

HOW WE MAKE MASS TRANSFER SEEM DIFFICULT

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MASS TRANSFER IS ONE of the few subjects taught only in chemical engineering. It is taught because it is important in the chemical process industries, basic to such operations as gas treating, liquid-liquid extraction, catalyst effectiveness, and cooling tower design. Fluid mechanics is, of course, taught in chemical engineering, but it is also carefully covered in civil engineering and in applied physics. Heat transfer is also taught in mechanical engineering, and thermodynamics is a core course in chemistry, just as it is in chemical engineering. However, mass transfer is taught in detail only in chemical engineering, and as such, is a unique focus of our profession.

I have no trouble explaining mass transfer at neighborhood parties. Over the years, I have learned to dread the social question, "What do you do, Ed?" If I answer, "I teach chemical engineering," the conversation switches to cars or sports. If I say, "I teach mass transfer," I get the response, "What on earth is mass transfer?" After I reassure my listener that mass transfer has nothing to do with the Teamsters, I can generally give a good brief answer.

However, most students think mass transfer is difficult, much harder than fluid mechanics or chemical thermodynamics. This is not true: mass transfer is easier. To prove this to yourself, try explaining a mass flux, a momentum flux, and a chemical potential at that same neighborhood party. Explaining the mass flux is easy: it is the mass transferred per area per time. Explaining the momentum flux is harder: no non-engineer will easily accept "the flux of y-momentum engendered

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by z-motion." Explaining the chemical potential is almost impossible: if it were not, there would not be such a glut of thermodynamics books.

But if mass transfer is easy, why do students think that it's hard? I believe it's because you and I teach it badly. We take a relatively simple subject and make it a nearly incomprehensible tangle of subscripts, superscripts, unit conversions, dimensionless correlations, correction factors, archaic graphs, and turgidly-written textbooks. We make a mess of it, so students think it's hard.

I think we should stop this tangled teaching, stop making students hate this cornerstone of our profession. In this paper, I have suggested three changes by which we may begin. First, we need to spend more time on simple problems which use a single, simple definition of mass transfer coefficients. Second, we need to use a different approach for describing analogies between mass, heat, and momentum transfer. Finally, we need a more coordinated approach for problems of diffusion and chemical reaction. Each of these changes is discussed in more detail below.

SIMPLE PROBLEMS (for Undergraduate Courses)

Undergraduates have the same problems with simple mass transfer problems as they do in

thermodynamics: they define the system carelessly and they get the units fouled up. We all do this. When I presented this material at the ASEE summer school, the professors present made the same mistakes. I do, too.

I have found that the best way to overcome the carelessness in defining a system is a lot of drill on simple problems. I don't devote a single lecture exclusively to this, but try to sprinkle simple examples throughout the course. In this, I always choose problems in dilute solution, for these are basic. Extensions to concentrated solution can wait until these basics are mastered.

Three problems which I find useful in introducing the idea of a mass transfer coefficient are given in the Appendix. Try to solve at least one without looking at the solutions. They are easy, but you can't depend on having taught them before. Problems like these humble me, making me more sympathetic with my students' difficulties.

The three problems in the Appendix use a common definition of the mass transfer coefficient k , given by

$$N_1 = k\Delta c_1 = \left(\frac{D}{l}\right)\Delta c_1 \quad (1)$$

where N_1 is the flux, Δc_1 is the concentration difference, D is a diffusion coefficient, and l is a characteristic distance, sometimes called a "film thickness." I know that this distance is hypothetical and that it depends on the diffusion coefficient. Still, I find its use a tremendous advantage for two reasons. First, it establishes a clear connection between mass transfer and diffusion, one which

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the students easily remember. Second, it avoids the curse of units, which is a chief reason that mass transfer seems hard.

To illustrate this curse of units, compare the definition of the mass transfer coefficient in Eq. (1) with the other common definitions summarized in Table 1. Obviously, the units for k vary widely because the units used for concentration vary widely. This variation confuses students, clouding the physical meaning of the problem. Moreover, it is unnecessary. It is as if we wrote the ideal gas law as

$$pV = nT \quad (2)$$

Then we would describe the temperature T not only as $^{\circ}\text{K}$ and $^{\circ}\text{R}$ but as ($l\cdot\text{atm}/\text{gmol}$), as (kJ/kgmol), as ($\text{psia}\cdot\text{ft}^3/\text{lbmol}$), as ($\text{m}^3\cdot\text{Pa}/\text{kgmol}$). Such a description would compromise any physical intuition which students had about the meaning of temperature. In the same way, if we use the plethora of mass transfer coefficients in Table 1, we obscure the student's intuition about mass transfer.

