MULTIVARIABLE CONTROL METHODS

PRADEEP B. DESHPANDE
University of Louisville
Louisville, KY 40292

DURING THE LAST several years numerous promising approaches to the solution of multivariable control problems have become available. These control strategies are likely to play an important role in coming years as the processes become more complex and the demands for more efficient operation grow in the light of competitive pressures and environmental considerations. Taking these trends into consideration, we have developed a new graduate course in multivariable control methods. The multivariable control concepts were covered in an intensive four-day short course offered recently, and the responses of the industrial participants were very favorable. The concepts have also been taught in existing graduate courses. An overview of the proposed course is being given in this paper, accompanied by pertinent comments and literature references. It is hoped that it will serve as an impetus for instructors in the area of process control.

Pradeep B. Deshpande is currently Professor and Chairman of the Chemical Engineering Department at the University of Louisville. His specialization is in the area of process dynamics and control. He has approximately seventeen years of academic and full-time industrial experience and has published three textbooks in control and over forty papers. He has consulted for several major companies in this country and abroad and has done collaborative research with them.

THE COURSE

There are four major topical areas of concentration. They are

- **Interaction Analysis**
- **Multiloop Controller Design**
- **Decoupling**
- **Multivariable Control Strategies**

Table 1 shows these areas further subdivided to provide greater detail. The contents can be comfortably covered in a standard one-semester graduate course. The prerequisites for the course should be a course in linear control theory and Laplace transforms, and a course in z-transforms and digital control concepts. More details about the topics are provided in the following paragraphs.

Interaction Analysis

Interaction analysis is the first phase of multivariable control systems design. The objective of interaction analysis can be twofold. The first objective is to select a suitable set of controlled and manipulated variables from competing sets. In a distillation control system, for example, there can be three (or more) possibilities: D, V; R, V; and R, B (first variable controls top composition, second controls bottoms composition). The second objective is to select controlled and manipulated variables within a given set; for example, should D be manipulated to control X_D and V to control X_B or should the reverse pairing be used? For small dimensional, say 2x2 systems, this step could perhaps be skipped if detailed dynamic information about the process is available. Then the available multivariable techniques could be tried through simulation, and a final pairing and control methodology could be selected based on the closed-loop simulation results. For large dimensional systems this is not feasible, and interaction analysis would have to be carried out.

Numerous techniques for carrying out interaction analysis are available. Some utilize steady-state gain

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TABLE 1
Multivariable Control Methods Course Outline

1. *Introduction to Multivariable Control*
   - **Incentive for Multivariable Control**
   - **Why Multivariable Systems are Difficult to Control**
   - **Industrial Examples**

2. *Interaction Analysis*
   - **Relative Gain Arrays**
   - **Singular Value Decomposition**
   - **Other Interaction Measures.**

3. *Multiloop Controller Design*
   - **Design of Multiloop PID-Type Controller**
   - **IMC Multiloop Controller**

4. *Decoupling (Explicit)*
   - **Decoupling in the Framework of RGA**
   - **Decoupling in the Framework of SVD**

5. *Multivariable Control Strategies*
   - *a.* **Nyquist Arrays**
     Direct Nyquist Arrays
     Inverse Nyquist Arrays
   - *b.* **Model Predictive Control**
     Internal Model Control
     Dynamic Matrix Control
     Model Algorithmic Control
     Simplified Model Predictive Control
   - *c.* **Modern Control Theory**
     Introduction to State-Space Models
     The Linear Quadratic Problem

information, while others require detailed knowledge of process dynamics. Clearly, there are incentives for wanting to determine the extent of interaction based on steady-state information. In many instances this is the only type of information available. Unfortunately, the interaction measures which utilize only steady-state gain data sometimes give wrong results. The methods of interaction analysis include relative gain arrays (RGA), singular value decomposition, IMC interaction measure, and inverse and direct Nyquist arrays, among others.

Multiloop Contoller Design

If interaction analysis reveals "modest" interaction, a multiloop control structure may be adequate. Cost-to-performance ratios could perhaps be considered in deciding whether a multiloop control structure should be employed or whether a full multivariable control system would be preferable. If PID-type controllers are employed, then a relatively simple tuning procedure is available. As an alternative to PID control, one may consider using the IMC multiloop controller. The PID tuning procedure is based on the Nyquist stability criteria, while the IMC multiloop controller design procedure neglects the off-diagonal elements of the process transfer function matrix.

Decoupling

If the extent of interaction is such that a multiloop controller structure is deemed to be inadequate, then there are two alternatives. The first is to carry out explicit decoupling in the framework of RGA or SVD, and the second is to use a full multivariable controller.

The multivariable control concepts were covered in an intensive four-day short course . . ., and the responses of the industrial participants were very favorable. The concepts have also been taught in existing graduate courses.

Explicit decoupling is covered here, and multivariable control strategies are the topics that follow. In explicit decoupling in the framework of RGA, one designs decoupling elements such that one pseudo manipulated variable affects only one controlled variable. In the SVD decoupling approach, one carries out a singular value decomposition of the process gain matrix (or process transfer function matrix, depending on whether only steady-state decoupling is desired or dynamic decoupling is desired) and then multiplies the resulting expression by appropriate left and right singular vectors to give a decoupled system and a set of "structured" manipulated and controlled variables. These variables are connected via PID-type controllers to give decoupled responses. Two points are worth mentioning here. One is that modeling errors will degrade performance, and the second is that complete decoupling is not always the best approach if the goal is to achieve minimum ISE or minimum settling times. Better results can sometimes be achieved by allowing interactions in the closed-loop system.

