ANOTHER WAY OF LOOKING AT ENTROPY

Entropy and Aging, Evolving Systems

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I TEACH A DUAL-LEVEL thermodynamics course which reflects my non-traditional interest in the subject, *i.e.*, I am fascinated with the concept of entropy, dissipative systems, stability through fluctuations, stationary states and equilibrium. I apply these ideas to my major preoccupation—aging, evolving systems, longevity analysis, and structural complexity. I examine living systems and so-called inanimate systems such as corporations. I find in the classroom that teaching this "new" thermodynamics engages student interest and enthusiasm to a degree which, amazingly, seems to match my own. I challenge the class to analyze the aging process in humans, in corporations and cities, from an entropic point of view. We examine the structure of organizations, as to which are most entropically efficient. We extend the boundaries of knowledge into regions where there are no apparent right and wrong answers, which means that sometimes I give the entire class an A grade to reflect my pleasure in their accomplishments.

Entropy, of course, is not something tangible, not something capable of being seen or touched. Hence, entropy is a difficult concept to grasp. It is not a solid, it is not hot or cold and does not have a physical consequence, such as temperature. We can say that entropy is a measure of the disorder of a system and show that more disorder means higher entropy content [3]. Real processes tend to go in the direction of increasing entropy. Aging can be envisioned as an irreversible process of entropy accumulation. Getting older means having less control of body functions, being more disordered. Death is the ultimate disorder, a state of maximum entropy. This point has been speculated upon by many scientists, including Jones [18] and Von Bertalanffy [35].

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The second law of thermodynamics essentially says that systems will run down of their own volition if left to themselves. In other words, the entropy content tends towards a maximum. Thus, increasing entropy could be an indication of the direction in which the system is inclined to go. Unless there is outside intervention, the second law of thermodynamics codifies the one-sidedness of time, or time's arrow. We can only move forward; that is, time is irreversible.

Everything we know is tending towards chaos (unless there is outside intervention), towards an equilibrium with the environment. To quote Von Bertalanffy [35]:

> **The significance of the second law can be expressed in another way. It states that the general trend of events is directed toward states of maximum disorder, the higher forms of energy such as mechanical, chemical, and light energy being irreversibly degraded to heat, and heat gradients continually disappearing. Therefore, the universe approaches entropy death when all energy is converted into heat of low temperature, and the world process comes to an end.**

AGING, EVOLVING LIVING SYSTEMS

It was not until the 1950s that entropy started to seriously emerge in discussions of living phenomena. Complicated biological processes such as cell differen-

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tiation, growth, aging, *etc.*, were now analyzed from the second law of thermodynamics and entropy calculations made [6, 36]. Bailey [1] wrote: "Entropy is a very viable concept for the biological and social sciences. It applies to both open and closed systems. It can be discussed in terms of organization or order." Jones [18] said: "One common feature of biological processes is their unidirectionality in time, that is, they are not reversible, except under special circumstances. Since entropy is the only physical variable in nature which generally seems to parallel the direction and irreversibility of time, these should be fertile areas for the effective use of entropic models."

Much of the historical development of entropy has dealt with isolated or closed systems. A closed system is one which can exchange energy but not matter with the surrounding environment. The second law of thermodynamics states that a closed system must evolve to a state in equilibrium with its environment—a condition of maximum entropy. Open systems are those which can exchange both matter and energy with the surroundings. Obviously we, the living, are examples of open systems. Open systems must maintain the exchange of energy and matter in order to sustain themselves, or slow the approach to the final state, death.

We can say that entropy accumulation within the living system is composed of two parts, one being the internal entropy production based on the myriad of irreversible chemical reactions which constitute the chemistry of life. Secondly, there is the entropy flow through our boundaries, such as in the food consumed, air inhaled and exhaled, biological waste products, heat from the skin, *etc.* The internal entropy production derived from our chemical reactions always proceeds in the direction of increasing our entropy content, since the chemistry of life is inherently irreversible. However, the food in, air in and out, waste products out, and heat out may in total contribute positive or negative entropy flows through our boundaries, which then affect the rate of accumulation of entropy in our living body.

Zotin [37] proposed that we evolve towards a final state, death, by a series of changes, each change called a stationary state. We settle into a stationary state and stay for a while until pushed to the next, and the next. This is clearly seen in the transformation of butterfly larvae and pupae, dramatic physical changes. Balmer [2] applied entropy concepts to the study of an aging annual fish. This species displays all the characteristics of birth, growth, aging, and senile death, over a short twelve-month period. Balmer identified entropy flows into and out of the fish, such as food, excrement and body heat dissipation.