ANALOGIES (for Undergraduate Courses)

Most undergraduate courses on transport processes discuss fluid mechanics first, then describe

TABLE 1
Common Definitions of Mass Transfer Coefficients*

Basic Equation	Typical Units of k^{**}	Remarks
$N_1 = k\Delta c_1$	cm/sec	Common in the older literature; best because of its simple physical significance (Welty et al. 1969; Fahien, 1983).
$N_1 = k'\Delta p_1$	mol/cm ² sec atm	Common for gas absorption; equivalent forms occur in biological problems (Sherwood et al., 1979; McCabe and Smith, 1975; Treybal, 1980).
$N_1 = k''\Delta x_1$	mol/cm ² sec	Preferred for some theoretical calculations, especially in gases (Bennett & Myers, 1974; Geankoplis, 1978; Edwards et al., 1979).
$N_1 = k''' \Delta c_1 + c_1 v$	cm/sec	Used in an effort to include diffusion-induced convection in concentrated solutions (Bird, et al., 1960).

*In this table, the interfacial flux N_1 is defined as mol/L²t and the concentration c_1 as mol/L³. Parallel definitions where N_1 is in terms of M/L³ are easily developed. Definitions mixing moles and mass are infrequently used.

**For a gas of constant molar concentration c , $k = RTk' = k''/c$. For a dilute liquid solution $k = (\bar{M}_2 H/\rho)k' = (\bar{M}_2/\rho)k''$, where \bar{M}_2 is the molecular weight of the solvent, H is Henry's law constant of the solute and ρ is the solution density.

heat transfer, and conclude with mass transfer. These courses sensibly outline analogies between these processes, and conclude with a gaggle of dimensionless groups codifying the analogies. This often confuses all but the most mathematically adroit students.

The usual way in which the analogies are taught may be summarized as follows. First, diffusion in one dimension is described by Fick's law

$$-j_1 = D \frac{dc_1}{dz} \quad (3)$$

where D is the diffusion coefficient. If this diffusion takes place into a semi-infinite slab, the concentration profile is

$$\frac{c_1 - c_{10}}{c_{1\infty} - c_{10}} = \operatorname{erf} z / \sqrt{4Dt} \quad (4)$$

where c_{10} and $c_{1\infty}$ are the concentrations at the slab's surface and far within the slab, respective-

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ly. Similarly, thermal conduction is described by Fourier's law

$$-q = k \frac{dT}{dz} \quad (5)$$

where k is the thermal conductivity. If conduction takes place into a similar semi-infinite slab, the temperature profile is

$$\frac{T - T_0}{T_\infty - T_0} = \operatorname{erf} z / \sqrt{4\alpha t} \quad (6)$$

where $\alpha (= k/\rho C_p)$ is the thermal diffusivity and T_0 and T_∞ are the temperatures of the surface of the slab and far within the slab, respectively. Finally, momentum transport follows Newton's law

$$-\tau = \mu \frac{dv}{dz} \quad (7)$$

where τ is the momentum flux or the shear stress and μ is the viscosity. If a flat plate is suddenly moved in an initially stagnant fluid, the velocity v of the fluid is

$$\frac{v - V}{0 - V} = \operatorname{erf} z / \sqrt{4\nu t} \quad (8)$$

where the plate's velocity is V , the fluid's velocity

far from the plate is zero, and the fluid's kinematic viscosity is $\nu (= \mu/\rho)$.

At this point, we professors draw the analogy between mass, heat, and momentum transfer. If each process is subject to mathematically equivalent boundary conditions, then each leads to results of the same mathematical form. Many say this is a more intimidating way: each process depends on combining a linear constitutive equation and a conservation relation to yield mathematically congruent results. The phenomenological coefficients of diffusion (D), of thermal conductivity (k), and of viscosity (μ) are thus analogous.

Many students find this conventional analogy confusing. Sure, Eqs. (3), (5), and (7) all say a flux varies with a first derivative. Sure, Eqs. (4), (6), and (8) all have an error function in them. But D , k , and μ do not have the same physical dimensions. Moreover, D appears in both Eqs. (3) and (4), but k in Eq. (5) must be replaced by α in Eq. (6). The viscosity μ in Eq. (7) is replaced by the kinematic viscosity ν in Eq. (8). These changes frustrate many students, and undercut any value which the analogies have.

