THE BURNING OF A LIQUID OIL DROPLET

A SIMPLE MATHEMATICAL ANALYSIS FOR TEACHING PURPOSES

A. N. HAYHURST, R. M. NEDDERMAN
University of Cambridge
Cambridge, CB2 3RA England

THE QUASI-STEADY STATE combustion of a single isolated liquid oil droplet has been considered by several authors [1-5]. We have found it necessary in our teaching to seek a simpler treatment than those usually presented. Below we consider such a single drop, burning in stagnant air without any effects from natural convection or radiative heat transfer. Of course, once this, the simplest situation, has been described mathematically, it is possible to treat more realistic and complicated cases. The experimental observations are clear, and excellent reviews are available [1]. The fundamental empirical fact is that the radius, a, of a liquid oil droplet decreases with time, t, according to

$$-\frac{d(a^2)}{dt} = \text{constant, } k_b \tag{1}$$

during its burning. This paper gives a rigorous derivation of k_b. Unlike the standard textbooks [2, 3], the approach below uses entirely molar, rather than mass, units because of the much simpler description of diffusion which results. Like Long's analysis [4], but in contrast with Spalding's [2], this treatment takes the products of combustion into account. However, unlike Long, we calculate, from first principles, the temperature of the flame front surrounding the burning droplet. All these approaches rely on Godsave's early experimental and theoretical work [5].

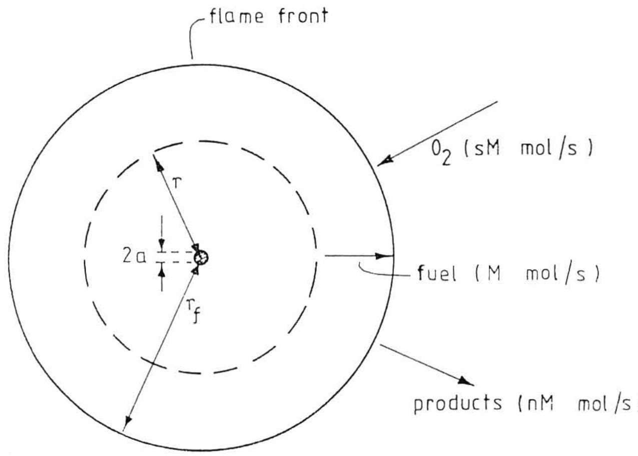
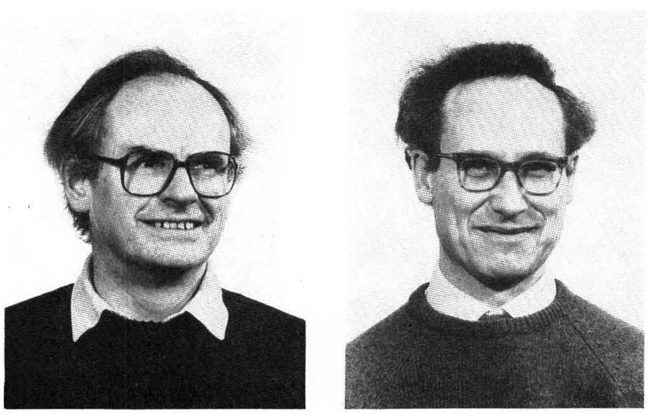


***FIGURE 1.** Sketch of burning oil droplet surrounded by spherical flame front and sphere of general radius r. The total radial flow rates of fuel, O_2 and products are as shown.*



Allan Hayhurst is a lecturer in chemical engineering at Cambridge University and a Fellow of Queens' College. He received his PhD degree from Cambridge where he has spent most of his career, apart from seven years at Sheffield University. His research interests include combustion and reactions in fluidised beds. (**L**)

Ronald Nedderman is a lecturer in chemical engineering at Cambridge University and a Fellow of Trinity College. His research interests include stress and velocity predictions in flowing granular materials with particular emphasis on silo design. He has never done any research on combustion but enjoys interfering in other people's algebra. (**R**)

DIFFUSION

To formulate the equations for diffusion we first consider a binary mixture of two gases A and B moving together in a one-dimensional situation. The molar flux of A in the x-direction (for a stationary observer) is given by

The oil droplet, depicted in Figure 1, is assumed to be at a constant temperature which is just less than, but close to, the boiling-point of the liquid fuel. While the droplet evaporates, vapour of the fuel diffuses outward; oxygen from the surrounding air diffuses inward.

