ChE laboratory

DIRECT DIGITAL CONTROL LIQUID LEVEL EXPERIMENT

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DIRECT DIGITAL CONTROL OF liquid level was developed at the New Jersey Institute of Technology as an experiment in the undergraduate process control laboratory. The objective of the experiment is to give the students exposure to the area of digital control which is widely used by industry, although not much attention is given to it in most undergraduate process control curricula. Students can be exposed to writing their control program, and can experimentally determine the transfer functions for all system components. They can compare the actual response of the controlled system with the theoretically predicted response following a step disturbance in the feed rate and also study the effect of sampling time on system response.

APPARATUS

The experimental set-up consists of a plexiglass tank 0.14 meter in diameter, fed with water from the bottom through a rotameter with a maximum capacity of 0.0015 m^3/s or 24.5 gpm. A Fisher Governor control valve type 657A, 3-15 psi is located on the tank outlet pipe. The original set-up, designed for pneumatic analog control includes a Foxboro Model 58P5 controller, and a 3-15 psi type 13A-1 Foxboro transducer. To provide direct digital control, a Devar Inc. air to electric converter (A/E) type 18-118L, 0-5 volts was connected to the transducer, and a Devar Inc. electric to air converter (E/A) type 18-150, 0-5 volts was connected to the control valve. Plumbing was done in such a way that the system can either be operated by the controller or by the digital computer. The digital computer installed was a

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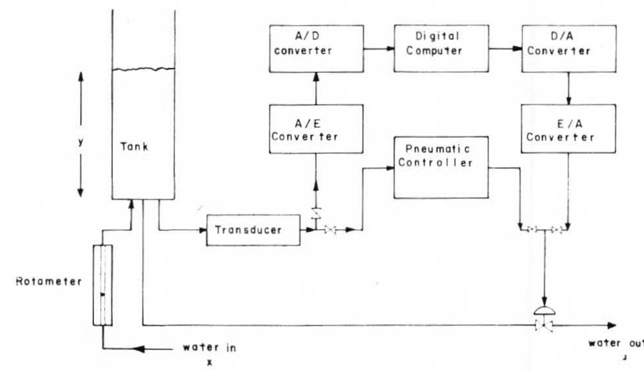

FIGURE 1. Schematic drawing of System.

Digital MINC-11 minicomputer equipped with four laboratory modules: a preamplifier, an analog to digital (A/D) converter, a clock, and a digital to analog converter (D/A). A schematic drawing of the system is shown in Fig. 1.

PROCESS TRANSFER FUNCTION

A material balance in terms of deviation values can be written as

$$A\frac{dY}{dt} = X - U \qquad (1)$$

where A = tank cross sectional area; X = feed flow rate; U = outlet flow rate.
The outlet flow rate "U" is a function of the pressure to the valve "P" and the liquid level "Y". This can be written as

$$U = (\partial u/\partial y)_{ss}Y + (\partial u/\partial p)_{ss}P \qquad (2)$$

where the subscript "ss" represents steady state value. Combining the above equations, transforming to the Laplace domain and rearranging gives

$$Y(s) = \frac{k_p}{(\tau s + 1)}[X(s) - k_v P(s)] \qquad (3)$$

where $\tau = A/(\partial u/\partial y)_{ss}$; $k_p = 1/(\partial u/\partial y)_{ss}$; $k_v = (\partial u/\partial p)_{ss}$.

CALIBRATION OF CONTROL ELEMENTS

1. τ, k_p can be determined by making a step change of magnitude "a" in the feed rate while the analog controller is placed on manual status.

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The two parameters can be calculated from the equation

$$Y = ak_p[1 - \exp(-t/\tau)] \quad (4)$$

2. With liquid level being kept constant, and the controller on manual status, Eq. (3) can be written as

$$\frac{P(s)}{X(s)} = k_v \quad (5)$$

The valve transfer function k_v can be obtained by changing the flow rate to the tank and varying the pressure to the valve to keep the liquid level at a specified value.