The source of confusion stems from the ways in which the basic laws are written. In Fick's law (Eq. 3), the molar flux varies with the gradient in moles per volume. To be analogous, the energy flux q should be proportional to the gradient of the energy per volume ($\rho C_p T$). In other words, Eq. (5) should be rewritten as

$$-q = \frac{k}{\rho C_p} \frac{d}{dz} (\rho C_p T) = \alpha \frac{d}{dz} (\rho C_p T) \quad (9)$$

Now mass and heat transfer are truly analogous. Just as Eq. (4) follows from Eq. (3), so Eq. (6) follows from Eq. (9). Similarly, Newton's law for momentum transport can also be rewritten so that the momentum flux is proportional to the gradient of the momentum per volume (ρv)

$$-\tau = \frac{\mu}{\rho} \frac{d}{dz} (\rho v) = \nu \frac{d}{dz} (\rho v) \quad (10)$$

This new form leads directly to Eq. (8).

Just as the fundamental laws for mass, heat, and momentum transfer can be made more parallel, so can expressions for mass transfer coefficients and heat transfer coefficients. The interfacial mass flux already varies with the difference in moles per volume.

$$N_1 = k \Delta c_1 \quad (11)$$

The interfacial heat flux must be modified so that the energy flux varies with the energy difference

TABLE 2
Analogies Between Processes Are Clearer With Flux Equations In An Uncommon, But Dimensionally Analogous Form.

	MASS TRANSFER		HEAT TRANSFER		MOMENTUM TRANSFER	
	Common Form	Analogous Form	Common Form	Analogous Form	Common Form	Analogous Form
Variable	concentration c_1	moles/volume c_1	temperature T	energy/volume $\rho C_p T$	velocity v	momentum/ volume ρv
Physical Property	diffusion coef. D (Eq. 3)	diffusion coef. D (Eq. 3)	thermal conduct. k (Eq. 4)	thermal dif. α (Eq. 9)	viscosity μ (Eq. 7)	kinematic visc. ν (Eq. 10)
Transfer Coefficient	mass transfer coefficient k (Eq. 11)	mass transfer coefficient k (Eq. 11)	heat transfer coefficient h (Eq. 12)	$(h/\rho C_p)$ (Eq. 12)	friction factor f (Eq. 13)	$(fv/2)$ (Eq. 13)
Dimensionless Groups	Schmidt number $\mu/\rho D$	Schmidt number ν/D	Prandlt number $\mu C_p/k$	Prandlt number ν/α		
	Sherwood number kd/D	Stanton number k/v	Nusselt number hd/k	Stanton number $(h/\rho C_p)/v$		
	Lewis number $k/(\rho C_p D)$	Lewis number α/D				

per volume

$$q \Big|_{z=0} = h\Delta T = \frac{h}{\rho C_p} \Delta(\rho C_p T) \quad (12)$$

Thus the mass transfer coefficient k corresponds less directly to the heat transfer coefficient h than to the quantity $h/\rho C_p$. The appropriate parallel for momentum transfer is the dimensionless friction factor f , defined as

$$\tau = f \left(\frac{1}{2} \rho v^2 \right) = \left(\frac{fv}{2} \right) (\rho v - 0) \quad (13)$$

Thus $(fv/2)$ is like k and $(h/\rho C_p)$.

When these equations are written in these parallel forms, they automatically suggest the most common dimensionless groups. For example, the ratio of coefficients in Eqs. (3) and (10) is ν/D , the Schmidt number. The ratio of coefficients in Eqs. (11) and (13) is $(k/v)(2/f)$. Because $(2/f)$ is itself dimensionless, this is equivalent to k/v , the Stanton number.

These analogies, summarized in Table 2, have made my teaching of mass transfer more effective. I can see the light dawn on dozens of student faces: that quite pause, and then a slow "Oh, I see . . ." Try this yourself the next time you lecture on a problem like the wet bulb thermometer.

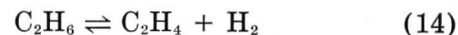
CHEMICAL REACTIONS (for Graduate Courses)

A third area in which we can improve our teaching concerns the coupling of mass transfer and chemical reaction. I have had trouble teaching

this material (even to graduate students) for two reasons. The first is that, in most curricula, courses in transport phenomena are very separate from those in chemical kinetics and reactor design. I could do a better job if the mass transfer and reaction courses were more completely integrated. We haven't solved this problem.