$$N_A = N\frac{c_A}{c} - D\frac{dc_A}{dx} = Ny_A - D\frac{dc_A}{dx} \qquad (2)$$

where c_A and c_B are the molar concentrations of A and B, c (= c_A + c_B) is the total molar concentration, N is the total molar flux, *i.e.*, $N_A + N_B$, and y_A is the mole fraction of A. Eq. (2) derives from the fact that the flux of A consists of a convective term, Ny_A, and a diffusive term, $-D\,dc_A/dx$. If A is in a multicomponent mixture of gases (temperature T and total pressure P) and confined to a one-dimensional situation, then Eq. (2) becomes

$$N_A = Ny_A - \frac{D_A P}{RT} \cdot \frac{dy_A}{dx} \qquad (3)$$

This assumes that multicomponent diffusion can be simplified to a pseudobinary description with a constant effective diffusivity, D_A. Eq. (2) is identical with the possibly more familiar [6] form:

$$N_A c_B - N_B c_A = -cD \cdot \frac{dc_A}{dx}$$

By analogy with Eq. (2) the equation for the enthalpy flux, Q, is

$$Q = Nh - k\frac{dT}{dx} \qquad (4)$$

where k is the thermal conductivity and h is the molar enthalpy of the particular gas mixture.

THE MODEL

The oil droplet, depicted in Figure 1, is assumed to be at a constant temperature, T_o, which is just less than, but close to, the boiling-point of the liquid fuel. While the droplet evaporates, vapour of the fuel diffuses outward; oxygen from the surrounding air diffuses inward. Fuel and oxygen meet in stoichiometric amounts and react very rapidly in a thin reaction zone, or flame front, at a distance r_f from the centre of the dropet. Here the oxidation reaction is assumed to be

$$1 \text{ fuel} + s\, O_2 \rightarrow n \text{ products}$$

and is taken to occur almost instantaneously. Consequently, the ratio of the molar fluxes of O_2 and fuel into the flame front is s. No distinction will be made at this stage between the products CO_2 and H_2O, although this is a straightforward thing to do, if it were considered necessary. Thus a total of n moles of products diffuses radially outwards for each mole of fuel consumed. The situation in Figure 1 has spherical symmetry.

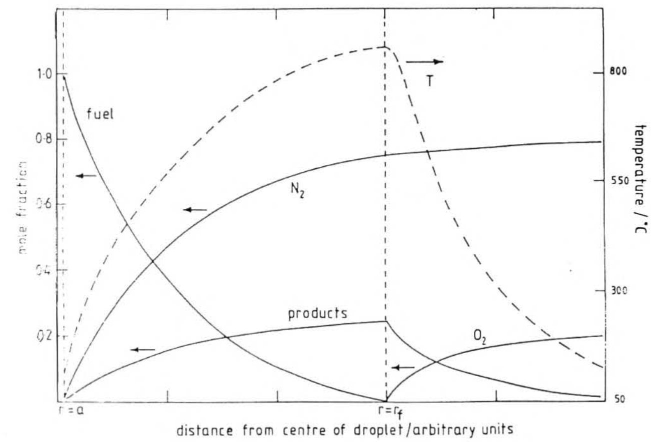


FIGURE 2. Approximate sketch of mole fractions of fuel vapour, O_2, N_2 and combustion products around liquid droplet, together with gas temperature.

Figure 2 sketches the expected concentration profiles of fuel, O_2, N_2 and products. It will be noticed that for a radius $r < r_f$ there is no oxygen, and for $r > r_f$ there is no fuel. The concentrations of both oxygen and fuel in the reaction zone are taken to be in effect zero, *i.e.*, reaction is rapid. As the droplet contracts, both N_2 and products diffuse inward to fill part of the space formerly occupied by the liquid fuel. The fluxes of N_2 and products toward the shrinking droplet's surface must be small, since the upper limit of their sum is (ρ_{gas}/ρ_{liq}) times the flux of fuel outwards away from the drop. Here (ρ_{gas}/ρ_{liq}) is the ratio of the molar density of the gas around the drop to that of the liquid fuel; its value will be roughly 1.5×10^{-3}. These considerations of the limiting case, when all the evaporating liquid is replaced by N_2 and products, establish that the flux of, *e.g.*, N_2 is everywhere negligible, as also is the flux of products in the region $a < r < r_f$.

Of the heat liberated in the reaction zone, some is conducted back to the droplet and provides the latent heat of vaporisation. The rest is convected or conducted outward from the flame front. Radiative effects are ignored here. A rough sketch of the temperature of the gases around the droplet is also given in

Figure 2. The temperature is assumed to have a maximum value of T_f in the reaction zone, which is well above T_∞, the temperature of the surrounding air. A calculation given below indicates that T_f can be as high as 900 °C.

The above model is one controlled by diffusion and heat transfer, with chemical kinetics being relatively fast. Suppose that M mol/s of fuel evaporate from the drop. The steady state rate of heat production in the flame front, where the temperature is a maximum, is accordingly $-M\Delta H°$ (*N.B.* the molar heat of combustion of the fuel, $\Delta H° < 0$). Of this heat an amount $M\lambda$ is conducted inward to the droplet surface, λ being the molar latent heat of evaporation. Thus for $r > r_f$, the total net outflow of heat past any spherical surface is

$$-(\Delta H° + \lambda)\, M$$

In addition, the total inflow of oxygen toward the reaction zone is sM mol/s and the total outflow of products is nM mol/s (see Figure 1).