Aging as an Entropy Driven Process

We can consider the aging process as a series of steps, proceeding from one stationary state to the next and the next. We can talk of an entropy driving force, causing the evolution from one stationary state to the next. Each time we achieve a stationary state, we "rest" for a "moment" and then go on to the next. In each stationary state we hunker down, collect ourselves, minimize our entropy production, smooth out our chemistry, and await the next push. And so we age. Thus the stationary state can be characterized as an entropy (disorder) producing state, where this entropy production is drawn down to a minimum before going on to the next stationary state where a new minimum is established. The theorem of minimum entropy production is a fundamental concept of the stationary state, first developed by Prigogine and Wiame [30]. Death is the final state to which we are drawn, where there no longer exists a tension for life; the driving forces have been reduced to nothing or some minimum level, below which life cannot be supported.

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Excess Entropy (EE) and Excess Entropy Production (EEP) Driving Forces for Aging

Having introduced the idea of longevity as an entropy driven process involving an evolution through innumerable stationary states, we can now explore the nature of the entropy driving force for life. If death represents the ultimate disorder (and our maximum entropy content) then we can characterize our longevity potential and vitality by the distance we are from the "black hole" of death. We can calculate a difference in entropy content, from the present (where we are) to where we're heading (towards death). This difference is called Excess Entropy (EE) and may be a driving force for life. By tracking EE versus age, we can see our life unfolding, or winding down, as EE approaches zero, the end of the journey. Not only is EE an important parameter in tracing our lifespan, but so too is the rate of change of EE. In other words, how quickly EE is diminishing with age is another key marker. We call this second parameter Excess Entropy Production (EEP).

EEP is the rate of change of EE with time. From Prigogine's theory of minimum entropy production in the stationary state, we can surmise that EEP should not only become a minimum in the vicinity of death (a stationary state) but EEP should indeed become zero since that final stationary state is also the final equilibrium state where all thermodynamic forces and flows become zero. Thus we can trace life's course by EE and EEP tracks [11].

Our life's course is considered stable, so long as the positive-valued EEP is descending to zero and the negative-valued EE is ascending to zero. From the stability theory developed by Liapunov [19], we can say that if both EE and EEP are positive, the living system becomes unstable. Thus we have another criterion for death: that is, when the EE line passes into the positive-valued domain (it crosses the zero axis). In other words, instability in the life process prevents us from exercising the control required to maintain a proper tension of life. And so we die.

Internal entropy production for a chemical reaction system is given by Prigogine and Wiame [30]

$$\frac{dS}{dt} = \frac{Ar}{T} \tag{1}$$

where
S = internal entropy content
A = chemical affinity, a chemical driving force
r = reaction velocity, a chemical flow
T = temperature
t = time

Eq. (1) can be more generally written as

$$\sigma(S) = \frac{dS}{dt} = \sum_{j=1}^{n} J_j X_j \tag{2}$$

where
$\sigma(S)$ = internal entropy production at any time, t
J_j = a thermodynamic flow, for component, j
X_j = a thermodynamic driving force, for component, j

For a reference state, from Eq. (2), we can write

$$\sigma^0(S) = \sum_{j=1}^{n} J_j^0 X_j^0 \tag{3}$$

and

$$\sigma(S) = \sigma^0(S) + \delta\sigma(S) \tag{4}$$

where
$\delta\sigma(S)$ = a small deviation of internal entropy production from the reference state. Prigogine has shown [28] that $\sigma^0(S)$ in Eq. (3) is a minimum in the reference state (the equilibrium or stationary state).

From Eq. (2), Lee [19] showed

$$\frac{d(\delta^2 S)}{dt} = 2\sum_{j=1}^{n} \delta J_j \, \delta X_j \tag{5}$$

and

$$S - S^0 = \frac{1}{2}\delta^2 S = \text{Excess Entropy} = \text{EE} \tag{6}$$

where
δJ_j = $J_j - J_j^0$, the deviation of the flow from the reference state
δX_j = $X_j - X_j^0$, the deviation of the force from the reference state
S^0 = entropy of the system, in the reference state
$\delta^2 S$ = second entropy deviation from the reference state

It is this difference between the entropy content, S, at any time and that of the reference state, S^0, which is defined as Excess Entropy, *i.e.*, EE = $S - S^0$, a thermodynamic driving force for the life process. If the reference state is an equilibrium or final stationary state, S^0 is a maximum. Thus EE is always negative and approaches zero in the negative domain as the system ages and evolves towards the final reference state.