3. In a closed loop experiment with proportional control only, the proportional band was set at 25, and the set point on the recorder controller was set at 9 psi. From the chart recorder, values of the error and pressure to the valve were obtained and the proportional control gain k_c can be calculated.
4. The transducer transfer function was obtained by changing the water level in the tank and recording the resulting transducer output (in psi as obtained from the chart recorder).

THEORETICAL RESPONSE FOR CONTINUOUS SAMPLING

The closed loop transfer function for a proportionally controlled system can be written as

$$\frac{Y(s)}{X(s)} = \frac{k_p}{(\tau s + 1) - k_c k_p k_v T} \quad (6)$$

For a step change from 18% to 30% on the rotometer scale and utilizing the following experimentally determined parameters.

k_p = 17.85 min/ft^2, (1.153 s/m^2)
k_v = −0.151 ft^3/min·psi, (−1.034 x 10^{-3}m^3/s·Pa)
k_c = 3.53 psi/psi, (3.53 Pa/Pa)
T = 2.98 psi/ft, (6.74 x 10^4 Pa/m)
τ = 176.40 s

The solution of the above equation for liquid level deviation in the units of meters can be written as

$$Y = 0.0729\,[1 - \exp(-0.1664t)] \quad (7)$$

where the time units are seconds.

DIRECT DIGITAL CONTROL

Liquid level was converted to an air pressure signal through the transducer, then to an electric voltage signal through the A/E converter, and finally to a digital signal through the A/D converter. The developed computer program compares the measured liquid level to the set point, thereby generating the error. Control actions provided in the software were; proportional (P), proportional-integral (P-I), proportional-derivitive (P-D), and proportional-integral-derivative (P-I-D).

For continuous sampling an ideal P-I-D controller is described by the following equation.

$$V = V_o + k_c \left[e + \frac{1}{\tau_I}\int_0^t e\,dt + \tau_D \frac{de}{dt} \right] \quad (8)$$

Where

V = controller output signal at time t
V_o = controller output signal at t = 0
e = error signal
τ_I = integral time
τ_D = derivative time

The discrete equivalent of the above equation can be obtained by replacing the derivative with finite difference, and using rectangular integration for the integral. The computer output at the nth sampling instant can, therefore, be written as

$$V_n = V_o + k_c \left[e_n + \frac{T}{\tau_I}\sum_0^n e_n + \frac{\tau_D}{T}(e_n - e_{n-1}) \right] \quad (9)$$

where

T = sampling periods, seconds
V_n = computer output at the nth sampling instant

e_n = error at the nth sampling instant
e_{n-1} = error at the (n-1) sampling instant

Similarly at the (n-1)th sampling instant, the equation for the output is

$$V_{n-1} = V_o + k_c \left[e_{n-1} + \frac{T}{\tau_I} \sum_{o}^{n-1} e_n + \frac{\tau_D}{T}(e_{n-1} - e_{n-2}) \right] \quad (10)$$

Substracting Eq. (10) from Eq. (9) gives

$$V_n = V_{n-1} + k_c \left[(e_n - e_{n-1}) + \frac{T}{\tau_I} e_n + \frac{\tau_D}{T}(e_n - 2e_{n-1} + e_{n-2}) \right] \quad (11)$$

In the sampling method adapted in this experiment one point (y_2) was collected at the start of each sampling loop and the error e_n was calculated. This error was used along with the errors e_{n-1} and e_{n-2} according to Eq. (11) to formulate the computer output signal. The next loop began with the collection of a new data point (y_2) and calculating a new error e_n the old value of e_n was transferred to e_{n-1}, the old value of e_{n-1} was transferred to e_{n-2}, and the old value of V_n was transferred to V_{n-1}. The fastest sampling rate was approximately one sample every 0.12 second for proportional control. A pause statement was also incorporated in the program to introduce a time delay, thus varying the sampling rate. The software also contained a program for calibrating the "transducer + A/E" converter, and a program for calibrating the "E/A converter + valve." Fig. 2 represents the block diagram for the digital control loop. The output of the A/E converter which is a continuous electric signal of the measured variable is fed to the A/D converter. The A/D converter changes the continuous signal to a discrete form (series of impulses of varying