In principle, there are five basic unknowns in this problem: the temperature of the drop, T_o, and the corresponding vapour pressure, P_v, of the fuel; the evaporation rate, M; the radius, r_f; and temperature, T_f, of the flame front. These can be evaluated by considering

1. **Heat transfer from the flame front to the drop**
2. **Heat transfer outward from the flame front**
3. **Mass transfer of fuel from the droplet to the flame front**
4. **Mass transfer of oxygen inward to the flame front**
5. **The dependence of vapour pressure of the fuel on temperature.**

In fact, it turns out that the drop is close to its boiling point, so that P_v is approximately atmospheric pressure. Consideration of 1, 2 and 4 is now sufficient to predict M, r_f and T_f and the remaining two relationships can be used to confirm the starting assumption that the drop is very close to its boiling point.

DIFFUSION OF OXYGEN OUTSIDE THE FLAME FRONT

Here we consider oxygen diffusing from the surrounding air to the reaction zone. The flux of O_2 past a sphere of radius r is

$$-(sM)/(4\pi r^2)$$

with the negative sign denoting an inward flux. Substitution into Eq. (3) gives

$$-\frac{sM}{4\pi r^2} = \frac{(n-s)My_{O_2}}{4\pi r^2} - \frac{D_{O_2}P}{RT}\cdot\frac{dy_{O_2}}{dr}$$

This equation assumes the total flux of all species to be

$$(n-s)M/4\pi r^2$$

i.e., the difference between the fluxes of products outward and O_2 inward. That the flux of N_2 in this region of space is negligible was demonstrated above. The above equation gives

$$\frac{M}{4\pi}\int_{r_f}^{\infty}\frac{dr}{r^2} = \frac{D_{O_2}P}{RT}\int_0^{y_\infty}\frac{dy_{O_2}}{(n-s)y_{O_2} + s}$$

for a mole fraction, y_∞, of O_2 far away from the droplet. Integration yields

$$\frac{M}{4\pi r_f} = \frac{D_{O_2}P}{RT(n-s)}\,\ell n\left(1 + \frac{(n-s)y_\infty}{s}\right) \tag{5}$$

In deriving Eq. (5) it has been assumed that D_{O_2}/T is independent of temperature, so that D_{O_2} is a mean value for the range $T_\infty < T < T_f$. In fact, $D_{O_2} \alpha T^{5/3}$ would have been a better approximation [7], but the need for simplicity is paramount and a careful averaging is not attempted at this stage.

ENTHALPY BALANCE FOR $r < r_f$

There is only fuel (M mol/s) diffusing outward in the region $a < r < r_f$, with heat being conducted inward to the droplet at a rate $M\lambda$. In the steady state the total enthalpy flow rate past an arbitrary surface of radius r is given by Eq. (4) as

$$4\pi r^2\, Q = 4\pi r^2\left(-k_f\frac{dT}{dr}\right) + M(H_f^\circ + c_fT)$$

Here k_f is the thermal conductivity of the fuel-rich gases for $r < r_f$, c_f is the molar specific heat of the fuel and $h = H_f^\circ + c_fT$ is the molar enthalpy of the fuel at temperature T. If T is in °C, then H_f° is the fuel's standard molar enthalpy referred to 0°C as the datum for enthalpy. It is convenient to use 0°C as the reference temperature for enthalpy, as it avoids an explicit statement of where the enthalpy datum lies. The above expression for the net energy flow rate past any sphere of general radius r can be calculated at the droplet's surface. Here the rate of heat conduction inward to the droplet from the surrounding gases is $M\lambda$ (see above) and the molar enthalpy of the vapour leaving the droplet is $(H_f^\circ + c_fT_o)$, where T_o can be taken to be the boiling point of the fuel in °C. Equating the enthalpy flow rates past spheres at r = r and r = a gives

$$4\pi r^2 \left(-k_f \frac{dT}{dr}\right) + M(H_f^\circ + c_f T) = -M\lambda + M(H_f^\circ + c_f T_o)$$

This simplifies to

$$4\pi r^2 k_f \frac{dT}{dr} = M\{\lambda + c_f(T-T_o)\}$$

or

$$\frac{M}{4\pi}\int_a^{r_f} \frac{dr}{r^2} = \int_{T_o}^{T_f} \frac{k_f\, dT}{\lambda + c_f(T-T_o)}$$

i.e.,

$$\frac{M}{4\pi}\left\{\frac{1}{a} - \frac{1}{r_f}\right\} = \frac{k_f}{c_f}\,\ell n\left\{1 + \frac{c_f(T_f-T_o)}{\lambda}\right\} \qquad (6)$$

Eq. (6) assumes mean values of k_f and c_f for the interval T_o to T_f.