From Eqs. (5) and (6), Excess Entropy Production (EEP) can be written [12] as

$$\text{EEP} = \frac{d\text{EE}}{dt} = \frac{d\left(\frac{1}{2}\delta^2 S\right)}{dt} = \sum_{j=1}^{n} \delta J_j \, \delta X_j \tag{7}$$

Lee [19] demonstrated that EEP approaches a minimum, or zero, as the system approaches the final stationary state or equilibrium. EEP describes the rate of approach of EE to the final state.

For a chemically reacting system

$$Z + Y \rightarrow C + D \tag{8}$$

and with the definitions

A = chemical affinity $\cong \log[ZY/CD]$
= chemical force, X

and

r = chemical reaction velocity $\cong ZY$
= chemical flow, J

we can obtain Eq. (9), using Eq. (7) and assuming Y, C, D are constants [19]

$$\text{EEP} \cong \frac{(\delta Z)^2}{Z} \tag{9}$$

Although the influences of free radicals, vitamins, minerals, and other nutrients are essential in establishing longevity, nevertheless in a living system the chemistry of life is basically the metabolism of carbohydrates, fats, and protein. In general, in a homogeneous population the proportions of these food components in the diet tend to remain approximately fixed. Thus, it is useful to focus on only one of these, protein, for example. Eq. (8) now becomes: protein + oxygen → carbon dioxide + water + urea + energy, and Eq. (9) is as follows:

$$\mathrm{EEP} \cong \frac{(\delta[\mathrm{Protein}])^2}{[\mathrm{Protein}]} \qquad (10)$$

where
[Protein] = Daily Protein Consumption
δ[Protein] = Daily Protein Consumption minus the Minimum Protein Required (in the reference, equilibrium or final stationary state of maximum disorder)

Life and Death

Aging may be the evolution towards a more probable state, the equilibrium state. As the body ages (returns to equilibrium) the EE driving force weakens or diminishes. Schrodinger [32] wrote, "Living systems survive by avoiding the rapid decay into the inert state of equilibrium." Jones [18] proposed, "The approach to equilibrium is a sign of death. Death may also be thought of as the attaining of a critical, maximum state of entropy during our journey towards equilibrium." Schrodinger further stated, "Thus a living organism continually increases its entropy . . . and tends to approach the dangerous state of maximum entropy, which is death."

Death can be when a critical amount of randomness is attained, when a certain amount of disorganization is suffered. Thus aging is a randomizing process, a disorganizing process. In terms of stability theory, equilibrium is the point or region of attraction; we are drawn relentlessly towards equilibrium. Life may be considered analogous to the spring-wound watch, where the timepiece may stop by one of two possible mechanisms. It can simply wind down (approach equilibrium), or the internal mechanism can somehow fail and the watch "dies" prematurely. Thus we can describe equilibrium deaths and instability or a catastrophic deaths.

Life may be considered a temporary upset from or perturbation of equilibrium. Equilibrium is absolutely stable, a universal attractor. Equilibrium always wins. Aging is a spontaneous process, where the body dissipates its EE. The various theories of aging (cross-linking, wear and tear, free radical, *etc.*) all imply a declining metabolic rate with age, and by extension, a diminishing EE and EEP. Evolution may be the natural process of prolonging the time that an organism spends in the far-from-equilibrium state. Genetics still plays its role in determining longevity potential, but is not in conflict with the ideas presented here. The tendency to return to equilibrium will always apply. Death will always have a probability of absolute certainty.

We can consider the aging process as a series of steps . . . In each stationary state we hunker down, collect ourselves, minimize our entropy production, smooth out our chemistry, and await the next push. And so we age.

AGING, EVOLVING, INANIMATE, CORPORATE-STYLE SYSTEMS

The concept of entropy has been widely discussed in many scientific and social arenas [4, 7, 14, 15, 20, 27, 33] and its application to organizational structure is a logical extension of the work done previously by Prigogine [24, 28], Georgescu-Roegen [5], Shannon [33], Quastler [31], Horowitz [15, 16, 17] and others [14, 24, 25].