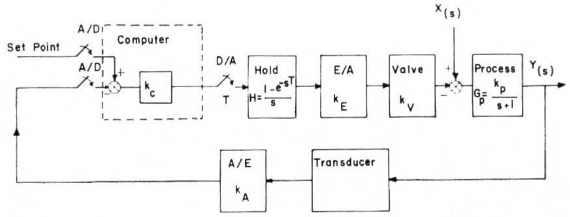

FIGURE 2. Block diagram representation of the digital control loop.

strength). The computer compares the discrete forms of set point and measured value to produce the error, and acts on the error by the appropriate control algorithm. The output of the computer generated every "T" seconds is sent to the D/A converter where a continuous signal is produced. In effect, the D/A converter clamps on the signal until the next one comes along; i.e. the output voltage of the D/A converter remains at a constant value over the sampling period. The D/A converter, therefore, acts as a holding device. For a zero-order holding device, the transfer function is

$$H = \frac{1 - \exp(-sT)}{s}$$

The analysis of a discrete system is conveniently done in terms of z-transforms, defined as

$$F(z) = Z[f(t)] = \sum_{n=0}^{\infty} f(nT)\, z^{-n} \quad (12)$$

The use of the z-transform in discrete systems is analogous to the use of Laplace transform for continuous sampling, and the Laplace transforms are usually tabulated alongside the time function and the corresponding z-transforms [1]. This allows direct conversion from the Laplace transform of a continuous function to the z-transform of that same function.

The closed loop transfer function for the system

$$\frac{Y(z)}{X(z)} = \frac{G_p(z)}{1 - (Tk_A k_c k_E k_v H G_p)(z)} \quad (13)$$

Since T, k_A, k_c, k_E, k_v are constants, they can be combined in one term as K, and the output variable can be written as

$$Y(z) = \frac{(XG_p)_z}{1 - K(HG_p)_z} \quad (14)$$

What is needed are the values of $(XG_p)_z$ and $(HG_p)_z$ to plug in the above equation

$$Z(XG_p) = Z\left[\frac{a}{s}\,\frac{k_p}{\tau s + 1}\right] = Z\left[\frac{ak_p}{s} - \frac{ak_p}{s + 1/\tau}\right]$$

$$= ak_p\left[\frac{z}{z-1} - \frac{z}{z - \exp(-t/\tau)}\right]$$

Let $e(-t/\tau) = b$. Then

$$Z(XG_p) = \frac{ak_p(1-b)\,z}{(z-1)(z-b)} \quad (15)$$

Also

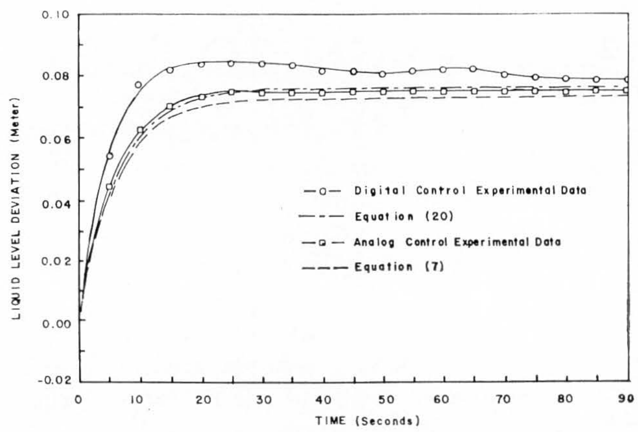

FIGURE 3. Comparison between experimental and theoretical responses.

$$Z(HG_p) = Z\left[\frac{k_p}{\tau s + 1}\;\frac{1 - \exp(-Ts)}{s}\right]$$
$$= Z\left[\frac{k_p[1 - \exp(-Ts)]}{(\tau s + 1)\, s}\right]$$
$$= k_p(1 - z^{-1})\, Z\left[\frac{1}{s(\tau s + 1)}\right]$$
$$= \frac{k_p(1-b)}{(z-b)} \qquad (16)$$