ENTHALPY BALANCE FOR $r > r_f$

If combustion in the flame front is adiabatic, so that, *e.g.*, there are no gains or losses of energy by radiation, then the rate of energy flow past any surface r = r equals the rate of flow across the droplet's surface at r = a. In the steady state this can be expressed as

$$-M\lambda + M(H_f^\circ + c_f T_o) = -4\pi r^2 k_o \frac{dT}{dr} + M\{n(H_p^\circ + c_p T) - s(H_{O_2}^\circ + c_{O_2} T)\} \qquad (7)$$

In Eq. (7) positive signs represent outward radial flows. Again, $H_{O_2}^\circ$ and H_p° are, respectively, the standard molar enthalpies of oxygen and products (lumped together, with no distinction between the triatomic species CO_2 and H_2O), but referred to 0°C. The thermal conductivity of this oxygen-rich mixture is k_o. By definition ΔH°, the heat of combustion at 0°C, equals

$$nH_p^\circ - sH_{O_2}^\circ - H_f^\circ$$

so that Eq. (7) simplifies to

$$-M(\Delta H^\circ + \lambda) = M\{T(nc_p - sc_{O_2}) - c_f T_o\} - 4\pi r^2 k_o \frac{dT}{dr}$$

The left-hand side of this equation corresponds to there being an outward flow rate of heat equal to $-M(\Delta H^\circ + \lambda)$ past a surface of given r ($> r_f$), as discussed above. Rearrangement yields

$$\frac{M}{4\pi}\int_{r_f}^{\infty} \frac{dr}{r^2} = \int_{T_f}^{T_\infty} \frac{k_o\, dT}{(nc_p - sc_{O_2})T + \Delta H^\circ + \lambda - c_f T_o}$$

i.e.

$$\frac{M}{4\pi r_f} = \frac{k_o}{(nc_p - sc_{O_2})}\,\ell n\left\{\frac{(nc_p - sc_{O_2})T_\infty + \Delta H^\circ + \lambda - c_f T_o}{(nc_p - sc_{O_2})T_f + \Delta H^\circ + \lambda - c_f T_o}\right\} \qquad (8)$$

This assumes that specific heats and k_o do not vary with temperature. Addition of Eqs.(6) and (8) gives

$$\frac{M}{4\pi a} = \frac{k_f}{c_f}\,\ell n\left\{1 + \frac{c_f(T_f - T_o)}{\lambda}\right\} + \frac{k_o}{(nc_p - sc_{O_2})}\,\ell n\left\{\frac{(nc_p - sc_{O_2})T_\infty + \Delta H^\circ + \lambda - c_f T_o}{(nc_p - sc_{O_2})T_f + \Delta H^\circ + \lambda - c_f T_o}\right\} \qquad (9)$$

The subsequent calculation can be simplified by assuming that

$$|\Delta H^\circ| \gg (\lambda - c_f T_o)$$

as is borne out in reality. Also, if a mean specific heat, c_o, for the fuel-lean gases at $r > r_f$ is defined, so that

$$(n-s)c_o = nc_p - sc_{O_2}$$

then Eqs. (8) and (9), respectively, become

$$\frac{M}{4\pi r_f} = \frac{k_o}{c_o(n-s)}\,\ell n\left\{\frac{(n-s)c_o T_\infty + \Delta H^\circ}{(n-s)c_o T_f + \Delta H^\circ}\right\} \qquad (10)$$

and

$$\frac{M}{4\pi a} = \frac{k_f}{c_f}\,\ell n\left\{1 + \frac{c_f(T_f - T_o)}{\lambda}\right\} + \frac{k_o}{(n-s)c_o}\,\ell n\left\{\frac{(n-s)c_o T_\infty + \Delta H^\circ}{(n-s)c_o T_f + \Delta H^\circ}\right\} \qquad (11)$$

Eqs.(5) and (10) can now be equated to eliminate $(M/4\pi r_f)$ and give

$$\frac{PD_{O_2}}{RT}\,\ell n\left\{1 + \frac{(n-s)y_\infty}{s}\right\} = \frac{k_o}{c_o}\,\ell n\left\{\frac{(n-s)c_o T_\infty + \Delta H^\circ}{(n-s)c_o T_f + \Delta H^\circ}\right\}$$

which gives the value of T_f. The last, but strictly-speaking unnecessary, assumption can now be made to simplify the algebra. This is to recognise the dimensionless group

$$(k_o RT/PD_{O_2} c_o)$$

as the Lewis number, or the ratio of the Schmidt and Prandtl numbers for the oxygen-rich gases at $r > r_f$. Simple versions of the kinetic theory of gases give a value of unity for the Lewis number. This is often a

Continued on page 149.

