqns. (9) - (13) apply to running on a treadmill for which there is no work against wind resistance. As shown in the Appendix, the mechanical power necessary to overcome self-generated wind resistance is:

$$P_{mech} = \frac{2}{\sqrt{\pi}} C_d \rho_a v^3 \sqrt{\frac{MH}{\rho_b}} \qquad (14)$$

where C_d = drag coefficient of the body = 0.50, ρ_a = density of surrounding air, and ρ_b = density of body $\simeq$ 1000 kg/m^3. Therefore the total mechanical power required for running in still air is given by the sum of Eqns. (12) and (14):

$$P_{mech} = W^*Mgv + \frac{2}{\sqrt{\pi}} C_d \rho_a v^3 \sqrt{\frac{MH}{\rho_b}} \qquad (15)$$

The total energy expenditure of running is obtained by dividing the mechanical work by the efficiency (29%).

Another interesting variable is the total energy expenditure per unit distance:

$$\frac{E}{L} = \frac{W_{int}}{L\epsilon} = \frac{W^*Mg}{\epsilon} + \frac{2}{\sqrt{\pi}} \frac{C_d \rho_a v^2}{\epsilon} \sqrt{\frac{MH}{\rho_b}} \qquad (16)$$

Eqn. (16) shows that the energy expenditure of running (per unit distance) versus velocity is the equation of a parabola with its vertex at v = 0, as shown on Fig. 2. Eqn. (16) is in good agreement with data reported in the literature [6].

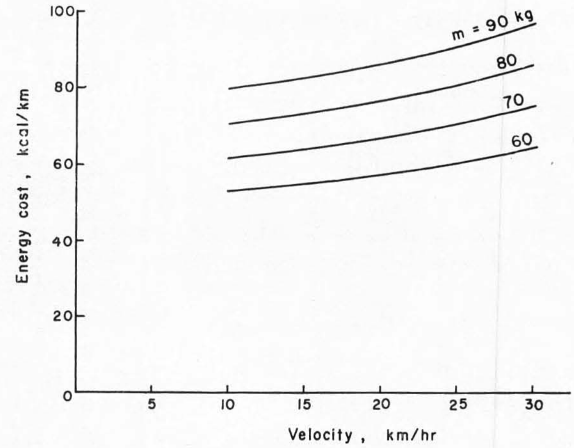


FIGURE 2. Net energy cost for running (less resting metabolism) for adult human.

Example 4. For the person described in Example 1, calculate the mechanical power and the total energy expenditure for running at a velocity of 15 km/hr.

The mechanical power is given by Eqn. (15). For MKS units:

$$P_{mech} = (0.107)(75)(9.8)(4.17) + \frac{2}{\sqrt{\pi}}(0.5)(1.184)(4.17)^3\left(\frac{(75)(1.8)}{1000}\right)^{1/2} = 346 \text{ w.}$$

The energy expenditure per unit distance is given by Eqn. (16) or by:

$$\frac{E}{L} = \frac{W_{int}}{L\epsilon} = \frac{P_{mech}}{v\epsilon} = \frac{(346)}{(4.17)(0.29)} = 286 \text{ J/m}$$

or in more familiar units:

$$\frac{E}{L} = 286\frac{J}{m}\left(\frac{kcal}{4184\ J}\right)\left(\frac{10^3 m}{km}\right) = 68.4 \text{ kcal/km} = 110 \text{ kcal/mile}$$

In summary, it has been shown how conventional methods of thermodynamic analysis may be applied to a living system. The example has been simplified and it is worthwhile to point out some of the refinements which might be introduced. Other variables such as body proportion and training affect the mechanical motion and the thermodynamic efficiency. Other modes of internal work could be considered, such as basal metabolism and the work performed by the lungs (according to Comroe [3] the power expenditure of the lungs is about 20 watts during running). Fig. 1 and Eqn. (16) are for the case of running at steady state on a hard, flat surface with no wind except that generated by the motion of the runner. Running on hills requires more work because the energy expended climbing a hill is only partially recovered while running downhill. Running against a headwind or running on a spongy surface like wet sand would increase the work of running and alter the stride as well.

HOMEWORK PROBLEMS

In addition to Examples 1-4, there are several interesting thermodynamic problems which can be assigned to enable students to evaluate their own performance during aerobic running:

PROBLEM 1. Plot the energy expenditure of running per unit distance for yourself in units of kcal/mile versus velocity.