The second reason that mass transfer and reaction are poorly taught, which I struggle to avoid, hinges on the distinction between heterogeneous and homogeneous chemical reactions. I make this distinction either in chemical terms or in mathematical ones, and I often forget to tell the students which terms I am using.

I can best illustrate this by an example. Imagine we are discussing that old warhorse of reaction kinetics, ethane dehydrogenation on platinum. The overall reaction is simple.



My modeling of the reaction can be more subtle. If the platinum is a simple crystal, I treat this reaction as heterogeneous, occurring on the surface of the crystal. This is the route taken in chemistry courses or at the start of reaction engineering courses where mechanisms matter. However, if the catalyst is dispersed in a porous support, I discuss catalyst effectiveness factors as if the reaction were homogeneous. The chemistry is the same, but my mathematical treatment changes.

To look at this in more detail, consider the key

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results which I give for each case. I make the major approximation that the reaction is first order and irreversible, so that things stay simple. For the single crystal, I might begin with a mass balance on the ethane near the crystal

$$0 = D \frac{d^2 c_1}{dz^2} \quad (15)$$

The left hand side is zero because the reaction is assumed to be in steady state, and no reaction term appears because the reaction takes place only at the crystal's surface. After many machinations, I present the result

$$N_1 = \frac{c_{10}}{(1/k_1) + (1/\kappa_2)} \quad (16)$$

where N_1 is reaction rate in moles per crystal area per time and c_{10} is the bulk concentration. The quantity k_1 describes ethane mass transfer from the bulk to the surface: it is a mass transfer coefficient. The quantity κ_2 describes the reaction rate on the catalyst surface. Thus this process involves two steps in series, and is a chemical analog to the resistances in series found throughout transport phenomena.

In contrast, if I describe the same reaction in a porous catalyst, I begin with a mass balance on the ethane in the catalyst:

$$0 = D \frac{d^2 c_1}{dz^2} - \kappa_2' c_1 \quad (17)$$

Again, the value of zero on the left hand side indicates steady state, and the diffusion term accounts for ethane moving into the catalyst pores. Now, however, a reaction term is present, describing the same reaction, now being modeled as if it were homogeneous. The two rate constants are related, but in a non-trivial way

$$\kappa_2' = \frac{a}{(1/k_1) + (1/\kappa_2)} \quad (18)$$

where a is the surface area per volume of the catalyst, and k_1 now refers to mass transfer from the pores' core to the walls. The solution to Eq. (17) is

$$N_1 = (D\kappa_2')^{1/2} \coth\left(\frac{\kappa_2' d^2}{D}\right) c_{10} \quad (19)$$

where d is a length characteristic of the pellets' size. This is very different than the result reported in Eq. (16), even though the chemical reaction is identical. No wonder students get confused.

In my teaching, I try to reduce this confusion by first lecturing on heterogeneous reactions and then turning to homogeneous reactions. This split

is artificial, but seems pedagogically effective. In my discussion of heterogeneous reactions, I use examples from non-petrochemical areas like electrochemistry, fermentation, and detergency, since these nontraditional subjects will be more important to our present students than they have been in the past. In the lectures on homogeneous reactions, I cover the more familiar ground of catalyst effectiveness and gas treating with reactive solvents. My presentation is imperfect, but it seems better now than a few years ago.

Splitting the material on heterogeneous and homogeneous reactions does clarify the concept of "diffusion-control." This important area is often carelessly treated in research, even though it is incessantly quoted. It means three separate things. First, for a heterogeneous reaction, "diffusion-control" means that the reaction rate is not influenced by chemical kinetics, although it may be altered by chemical equilibria. The rate for a first order irreversible reaction is found by letting κ_2 become large in Eq. (16).

Second, for a homogeneous reaction, "diffusion-control" means that the reaction rate depends on both diffusion and kinetics. For the simple case above, the rate constant as κ_2' becomes large can be found from Eq. (19) to be $(D\kappa_2')^{1/2}$. Third, in chemistry, "diffusion-control" refers to a reaction governed by the Brownian motion in a well-mixed solution. Entering graduate students in engineering remember this third definition best.