One of the most famous uses of the entropic concepts in chemistry was by Ilya Prigogine [28, 29], recipient of the 1977 Nobel Prize in chemistry. He correlated non-equilibrium phenomena and disorder (entropy). He studied systems near equilibrium and systems with minimum entropy production. However, beyond a certain critical distance from equilibrim, entirely new structures could emerge. These new systems, far from equilibrium, Prigogine referred to as dissipative structures. In addition to his work in chemical thermodynamics, Prigogine has also been an innovator in the field of social thermodynamics [24]. He has adapted many of the principles of thermodynamics to social organizations, recognizing that each organization is itself an open system. Another contributer to the interdisciplinary analysis of open systems was economist and thermodynamicist, Nicholas Georgescu-Roegen [5] who believed that the notion of entropy had great utility in economics, that the earth is an open system with irreversible processes occurring within it.

Emerging States Driven by Nonequilibrium Conditions

The behavior of nations and other human organizations illustrates the recondite dynamics of the nonequilibrium open system [14]. States, living cells, economic processes, ecological systems, and even transportation networks demonstrate the bridges between the physical and social sciences [9]. Prigogine studied self-organization under conditions of fluctuations and change, an evolutionary process in systems pushed far from equilibrium. Whether the fluctuations impinge on a household or a nation, the belief is that they can be explained. He examined oscillating phenomena and sought a basis for predicting order from the perspective of entropy production (regarded by some as the reification of the arrow of time since entropy production describes in general the direction spontaneous processes must go). Use of the term, entropy, is derived from the Greek, meaning evolution [7]. Systems near equilibrium can be buffeted by small perturbations in energy and mass pressures, but no new organizations, no new structures, are formed. Imposing stronger gradients from the outside world could force the appearance of new, dissipative, nonequilibrium stationary states. Examples cited by Prigogine are a town and the living cell. Georgescu-Roegen wants economics, which ignores entropy, to begin to mark the existence and applicability of nonequilibrium dynamics and irreversibility. Economists, he says, have a blind faith in reversibility (we can restore original conditions by invoking the same laws, backwards and forwards, and ignoring time). Georgescu-Roegen states that matter is subjected to entropy degradation, from higher order to lesser, and hence becomes less useful. What he is stressing is an evolutionary philosophy. If driven hard enough we see new structures developing, nurtured by energy and matter fluxes [24]. Entropy, applied to the economic process, adds important new perceptions to the interactions of humans, technology, the market system, and limited resources—ideas spawned by Georgescu-Roegen. He proposed we think in terms of irreversibilities, limits on resource availability, and a more parsimonious society. Most current economic policy tinkers with prices, taxes, or the market in some way. Until now, we have stressed economic balances of energy and matter—what comes in must equal what goes out—rather than an understanding that something is lost in every transaction (in entropy terms). The real world dictates the transformation in one direction only: low entropy to high entropy. The consumer takes in high-grade, ordered energy and matter and exhausts low-grade, disordered wastes. The wastes must not injure or render inoperative the feedback and control mechanisms which affect the stability of the open, temporary state. Consumers may be individuals, cities, governments, corporations, or civilizations [10, 12, 13, 26].

Informational Entropy: Shannon's Approach

The mathematical definition of informational entropy was derived by C. E. Shannon [33] in 1949. With molecules in motion, colliding and rebounding, different molecules will occupy a given space at various times and hence many molecular arrangements, called microstates, are possible. We associate the concepts of greater freedom, uncertainty, and more configurational variety with an increased number of microstates and higher entropy. If one had to guess where a particular molecule would be at a given time, the probability of error would be greater in the higher entropy state. Thus ordering of a system implies a lower entropy, which carries with it a certain reliability and smaller probability of error.

From the preceding discussion we know that as the number of microstates, W, increases, the state entropy, H, increases. Hence we can write [4],

$$H = K \log_a W \tag{11}$$

where a is the logarithmic base.