Multivariable Control Techniques

In many instances a full multivariable controller may well be the preferred choice. This is especially true in those applications where constraints are present and perhaps in those which have an unequal number of inputs and outputs. (If a system is non-square, then singular value decomposition is an alternative to consider, although in this case external dead time compensation may have to be applied, making the approach somewhat cumbersome.) Additional benefits accruing from a multivariable controller include dead time compensation and decoupling.

There are several multivariable control techniques available. Three are included in Table 1. The first is based on Nyquist arrays. Direct and inverse Nyquist arrays are frequency domain techniques that require interactive computing with graphics for optimum benefits. Nyquist arrays can also be used for interaction analysis. Furthermore, they can be used to design precompensators and postcompensators such that interaction is greatly reduced. These compensators permit the designer to control an n x n interacting system by n SISO PID-type controllers.

The second of the three topics is on model predictive control methods. In model predictive control, a mathematical model of the process is used for identification/control. The discussion begins with internal model control design based on factorization of the transfer function matrix into two parts, one involving the nonminimum phase elements and the other containing the remaining terms. The latter, when inverted, leads to the IMC controller. A diagonal filter network insures robustness in the presence of modeling errors. In the next phase, the predictive formulation of IMC is discussed. The objective in this instance is to calculate a set of future control actions based on the actual and model outputs such that a suitable performance index is minimized. Only the first control action is applied and the computations are repeated at the next sampling instant. Since the optimization procedure yields future control actions, one can anticipate when constraint violations are likely to occur and therefore what actions to take to keep this from happening. The predictive formulations lead to dynamic matrix control and model algorithmic control. In the final phase, a technique known as simplified model predictive control is discussed. SMPC is a relatively simple multivariable control technique that utilizes an impulse response type model of the process for implementation. It insures some decoupling. SMPC is suitable for low dimensioned processes.

The final topic in multivariable control is on modern control theory. Here, the student is first introduced to the notion of state space models. Then the optimal control problem is formulated, and the methods of solving it are described. The solution of the optimal control problem gives a matrix of control actions which, when applied, leads to process responses that satisfy a quadratic performance index. Recent research indicates that the linear quadratic problem can be formulated in the context of IMC.

At this time research is in progress at various locations which is aimed at designing controllers in the presence of uncertainties. The concept of structured singular values has been employed for this purpose. These concepts have not been incorporated into the current version of the course.

IN CONCLUSION

A course on multivariable control methods has been described. Instructional tools, including a text and computer-aided instruction software (CAI), are available for effective teaching of this course. The material is suitable for full-time graduate students and for control engineers from industry. It is believed that this course will be a good addition to the control speciality, not only in the chemical engineering discipline, but also in other engineering disciplines such as electrical engineering.

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ChE book reviews

PROCESS FLUID MECHANICS
by Morton M. Denn
Prentice-Hall Publishing Co.,
Englewood Cliffs, NJ

Reviewed by
John Eggebrecht
Iowa State University

At Iowa State University "Momentum Transport" is required as the first of a three-semester sequence which continues with "Heat" and "Mass." The second-year student has, with adequate high school preparation, completed the introductory calculus and physics courses. Frequently students are concurrently enrolled in introductory ordinary differential equations.

As the instructor, I see the focus of the course, and of the engineering science curricula in general, as a development of analytical skills. The significant part of a section of text in support of this is not the derivation or the equation confined by a box at the end, but the physical principles, assumptions and approximations which are expressed by these. Many students, having restricted their intellectual objectives to those which they perceive as appropriate for a BS engineer, regard only the "formulae." Some students, enraptured by the mechanics of the calculus, only regard the derivation. To persuade both groups to my point of view I need a text which emphasizes the physics of fluid flow both in the development of topics and in their relations.

On the other hand, engineering practice is as much art, *viz.*, design, as it is science. A responsibility of the course is to introduce the jargon and operational empiricism of process equipment. It is not possible to find a single text on fluid mechanics which encompasses this range of material and conforms to my focus. However, Denn's text is superior to all others which I have considered in the treatment of the physical principles of fluid flow. It is much easier to compensate for the omission of material, which can be extracted from handbooks, than for a presentation which shares the students' bias for either formula or calculus. I am especially appreciative of the organization of the text. Topics appear in an order which reflects the evolution of understanding of fluid flow, and for that reason, I believe, the order which is most easily understood by the student.

The text opens with observation and experimentation on flow primitives; the cylindrical filled conduit and the submerged sphere. This can provide a framework for an appreciation of the analysis of simple systems by the identification of key physical dependencies and the analysis of complex systems by construction from primitives. Also, this introduction establishes the proper relationship between observation and analysis and may help to correct the mistaken perception that discovery is deductive. The prediction of the pressure drop in a straight pipe leads, through Reynolds, to the friction factor correlation and the viscous force on a falling sphere leads, through Stokes, to the drag coefficient correlation. The similarity of these two important results is striking and properly emphasized. Key discoveries are followed by extension to more complex systems and the presentation acknowledges this process by presenting reasonable, yet simple arguments, which lead to correlations for non-cylindrical conduits, partially filled conduits, rough pipes, non-spherical submerged objects and packed beds. These progressions allow me to highlight central themes; the importance of symmetry and frame invariance, the emergence of design correlations from the identification of the significant physics and the replacement of complex systems by simpler systems through judicious approximation. All of this is accomplished without ever taking a derivative.

While the first section of the text is the greatest strength, the following section must be supplemented as an introduction to the application of the conservation of energy to the analysis of macroscopic flows. The derivation of the mechanical energy balance equation is easily understood and very thorough in the statement of assumptions by which the conservation equation is simplified to a "formula." The conservation of linear momentum is combined with the energy conservation equation to analyze a sequence of increasing complexity; expansion, elbow, contraction, free jet and manifold. A logical parallel of the first section

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