LIQUID OIL DROPLET

Continued from page 129.

good approximation, as discussed below, in which case the above equation becomes

$$1 + \frac{(n-s)y_\infty}{s} = \frac{(n-s)c_o T_\infty + \Delta H^\circ}{(n-s)c_o T_f + \Delta H^\circ} \qquad (12)$$

or

$$T_f = \frac{s\ T_\infty}{[s + (n-s)y_\infty]} - \frac{\Delta H^\circ\ y_\infty}{c_o[s + (n-s)y_\infty]} \qquad (13)$$

This is the simplest expression for T_f, the temperature of the flame front and, in fact, on detailed examination shows that T_f is not equal to the adiabatic flame temperature, as assumed by Long [4]. Eqs. (12) and (13) can now be substituted into Eq. (11), giving

$$\frac{M}{4\pi a} = \frac{k_f}{c_f}\ \ell n \left\{1 + \frac{c_f[c_o T_\infty - \Delta H^\circ y_\infty/s - c_o T_o(1 + (n-s)y_\infty/s)]}{c_o\lambda\left[1 + (n-s)\dfrac{y_\infty}{s}\right]}\right\} + \frac{k_o}{c_o(n-s)}\ \ell n \left[1 + (n-s)\frac{y_\infty}{s}\right] \qquad (14)$$

This now enables a simple expression to be calculated for the burning constant k_b (see Eq. (1)) using

$$M = -\frac{d}{dt}\left(\frac{4}{3}\pi a^3 \rho_{liq}\right) = -4\pi a^2 \rho_{liq}\frac{da}{dt}$$

or

$$-\frac{d}{dt}(a^2) = k_b = \frac{M}{2\pi a \rho_{liq}}$$

giving

$$k_b = \frac{2k_f}{c_f \rho_{liq}}\ \ell n \left\{1 + \frac{c_f[c_o T_\infty - \Delta H^\circ y_\infty/s - c_o T_o(1+(n-s)y_\infty/s)]}{c_o\lambda\left[1 + (n-s)\dfrac{y_\infty}{s}\right]}\right\} + \frac{2k_o}{c_o(n-s)\rho_{liq}}\ \ell n \left[1 + \frac{(n-s)y_\infty}{s}\right] \qquad (15)$$

Of course, a somewhat more cumbersome form of Eq. (15) can be derived by not using the above approximations of an averaged specific heat, c_o, for O_2 and the products of combustion or of unit Lewis number, *etc.* In fact, Eqs. (5), (8), and (9) without these assumptions lead to the general expression

$$k_b = \frac{2k_f}{c_f\rho_{liq}}\ \ell n \left\{1 + \frac{c_f T_\infty + \dfrac{c_f(\Delta H^\circ + \lambda - c_f T_o)}{(nc_p - sc_{O_2})}\left[1 - \left(1 + \dfrac{(n-s)y_\infty}{s}\right)^\alpha\right] - c_f T_o\left(1 + \dfrac{(n-s)y_\infty}{s}\right)^\alpha}{\lambda\left(1 + \dfrac{(n-s)y_\infty}{s}\right)^\alpha}\right\} + \frac{2k_o\alpha}{\rho_{liq}(nc_p - sc_{O_2})}\ \ell n\left[1 + \frac{(n-s)y_\infty}{s}\right] \qquad (16)$$

Here the dimensionless group α is given by

$$\alpha = PD_{O_2}c_o/RTk_o$$

and is the reciprocal of a Lewis number. Of course, Eq. (15) is preferred over Eq. (16) for teaching purposes.

DISCUSSION

Eqs. (15) and (16) each contain two terms, and in that sense are similar to Long's result [4]. However, our second term is different from Long's, which we conclude to be in error. Substitution of numerical values into either Eq. (15) or (16) indicates that the second term is 1-2% of the first term. In this case k_b can be taken to be the first term of either Eq. (15) or (16), depending on the precision required. Fixing attention on Eq. (15) for a fairly heavy paraffin being burnt according to

$$C_i H_{2i+2} + \frac{(3i+1)}{2} O_2 \rightarrow iCO_2 + (i+1)\ H_2O$$

we have

$$n/s = 2(2i+1)/(3i+1) \approx 4/3$$

provided $i > 5$. As a result

$$(1 + (n-s)y_\infty/s) \approx 1 + y_\infty/3 \approx 1.07$$

which can be taken as unity. Consequently, the first term in Eq. (15) can be simplified to

$$k_b = \frac{2k_f}{c_f\rho_{liq}}\ \ell n \left\{1 + \frac{c_f[(T_\infty - T_o) - \Delta H^\circ y_\infty/c_o s]}{\lambda}\right\} \qquad (17)$$

which is similar to Spalding's result [2]. In fact, it is identical to Spalding's result, when $c_f = c_o$. Spalding's derivation [2] is interestingly different from that given above; in particular, it requires the possibly unpalatable assumption that diffusion for both $r < r_f$ and $r > r_f$ can be modelled by one composite substance of fuel and "negative" oxygen, with complete neglect of the products of combustion. As discussed below, Spalding's assumption that $c_f = c_o$ is often not a good one.