If we assume that all the microstates are equiprobable, then the probability of achieving each individual microstate, p_i, is simply one out of the total number of the microstates, W, or

$$p_i = \frac{1}{W} \tag{12}$$

or

$$H = -K \log_a (p_i) \tag{13}$$

We can extend this idea to non-equiprobable systems with the use of the Expectation Value, E_x [22], which is by definition the probability of each outcome, p_i, multiplied by the value of that individual outcome, X_i, summed over all possible outcomes, as indicated in Eq. (14)

$$E_x = \sum_i p_i \cdot X_i \tag{14}$$

With the probability, p_i, and the associated H value, $-K \log_a p_i$, from Eq. (13), E_x (also called S, the entropy of the system) is

$$S = -K \sum_i p_i \cdot \log_a p_i \tag{15}$$

Thus, Eq. (15) expresses the entropy of a system in terms of probabilities. It takes the concept of entropy from the thermodynamic setting to the domain of general probability theory. It can be shown that S in Eq. (15) achieves a maximum value if and only if all the p_i's are equal [26]. If for convenienoe the constant K is taken to be unity, then Eq. (15) reduces to

$$S = -\sum_i p_i \log_a p_i \tag{16}$$

This is Shannon's formula for informational entropy.

The Meaning of Stored Information

As Weaver [33] states, "... the word 'information' in communication theory relates not so much to what you say, as to what you can say." Potential message variety, freedom of choice, and a large vocabulary are the desired ends of communication and information transmission. A library obviously contains stored information. The information is stored in a linear sequence of symbols organized to the constraints of a language. The sequences are organized into books and periodicals, and these are carefully ordered on shelves. Everywhere order and constraints are associated with the information storage process. This is a state of low entropy. If we take each page of each book, cut it into single-letter pieces and mix them in one jumbled heap, the entropy would increase and stored information would decrease. In entropy terms, stored information is the divergence from the state of maximum disorder (when all p_i terms are equal). In other words, stored information is the difference between entropy content for the equally probable state, S^{max} (maximum disorder) and that for the unequally probable present, S, and is denoted by D. Therefore, stored information is

$$D = S^{max} - S \tag{17}$$

Entropy and Corporate Structure

Shannon's informational entropy formula [33] has in the past found application by Gatlin [4], who computed genetic stored information, by Horowitz and Horowitz [15, 16] and Herniter [8] in marketing, by Lev [21] in accounting, by Thiel [34] and Georgescu-Roegen [5] in economics, by Philipatos and Wilson [27] in securities analysis, and by Murphy and Hasenjaeger [23] in organizational decentralization.

These authors extended the definition of p_i from the probability that a system will be in a particular microstate to such related considerations as: (1) the probability that a customer will purchase a product; (2) the degree of competition in the marketplace; (3) a measure of the dispersion in a securities portfolio; (4) the degree of market share; (5) the degree of organizational decentralization; and (6) the bits of stored genetic information.

One can generate a Power Index [10, 12, 26] to be used in Shannon's formula, analogous to p_i, in order to characterize the overall structure of a corporation. Since each unit controls those beneath it, one can also define a Cumulative Power Index for each unit, C_i, where

$$C_i = P_i + \text{sum of all } P_i \text{ controlled by this unit, i}$$

Finally, a Fractional Cumulative Power Index, f_i, is introduced as the unit's C_i divided by the sum of all C_i in the organization. Thus Eq. (16) is transformed to

$$S = -\sum_i f_i \log_2 f_i \tag{18}$$

We can define an entropic distance from disaster, D, for the actual structure, by Eq. (19):

$$D = S^{max} - S \tag{19}$$

and the distance from disaster for an ideal structure, D^o, as

$$D^0 = S^{max} - S^0 \tag{20}$$

and a structural efficiency, η, by Eq. (21):

$$\eta = \frac{S^{max} - S}{S^{max} - S^0} \times 100 \tag{21}$$

The distance from disaster, D, is also stored information, Eq. (17), as well as EE for the inanimate system, the corporate-style organization,

$$EE = S - S^{max} \tag{22}$$

where S^{max} = maximum entropy content for the organization, if all the units have the same budget, are at the same level, and are completely independent. A prescription for disaster. ($S^{max} = \log_2 n$ and $f^{disaster} = 1/n$, where n is the number of units in the organization [10].)

From Eqs. (7), (18) and (22) and recognizing that EEP is expressed as the product of thermodynamic forces and flows, we can write [12]

$$EEP = \delta S \delta EE \tag{23}$$

where

J = flow = S
X = Force = EE

and where previously, for the living system, flow = r and force = A. It can be shown [12, 26] that

$$EE = S - S^{max} = S - \log_2 n \tag{24}$$

and

$$EEP = \left[\sum_{i=1}^{n} \left(f_i - \frac{1}{n}\right) \log_2 f_i\right]^2 \tag{25}$$

For each year of a corporation's history, f_i can be computed for each unit and the summing process of Eq. (18) accomplished to produce S and then efficiency, η, from Eq. (21). The EE is obtained from Eq. (24) and EEP from Eq. (25). Thus EE and EEP longevity tracks can be constructed, just as they can be (and have been) for the living system.