Substituting and rearranging gives

$$Y(z) = ak_p\,(1-b)z \Big/ \Big\{ z^2 + z[Kk_p(b-1) - (b+1)] + [b - K\,k_p(b-1)] \Big\} \qquad (17)$$

with values of

k_A = 0.358 volt/psi, (5.1924 v 10^{-5} volt/Pa)
k_E = 2.649 psi/volt, (1.8264 x 10^4 Pa/volt)
K = –1.51 ft^3/min•ft, (–2.3381 x 10^{-3} m^3/s•m)

To examine the effect of sampling time on the stability of the system one has to substitute different values of sampling time in the above equation and find the values of denominator roots. If the roots are located within the unit circle of the z-plane, the system is stable [2]. The following table shows the values of the roots at different sampling time.

TABLE OF ROOTS

Sampling Time	Roots
0.12 sec.	+1.000, +0.981
1.0 sec.	+1.000, +0.842
5.0 sec.	+1.000, +0.220
10.0 sec.	+1.000, —0.563
12.0 sec.	+1.000, —0.831
13.0 sec.	+1.000, —0.982
13.1 sec.	+1.000, —1.000
14.0 sec.	+1.000, —1.128

Notice that second root decreases in value as the sampling time increases and the system becomes theoretically unstable when the sampling time is above 13.1 seconds. It can therefore be stated that increasing the sampling time would destabilize the system.

For a step change from 18% to 30% on the feed rotameter scale, and for a sampling time of 0.12 seconds, Eq. (7) reduces to

$$Y = \frac{0.001452\, z}{z^2 - 1.98103\, z + 0.98103}$$
$$= 0.0759\left[\frac{z}{z-1}\;\frac{z}{z-0.981}\right] \qquad (18)$$

The pole 0.981 can be expressed as

$$0.981 = \exp(-aT) = \exp(-0.0192)$$

$$Y = 0.0759\left[\frac{z}{z-1} - \frac{z}{z - \exp(-0.0192)}\right] \qquad (19)$$

Inverting to the time domain gives

$$Y = 0.0759\,[1 - \exp(-0.0192n)] \qquad (20)$$

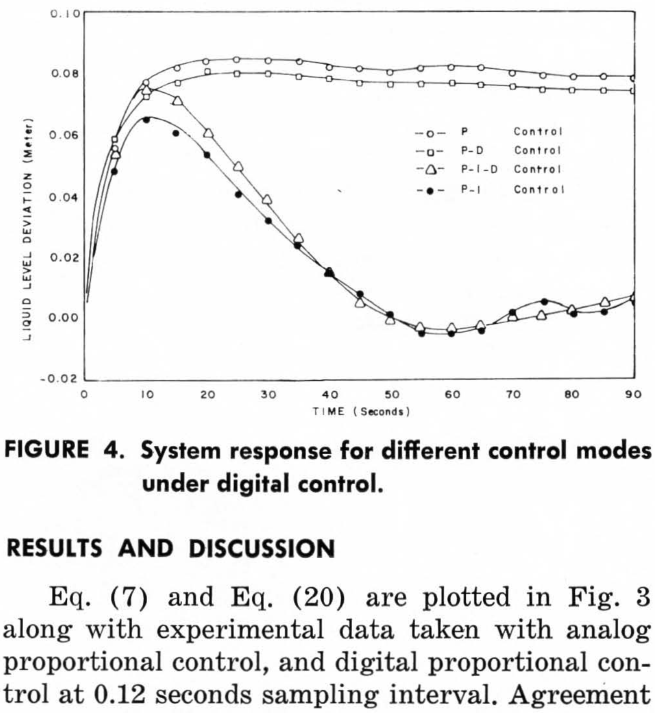

FIGURE 4. System response for different control modes under digital control.

RESULTS AND DISCUSSION

Eq. (7) and Eq. (20) are plotted in Fig. 3 along with experimental data taken with analog proportional control, and digital proportional control at 0.12 seconds sampling interval. Agreement between the equations and the actual response was quite good.