Some insight can be gained into the above conclusion that the ratio of the second term to the first in both Eqs. (15) and (16) is small. Comparison of Eqs. (6), (11), and (14) reveals that the ratio of the first and second terms in both Eqs. (15) and (16) is r_f/a. Substitution of the values of physical properties (see below) shows that r_f/a is roughly in the range 50-100. Thus neglect of the second term in Eqs. (15) or (16) generates an uncertainty of less than 2%. Perhaps the best expression for k_b from Eq. (16) neglects the second term and assumes

$$1 >> (n-s)y_\infty/s$$

but does not assume unit Lewis number. The result is

$$k_b = \frac{2k_f}{c_f \rho_{liq}} \ln \left[1 + \frac{c_f(T_\infty - T_o) - c_f \Delta H^\circ \alpha y_\infty / s c_o}{\lambda}\right] \tag{18}$$

NUMERICAL CONSIDERATIONS

1. First we estimate T_f by substituting values into Eq. (13). Consider toluene burning in $C_7H_8 + 9O_2 \rightarrow 7CO_2 + 4H_2O$, for which $\Delta H^\circ = -3.9 \times 10^6$ J/mol, $\lambda = 3.2 \times 10^4$ J/mol, $s = 9$, $y_\infty = 0.21$ and $n = 11$. At the quite arbitrary temperature of 700 K, the molar heat capacities of O_2, H_2O and CO_2 are [8], respectively, 33.0, 37.5 and 49.6 J/(mol K). This gives $2c_o = 7 \times 49.6 + 4 \times 37.5 - 9 \times 33.0$ or $c_o = 100$ J/(mol K). Also, these data give $T_f = 1149$ K for an ambient air temperature $T_\infty = 20°C$. This value of T_f is less than the adiabatic temperature of around 2300 K, in contrast with Long's assumption [4] that the two are identical. This discrepancy arises partly because c_o is much greater than the heat capacities of O_2, H_2O or CO_2. Even so, c_o is not as large as the specific heat of the fuel, with [8] *e.g.*, $c_f = 173$, 208 and 358 J/(mol K), respectively, for benzene, toluene and n-octane. Spalding's simplification [2] that $c_f = c_o$ accordingly should be treated with great care.

2. The above data also give $r_f/a = 49$ (again for toluene burning in air at 20°C), from a comparison of the two terms in Eq. (15). However, if the more precise Eq. (16) is used, then $r_f/a = 72$ is obtained. For these calculations values of k_f and k_o were taken [8] to be 0.065 and 0.050 J/(K s m), respectively, and $T_o = 383$ K (boiling point of toluene = 110.6°C) was assumed. Such a magnitude for r_f/a justifies the earlier neglect of the second term in Eqs. (15) and (16).

3. The previous assumption of unit Lewis number is worthy of comment. In fact, the value [9] for single gases at atmospheric pressure is close to unity at 0°C, but can rise to around 1.3 at 700 K. The magnitude of α (the reciprocal of a Lewis number) used in Eqs. (16) and (18) turns out to be 2.6 at 700 K, but this is due to the unexpectedly large value of c_o.

4. We are now in a position to calculate the burning constant, k_b. Eq. (16) is expected to yield the most precise value of it, and Eq. (17) the crudest. Eq. (18) is relatively simple and unlike Eq. (17), does not take the Lewis number to be unity. The resulting values of k_b are 1.7×10^{-7}, 1.2×10^{-7} and 1.8×10^{-7} m²/s, as calculated from Eqs. (16)-(18), respectively. The second term in Eq. (16) was ignored in this calculation, which assumed $\rho_{liq} = 9.6$ kmol/m³. The experimental value [3, 4, 5] of k_b is around 1.8×10^{-7} m²/s, which suggests that Eqs. (16) and (18) both provide satisfactory estimates. However, it will be noted that in each case the calculated k_b is proportional to the ratio k_f/c_f, which for pure toluene increases by a factor of 3.1 from its boiling point (110.6°C) to 1000°C. Hence, by a *judicious* choice of mean temperature for estimating k_f/c_f, a match between theory and experiment can always be obtained. Long [4] used 1300 K (*i.e.*, greater than T_f as calculated here for toluene), whereas 700 K was arbitrarily used above. Clearly, for any greater precision, the above theory has to be re-worked to cope with k_f and c_f varying with temperature and also composition, because traces of H_2 from pyrolysis of the fuel will have a significant effect on k_f. Otherwise, it is worth noting that Eq. (18) is a useful and more precise simplification of Eq. (16) than Eq. (17).