CONCLUSIONS

I believe that we can improve our teaching of mass transfer by making three simple changes. First, we need to use a single, simple definition of mass transfer coefficients. Such a definition lets students reinforce their intuition and reduces problem solution by number plugging. Second, we need to alter our use of analogies, which is most easily effected by writing flux equations in more parallel terms. Third, we need to repeatedly stress the differences between the mathematical models used for chemical reactions and the actual chemistry of these reactions.

These three changes have improved my teaching. Still, I use them with a convert's pious zeal, which is infecting. They may not work for you, but I hope that they will galvanize you into continuing efforts to make mass transfer seem easier. After all, the unique parts of our profession deserve our finest efforts. □

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NOTATION

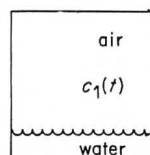
a	surface area per volume (Eq. 18)
c_1	concentration of species "1"
C_p	specific heat capacity
D	diffusion coefficient
f	friction factor (Eq. 13)
h	heat transfer coefficient (Eq. 12)
j_1	diffusion flux of species "1"
k	mass transfer coefficient
k', k'', k'''	alternative mass transfer coefficients (Table 1)
k	thermal conductivity
l	characteristic "film" thickness (Eq. 1)
n	number of moles
N_1	interfacial flux of species "1"
p	pressure
p_1	partial pressure of species "1"
q	heat flux
t	time
v	velocity
V	velocity of boundary (Eq. 8)
V	volume (Eq. 2)
x_1	mole fraction of species "1"
z	position
α	thermal diffusivity (Eq. 9)
κ_2, κ_2'	reaction rate constants (Eqs. 16-17)
μ	viscosity
ν	kinematic viscosity
ρ	density
τ	shear stress

APPENDIX: THREE SIMPLE EXAMPLES

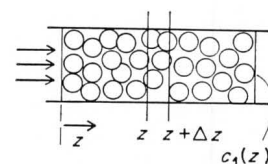
Example 1: Humidification

Water is evaporating into initially dry air in the closed vessel shown schematically in Fig. 1 (a). The vessel is isothermal at 25°C, so the water's vapor pressure is 23.8 mmHg. This vessel has 0.8 l of water with 150 cm² of surface area in a total volume of 19.2 l. After three minutes, the air is 5% saturated. What is the mass transfer coefficient? How long would it take to reach 90% saturation?

(a) Humidification



(b) Packed bed



(c) A gas bubble

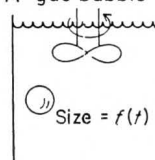


FIGURE 1. Three easy examples. Each situation leads to a simple problem. In (a) we assume that the air is at constant humidity except near the air-water interface. In (b) we assume that water flowing through the packed bed is well mixed except close to the solid sphere. In (c) we assume that the liquid is at constant composition except near the bubble surface.

Solution

The flux at three minutes can be found directly from the values given

$$\begin{aligned}
 N_1 &= \frac{(\text{vapor concentration}) (\text{air volume})}{(\text{liquid area}) (\text{time})} \\
 &= \frac{0.05 \left(\frac{23.8}{760} \right) 1 \text{ mol} \left(\frac{273}{298} \right) 18.4 \text{ l}}{150 \text{ cm}^2 (180 \text{ sec})} \\
 &= 4.4 \cdot 10^{-8} \frac{\text{mol}}{\text{cm}^2 \cdot \text{sec}} \quad (\text{A-1})
 \end{aligned}$$

The concentration difference is that at the liquid's surface minus that in the bulk solution. That at the liquid's surface is the value at saturation; that in the bulk at short times is essentially zero. Thus from Eq. (1) we have

$$\begin{aligned}
 4.4 \cdot 10^{-8} \frac{\text{mol}}{\text{cm}^2 \cdot \text{sec}} &= k \left(\frac{23.8}{760} \frac{1 \text{ mol}}{22.4 \cdot 10^3 \text{ cm}^3} \frac{273}{298} - 0 \right) \\
 k &= 3.4 \cdot 10^{-2} \frac{\text{cm}}{\text{sec}} \quad (\text{A-2})
 \end{aligned}$$

This value is lower than that commonly found for transfer in gases.