Fig. 4 shows the response of the system at the fastest sampling time to a step change in the feed rate using proportional control (k_c = 3.53 volt/volt), proportional-integral control (k_c = 3.53 volt/volt, τ_I = 20 seconds), proportional-deriva-

Continued on page 47.

actor design books. On the other hand, the authors have barely touched upon catalyst deactivation, and the discussion of intra particle diffusion limitation and multiplicity in catalyst pellets is very limited. The fifth and the last chapter is on Gas-Liquid and Liquid-Liquid Reaction. The material in this chapter is better presented than in most other books and is very useful to the design engineers.

The authors have completely neglected non-catalyzed gas-solid reactions and a very common class of reactors, namely slurry reactors.

The book can be useful as a reference book for chemical engineers in industry but its utility as an undergraduate or a graduate text seems very limited. This is due not only to the lack of mathematical approach but also because it contains no problems for students to solve. Finally, the book does not represent the state of the art since most of the references are pre-1975. □

J. M. SMITH

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in 1973 and 1980; and Pieter Stroeve and Dewey Ryu, are the most recent additions.

While Joe has always emphasized to his colleagues that undergraduate education is a vital part of the UC Davis chemical engineering program, he has also promoted the ideal of strong and varied graduate training. Although Joe is planning a partial retirement to begin in the fall of 1984, he expects to continue to teach and work with graduate students and postdoctoral scholars on chemical engineering research. In view of the personal characteristics he has so far exhibited, it is not anticipated that Joe will retire to a life of leisure and abandon. His single-minded pursuit of achievement in solving chemical engineering problems is probably the critical factor in Joe's success. Undoubtedly, the robust creativity and inexhaustible energy have been important ingredients as well, but his exacting commitment to getting the job done and done well has made the difference between routine and monumental accomplishment.

In all the aspects of chemical engineering education—exemplary research, the writing of textbooks, teaching classes, the guiding of students (graduate and undergraduate) in their research, and administration—Joe Smith has earned and will continue to deserve his Davis title, "Mr. Chemical Engineering". □

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tive control (k_c = 3.53 volt/volt, τ_D = 0.1 seconds), and proportional-integral-derivative control (k_c = 3.53, τ_I = 20, τ_D =0.1). Introduction of integral action eliminates the offset (at much longer time than shown in the graph), and less oscillation is shown with P-I-D control.

Fig. 5 represents the effect of sampling time on the system with only proportional control action.

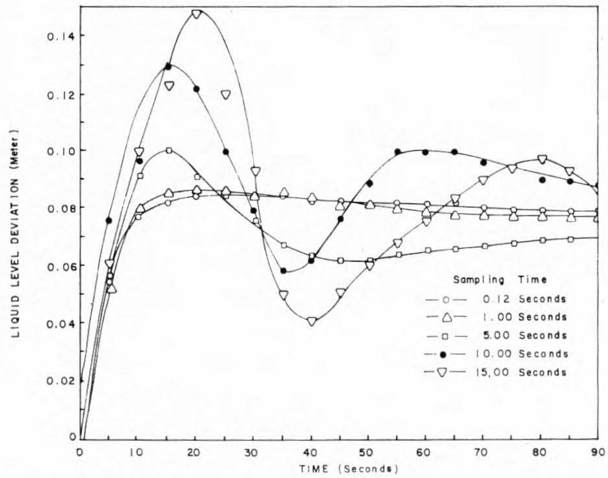

FIGURE 5. Effect of sampling time on system with proportional control.

Increasing the sampling time resulted in higher overshoot and more oscillation. What is interesting is that the system which is first order starts to act like a second order system with decreasing damping coefficient as the sampling time increases. Similar responses are obtained for P-I and P-I-D control.

It should be pointed out that the present undergraduate process control course at NJIT does not cover direct digital control. With this experiment, and the introduction of z-transforms, students can get a very good understanding of discrete sampling and direct digital control. □

ACKNOWLEDGMENT

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