5. It was assumed above that T_o, the droplet's temperature, was in effect the normal boiling point of the liquid concerned. This assumption can be checked by first considering mass transfer of fuel from the droplet (at $r = a$) to the flame front at $r = r_f$. Here the total outward flow of fuel vapour is M mol/s and Eq. (3) becomes

$$\frac{M}{4\pi r^2} = \frac{M}{4\pi r^2} y_f - \frac{D_f P}{RT} \cdot \frac{dy_f}{dr}$$

where y_f and D_f are, respectively, the mole fraction and diffusivity of the fuel. Hence

$$\frac{M}{4\pi} \int_a^{r_f} \frac{dr}{r^2} = - \frac{D_f P}{RT} \int_{y_{fo}}^{o} \frac{dy_f}{(1 - y_f)}$$

where $y_f = y_{fo}$ at the droplet's surface ($r = a$). The left hand side of this equation is given by Eq. (6), so that after integration, assuming D_f/T to be constant

$$\frac{k_f}{c_f} \ln \left(1 + \frac{c_f(T_f - T_o)}{\lambda}\right) = \frac{D_f P}{RT} \ln \left(\frac{1}{1 - y_{fo}}\right)$$

As a good approximation the Lewis number for the fuel ($k_f RT/PD_f c_f$) can be taken to be unity, leading to

$$y_{fo} = \frac{c_f(T_f - T_o)}{\lambda + c_f(T_f - T_o)} \tag{20}$$

Also the Clausius-Clapeyron equation gives the variation of y_{fo} with T_o as

$$\ell n\ y_{fo} = -\frac{\lambda}{R}\left(\frac{1}{T_o} - \frac{1}{T_{bp}}\right) \tag{21}$$

where T_{bp} is the normal boiling point of the liquid fuel. Eqs. (20) and (21) are two simultaneous equations in T_o and y_{fo}. Using the above data for a toluene droplet burning in atmospheric pressure air, the solution is that $y_{fo} = 0.898$ and $T_o = 380$ K for $T_{bp} = 384$ K. This confirms the previous assumption that $T_o = T_{bp}$.

6. Finally, as noted by Spalding [2] and Long [4], the above treatment can be used to calculate the diffusion-controlled burning rate and burn-out time for an involatile solid, such as carbon. In this case O_2 diffuses right up to the surface of the solid, and Eq. (5) has to be modified to make a = r_f, giving

$$\frac{M}{4\pi a} = \frac{D_{O_2} P}{RT(n-s)}\ \ell n\left(1 + \frac{(n-s)y_\infty}{s}\right) \tag{22}$$

which simplifies, for $1 >> (n\text{-}s)\ y_\infty/s$, to

$$\frac{M}{4\pi a} = \frac{D_{O_2} P\, y_\infty}{sRT} \tag{23}$$

Eq. (23) is exactly true (see the derivation of Eq. (5) for n = s) if the combustion reaction is $C_s + O_2 \rightarrow CO_2$, when in fact, n = s. Alternatively, if the surface reaction is $C_s + \frac{1}{2}O_2 \rightarrow CO$, then $(n\text{-}s)\ y_\infty/s = 0.21$, in which case Eq. (23) is true to within 10%. Eq. (23) is the Nusselt relationship [10] for the diffusion-controlled burning of a solid particle, such as coal char.

REFERENCES

1. Hedley, A. B., and G. F. Martin, *J. Inst. Fuel*, 1971, **43**, 38; Williams, A. *Combustion and Flame, 1973*, **21**, 1; *Prog. Energy Combust. Sci.*, 1976, **2**, 167; *The Combustion of Sprays of Liquid Fuels*, Paul Elek, 1976.
2. Spalding, D. B., *Combustion and Mass Transfer: A Textbook with Multiple-Choice Exercises for Engineering Students*, Ch. 7, Pergamon Press, 1979, and references therein.
3. Kanury, A. M., *Introduction to Combustion Phenomena*, Gordon and Breach, 1977; Glassman, I., *Combustion*, Academic Press, 1977.
4. Long, V. D., *J. Inst. Fuel*, 1964, **37**, 522.
5. Godsave, G. A. E., *Nature* 1949, **164**, 708; 1953, **171**, 86; *Fourth Symposium (Int.) on Combustion*, p. 818, Williams and Wilkins, Baltimore, 1953.
6. Kay, J. M., and R. M. Nedderman, *Fluid Mechanics and Transfer Processes*, p. 374, Cambridge University Press, 1985.
7. Fristrom, R. M., and A. A. Westenberg, *Flame Structure*, Ch. 12, McGraw-Hill, 1965.
8. *JANAF Thermochemical Tables*, 2nd ed, DOW Chemical Co., Midland, Michigan, 1970; Yaws, C.L., *Physical Properties*, McGraw-Hill, 1977; Reid, R. C., J. M. Prausnitz, and T. K. Sherwood, *The Properties of Gases and Liquids*, 3rd ed, McGraw-Hill, 1977.
9. Kanury, A. M., *Introduction to Combustion Phenomena*, Gordon and Breach, p. 253, 1977.
10. Nusselt, W., *Z. des Vereins Deutscher Ing.*, 1924, **68**. 124.