The time required for 90% saturation can be found from a mass balance: (accumulation in gas phase) = (evaporation rate)

$$\frac{d}{dt} V c_1 = A N_1 = A k [c_1(\text{sat}) - c_1] \quad (\text{A-3})$$

The air is initially dry, so

$$t = 0 \quad c_1 = 0 \quad (\text{A-4})$$

We use this condition to integrate the mass balance

$$\frac{c_1}{c_1(\text{sat})} = 1 - \exp[-(kA/V)t] \quad (\text{A-5})$$

Rearranging the equation and inserting the values given, we find

$$\begin{aligned} t &= -\frac{V}{kA} \ln \left(1 - \frac{c_1}{c_1(\text{sat})} \right) \\ &= -\frac{18.4 \cdot 10^3 \text{ cm}^3}{3.4 \cdot 10^{-2} \frac{\text{cm}}{\text{sec}} \cdot 150 \text{ cm}^2} \ln(1 - 0.9) \\ &= 8.3 \cdot 10^3 \text{ sec} = 2.3 \text{ hrs} \end{aligned} \quad (\text{A-6})$$

It takes over two hours to reach 90% saturation.

Example 2: Mass Transfer in a Packed Bed

Spheres of benzoic acid 0.2 cm in diameter are packed into a bed like that shown schematically in Fig. 1(b). The spheres have 23 cm² surface per cm³ bed. Pure water flowing at a superficial velocity of 5 cm/sec into the bed is 62% saturated with benzoic acid after it has passed through 100 cm of bed. What is the mass transfer coefficient?

Solution

The answer to this problem depends on the concentration difference used in the definition of the mass transfer coefficient. In every definition, we choose this difference as the value at the sphere's surface minus that in the solution. However, we can define different mass transfer coefficients by choosing the concentration difference at various positions in the bed. For example, we can choose the concentration difference at the bed's entrance and so obtain

$$\begin{aligned} N_1 &= \frac{0.62 c_1(\text{sat}) (v^0)}{a(lA)} = k(c_1(\text{sat}) - 0) \\ &= \frac{0.62 c_1(\text{sat}) 5 (\text{cm/sec}) A}{23 (\text{cm}^2/\text{cm}^3) (100 \text{ cm}) A} \end{aligned} \quad (\text{A-7})$$

where A is the bed's cross section. Thus

$$k = 1.3 \cdot 10^{-3} \frac{\text{cm}}{\text{sec}} \quad (\text{A-8})$$

This definition for the mass transfer coefficient is infrequently used.

Alternatively, we can choose as our concentration difference that at a position z in the bed, and write a mass balance on a differential volume AΔz at this position: accumulation = (flow in minus out) + (amount of dissolution)

$$0 = A \left[c_1 v^0 \Big|_z - c_1 v^0 \Big|_{z+\Delta z} \right] + (A\Delta z) a N_1 \quad (\text{A-9})$$

Substituting for N₁ from Eq. (1), dividing by AΔz, and taking the limit as Δz goes to zero, we find

$$\frac{dc_1}{dz} = \frac{ka}{v^0} [c_1(\text{sat}) - c_1] \quad (\text{A-10})$$

This is subject to the initial condition

$$z = 0 \quad c_1 = 0 \quad (\text{A-11})$$

Integrating, we obtain an exponential of the same form as in the first example

$$\frac{c_1}{c_1(\text{sat})} = 1 - \exp[-(ka/v^0)z] \quad (\text{A-12})$$

Rearranging the equation and inserting the values given, we find

$$\begin{aligned} k &= -\left(\frac{v^0}{az} \right) \ln \left(1 - \frac{c_1}{c_1(\text{sat})} \right) \\ &= -\frac{5 \text{ cm/sec}}{(23 \text{ cm}^2/\text{cm}^3) (100 \text{ cm})} \ln(1 - 0.62) \\ &= 2.1 \cdot 10^{-3} \frac{\text{cm}}{\text{sec}} \end{aligned} \quad (\text{A-13})$$

This value is typical of those found in liquids. This type of mass transfer coefficient definition is the same as a log-mean value.

Example 3: Mass Transfer from an Oxygen Bubble

A bubble of oxygen originally 0.1 cm in diameter is injected into excess stirred water as shown schematically in Fig. 1(c). After 7 min, the bubble is 0.054 cm in diameter. What is the mass transfer coefficient?