Ans. Eqn. (16) applies. Fig. 2 is a plot of this equation for several values of mass.

PROBLEM 2. It is sometimes said that it is cheaper to go by foot than by car. Examine this assumption by comparing the fuel cost (food) for running with the fuel cost for traveling by car (one passenger), for equal distances.

Ans. The cost of food prepared at home is about ten times the cost of gasoline on the basis of equal mass. Assuming reasonable mileages of 5 miles per pound of gasoline (car) and 20 miles per pound of food (person), the cost of running a given distance is more than twice the cost of traveling by car. Of course this calculation ignores capital investment.

PROBLEM 3. Determine the length of your stride and its frequency as a function of velocity for running.

Ans. The result can be estimated using Fig. 1 (see Example 1) and checked by timing the velocity and counting the steps on a measured distance.

PROBLEM 4. What is your mechanical power requirement for running, in units of watts as a function of speed?

Ans. Eqn. (15) applies.

PROBLEM 5. How far must you run at the reasonable jogging speed of one mile in eight minutes to trim off one pound of fat? The heat of combustion of fat is 9 kcal/g.

. . . it has been shown how conventional methods of thermodynamic analysis may be applied to a living system. The example has been simplified and it is worthwhile to point out some of the refinements which might be introduced.

Ans. The energy cost per unit distance is given by Eqn. (16). The required distance is about 40 miles for a person of 70 kg mass.

PROBLEM 6. Calculate your power-to-mass ratio for running in units of ml of oxygen (STP) per kilogram per minute. First find your maximum velocity for aerobic running by finding how far you can run in 12 minutes. This is the Cooper fitness test. Find the mechanical power for running at this velocity using Eqn. (15) and divide this power by the efficiency (0.29) to obtain the rate of combustion. Divide again by your body mass and then calculate your maximum oxygen capacity using the equivalency of 4.8 calories per ml of oxygen.

Ans. See Example 2 for a brief discussion of oxygen capacity.

PROBLEM 7. The dimensionless energy expenditure for aerobic running is

$$W^*/\epsilon = (0.107)/(0.29) = 0.37$$

Compare this figure to that for a compact car.

Ans. For a 1000 kg compact car which uses 6 liters of gasoline per 100 km, the dimensionless energy expenditure is 0.2 (heat of combustion of gasoline=11 kcal/g) or about half the value for running. For a larger car the dimensionless energy expenditure (0.3 to 0.4) is about the same as for running.

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CRITIQUE OF DESIGN COURSE

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On the other hand, should chemical engineers boldly strike out and endeavor to develop new forms for "the creative application of fundamentals to practical problems?" Or would another kind of course provide a better synthesis experience for our times? Do we see a candidate in a course based on the text "The Structure of the Chemical Process Industries," by Wei, *et al.* [13]? As stated in its preface, this book has the worthy purpose of making one understand "how chemical technology is mobilized to benefit society, and how chemical engineers can contribute effectively to it."

The design course may be in a rut. If so, changes for just the sake of change (a common motivation for curriculum redesign) should be avoided unless the contending schemes are superior to traditional programs. New directions are encouraged by the 1979 definition of the design experience in education [12] The book by Wei, Russell, and Swartzlander suggests a new kind of capstone experience. □

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APPENDIX

For running in still air at velocity v, the drag force of the wind is:

$$F_{wind} = C_d \rho_a v^2 A \qquad (1)$$

Assuming a cylindrical form of radius r and height H for the body, the projected area is:

$$A = 2rH \qquad (2)$$

and the volume is:

$$V = \pi r^2 H = \frac{M}{\rho_b} \qquad (3)$$

Elimination of r in Eqn. (2) using Eqn. (3) gives:

$$A = \frac{2}{\sqrt{\pi}} \sqrt{\frac{MH}{\rho_b}} \qquad (4)$$

The mechanical power for overcoming wind resistance is:

$$P_{wind} = F_{wind} \cdot v \qquad (5)$$

Substituting Eqns. (1) and (4) in (5):

$$P_{mech} = \frac{2}{\sqrt{\pi}} C_d \rho_a v^3 \sqrt{\frac{MH}{\rho_b}} \qquad (6)$$

The error resulting from the incorrect assumption of cylindrical form is cancelled by calculating C_d from experimental data [5] for the drag force on the body during running. Defining $\rho_b = 1000\ kg/m^3$, the drag coefficient C_d is found to be 0.50.