NOMENCLATURE

a	radius of burning droplet
c_A, c_B	concentration of gaseous species A and B (mol/m³)
c	$c_A + c_B$
c_f, c_{O_2}, c_p	molar heat capacity of fuel, oxygen and products
c_o	mean molar heat capacity defined by $(n\text{-}s)c_o = (nc_p - sc_{O_2})$
D	binary diffusion coefficient
D_A, D_{O_2}	diffusion coefficient of a species in a multicomponent mixture
$H_f^\circ, H_{O_2}^\circ, H_p^\circ$	standard molar reference enthalpies for fuel, oxygen and combustion products
k_b	burning constant (see Eq. (1))
k_f, k_o	thermal conductivity for fuel-rich ($r < r_f$) and fuel-lean gases ($r > r_f$), respectively
k	thermal conductivity
M	number of moles evaporating from fuel droplet per sec.
N_A, N_B	molar fluxes of species A and B (mol/(m² s))
N	total of all molar fluxes
n	number of moles of products formed by complete combustion of one mole of fuel
P	total pressure
P_v	saturated vapour pressure of fuel at temperature T_o
Q	enthalpy flux (per unit area)
R	universal gas constant
r	distance from centre of burning oil droplet
r_f	distance of flame front from centre of oil droplet
s	number of moles of O_2 required to burn completely one mole of fuel

T	temperature of gases around droplet at distance r
T_{bp}	normal boiling point of liquid fuel
T_o	temperature of oil droplet's surface
T_∞	temperature of surrounding air well away from droplet
T_f	temperature of gases in flame front
t	time
x	distance in one-dimensional situation
y_A, y_{O_2}	mole fractions of A and O_2
y_∞	mole fraction of O_2 in air = 0.21
α	dimensionless group = $D_{O_2}P(nc_p\text{-}sc_{O_2})/RTk_o(n\text{-}s)$
$\Delta H°$	heat of combustion per mole fuel at 0°C
λ	latent heat of vaporisation per mole fuel
ρ_{gas}	density of gas adjacent to droplet (mol/m³)
ρ_{liq}	density of liquid fuel (mol/m³) □

ChE book reviews

COAL SCIENCE: AN INTRODUCTION TO CHEMISTRY, TECHNOLOGY, AND UTILIZATION

by Rita K. Hessley, John W. Reasoner, and John T. Riley
John Wiley & Sons, New York, 1986. $35.00

Reviewed by
T. D. Wheelock
Iowa State University

This concise and easily read book provides a useful and basic introduction to the field of coal science and technology. The introductory chapter provides an overview of the coal mining/utilization industry and coal resources of the United States. Subsequent chapters deal with a series of varied and important topics. A description of the complex processes which form peat and convert peat into coal provides a basis for the physical and petrographic characterization of coal. A picture of the organic structure and chemical reactions of coal is built up through a review of coal treatments involving pyrolysis, solvent extraction, hydrogenation, and oxidation, and through a review of coal characterization by instrumental analysis. The book treats the chemistry and technology of a number of methods which have been proposed and sometimes used for converting coal into liquid and gaseous fuels. The final chapter is devoted to a review of standard methods (mainly ASTM) for determining the chemical and physical properties of coal. By seeing how such properties are measured, the reader is left with a greater appreciation for a number of empirical properties such as the proximate analysis, free-swelling index, and grindability. This appreciation justifies the greater coverage given to methods of analysis than to any other topic.

The book should serve as a useful reference for those seeking a broad rather than a penetrating introduction to coal science and technology. However, it could also serve those with more specialized interests by providing an entry into the technical literature. Frequent references are made to the literature throughout the text, and each chapter is furnished with a lengthy list of references.

The book has been used as a college text for coal chemistry courses taught by the authors. Students and others using this book would benefit from a prior knowledge of general chemistry and organic chemistry. □

ChE letters

THE "AYES" OF TEXAS . . .

Editor:

I want to express my appreciation and that of my colleagues for the very generous editorial entitled, "A Department That Serves," that appeared in the Spring 1987 issue of *Chemical Engineering Education*. It is most gratifying to us that you have so eloquently expressed our actual motives for developing the study on "Chemical Engineering for the Future" and disseminating its findings.

Howard F. Rase
The University of Texas, Austin

ChE books received

Adsorption Technology: A Step-By-Step Approach to Process Evaluation and Application, edited by Frank L. Slejko; Marcel Dekker, 270 Madison Ave., New York 10016; 240 pages, $55 (1985).

Heat Transfer and Fluid Flow in Rotating Machinery, Wen-Jei Yang. Hemisphere Publishing, New York, NY 10016 (1987). 553 pages, $95.00.

Computational Heat Transfer, Yogesh Jaluria and Kenneth E. Torrance. Hemisphere Publishing, New York, NY 10016 (1986). 366 pages, $49.00.

Aerothermodynamics of Low Pressure Steam Turbines and Condensers, M. J. Moore and C. H. Sieverding. Hemisphere Publishing, New York, NY 10016 (1987). 290 pages, $59.95.

Basic Cost Engineering, Second Edition, Revised and Expanded, Kenneth K. Humphreys and Paul Wellman. Marcel Dekker, New York, NY 10016 (1987). 368 pages, $34.75