Solution

This time, we write a mass balance not on the surrounding solution but on the bubble itself

$$\frac{d}{dt} (c_1 \frac{4}{3} \pi r^3) = AN_1 = -4\pi r^2 k (c_1(\text{sat}) - 0) \quad (\text{A-14})$$

where r is the radius of the bubble at any time, t. This equation is tricky: c₁ refers to the oxygen concentration in the bubble, (1 mol/22.4l) at standard conditions; but c₁(sat) refers to the oxygen concentration at saturation in water, about 1.5 · 10⁻³ mol/l under similar conditions. Thus

$$\frac{dr}{dt} = -k \frac{c_1(\text{sat})}{c_1} = -0.034 k \quad (\text{A-15})$$

This is subject to the condition

$$t = 0 \quad r = 0.05 \text{ cm} \quad (\text{A-16})$$

so integration gives

$$r = 0.05 \text{ cm} - 0.034 \text{ kt} \quad (\text{A-17})$$

Inserting the numerical values given, we find

$$\begin{aligned} 0.027 \text{ cm} &= 0.05 \text{ cm} - 0.034 \text{ k}(420 \text{ sec}) \\ &= 1.6 \cdot 10^{-3} \text{ cm/sec} \end{aligned} \quad (\text{A-18})$$

Remember that this coefficient is defined in terms of the concentration in the liquid, and would be numerically different if it were defined in terms of the gas phase concentration. \square

DEPARTMENT: Syracuse University

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voted to an investigation of transport mechanisms of small molecules in rubbery and glassy polymers, and to some important applications of these mechanisms. The applications include the development of high-selectivity, high-flux membranes for fluid separation processes, the design of controlled drug delivery systems, and the improvement of adhesion of integrated circuits.

Lawrence L. Tavlarides is conducting research on chemically reactive turbulent liquid dispersion and chemical reaction kinetics. The objective of the research in liquid dispersions is to provide a fundamental basis for the design and scale up of extractors and reactors. The microscopic droplet rate processes of coalescence and breakup are analyzed with population balance equations and Monte Carlo simulation techniques. Hydrodynamic turbulent flow models are also developed to predict the local turbulent kinetic energy and energy dissipation in mechanically agitated turbulent dispersions. Hydrometallurgical solvent extraction in Tavlarides' group is focussed on multiple metal chelation reactions and chemical equilibria. Intrinsic chemical kinetic models are developed for reactions occurring at or near the liquid-liquid interface using the novel liquid jet recycle reactor. Thermodynamic based heterogeneous equilibrium models are also developed. The above simulation models are employed with the kinetic and equilibrium models to predict conversion and selectivity. Other studies by this group include catalyst deactivation by surface carbon to determine kinetic rate models for synthesis reactions.

Chi Tien's major research activities are in three areas: fluid-particle technology, liquid phase adsorption, and biochemical engineering. The fluid-particle technology research covers a broad spectrum of topics including deep bed filtration of liquid suspensions, aerosol filtration in granular

and fibrous media, and in fluidized beds with or without magnetic stabilization, and stratification and segregation of particles in sedimentation and liquid fluidized beds. In filtration research a theoretical framework which incorporates all the important aspects of filtration process has been developed to quantitatively describe the dynamic behavior of the process.

In liquid phase adsorption studies, efficient algorithms are developed for exact and detailed adsorption calculations in various process configurations involving systems with arbitrarily large number of adsorbates, formulation of the species-grouping procedure for simplifying multicomponent adsorption calculations and establishment of a characterization procedure which describes gas solution with unknown adsorbates as solution with a fixed number of pseudo-species of adsorbates.

In biochemical engineering research, Tien's group is studying the interaction between adsorption and bacterial growth when granular activated carbon is used to treat liquid waste containing both adsorbable and biodegradable organic substrates. The work is applied to fluidized bed biofilm reactor design.

Professor Vook is studying various properties of current carrying and sliding electrical contacts. The work is carried out in an ultra high vacuum system where electrical contact resistance, friction coefficient, and the chemical composition of the slip ring surface (by Auger electron spectroscopy) are measured in situ as a function of contact force, current through the contact, and gaseous lubricating environment. The goal of this work is to understand the physical and chemical forces that limit the current-carrying capacity of the moving (rotating) electrical contact. Vook is also developing thin film coatings and surface pretreatments for preventing the out-of-core radioactive buildup that occurs on austenitic stainless steels used in boiling water nuclear reactors.

Chiu-Sen Wang is on leave for a few years and working at CalTech in the area of particle deposition in branched airways (e.g. the lungs), and in aerosols.

These projects illustrate the breadth and depth of the research interests of the faculty at Syracuse. There is breadth in the number and variety of research interests and depth in that several faculty members work in the key areas of separation and transport processes, chemical reaction engineering, fluid-particle technology, and materials science. \square