CONCLUSION

In the classroom I offer undergraduate and graduate students the opportunity to explore another meaning of entropy, applied to: open systems; dissipative systems; stationary states exhibiting minimum entropy production; stability through fluctuations; entropy driving forces; excess entropy and excess entropy production concepts; the entropy nature of living systems; the entropic evolution of inanimate systems such as corporations; the meaning of life as an entropy-driven process; the finality of death from an equilibrium viewpoint; change processes which can affect entropy content of systems; aging, evolving systems and increasing disorder and entropy.

This leads our discussions into diverse fields such as gerontology and geriatrics, systems research, corporate planning, history, economics and process control. We become interdisciplinary for one quarter at least. Or longer.

REFERENCES

1. Bailey, K. D., "Equilibrium, Entropy, and Homeostasis: A Multidisciplinary Legacy," *Syst. Res.* **1,** 25-43 (1984)
2. Balmer, R. T., "Entropy and Aging in Biological Systems," *Chem. Eng. Com.,* **17**, 171-181 (1982)
3. Fast, J. D., *Entropy,* Gordon and Breach, New York (1968)
4. Gatlin, Lila L., *Informational Theory and the Living System,* Columbia University Press, New York (1972)
5. Georgescu-Roegen, Nicholas, *Entropy and the Economic Process,* Harvard University Press, Cambridge, MA (1971)
6. Gump, F. E., "Whole Body Metabolism," in S. M. Altura *et al* (eds) *Handbook of Shock and Trauma, Vol 1: Basic Sciences,* Raven Press, New York (1983)
7. Hahn Jr., John, "An Application of Informational Entropy Concepts to Profit-Seeking Organizations; MS Thesis in Chemical Engineering, University of Cincinnati (1979)
8. Herniter, J. D., "An Entropy Model of Brand Purchase Behavior," *J. of Market. Res.,* **10**, 361-375 , Nov. (1973)
9. Hershey, D., and H. H. Wang, *A New Age-Scale for Humans,* Lexington Books, Lexington, MA (1980)
10. Hershey, D., V. Patel, and J. Hahn, "Speculations on the Use of Shannon's Formula to Characterize an Organization," *Systems Res.,* **4,** 211-212 (1987)
11. Hershey, D., and W. E. Lee, "Entropy, Aging, and Death," *Syst. Res.,* **4**, 269-281 (1987)
12. Hershey, D., "Excess Entropy (EE) and Excess Entropy Production (EEP) in Aging, Evolving Systems, *Sys. Res.* (in press)
13. Hershey, D., "Aging and Evolving Systems," *Syst. Res.,* **5**, 167-170 (1988)
14. Hershey, D., "Aging of Cities, Corporations, and Civilizations," *Syst. Res.,* **3**, 3-8 (1986)
15. Horowitz, A. R., and J. Horowitz, "Entropy, Markov Processes and Competition in the Brewing Industry," *J. Ind. Economics,* **16**, 196-211 (1968)
16. Horowitz, A. R., and I. Horowitz, "The Real and Illusory Virtues of Entropy-Based Measures for Business and Economic Analysis," *J. of the Amer. Inst. for Decision Sci.,* **7**, 1; 121-136 (1976)
17. Horowitz, I., "Employment Concentration in the Common Market: An Entropy Approach," *J. of the Royal Statistical Soc.,* **133**, part 3, 617-625 (1970)
18. Jones, D. D., "Entropic Models in Biology: The Next Scientific Revolution?" *Persp. Biol. Med.,* **20**, 285-299 (1977)
19. Lee, W., "Analysis to Understand the Thermodynamics of Living Aging Systems," PhD Thesis, Univ. of Cincinnati (1984)
20. Lepkowski, W., "The Social Thermodynamics of Ilya Prigogine," *Chem. Eng. News,* pp 30-33, April 16 (1979)
21. Lev, B., "The Aggregation Problem in Financial Statements: An Informational Approach," *J. of Account. Res.,* **6**, 247-261, Autumn (1968)
22. Miller, I., and Freund, J. E., *Probability and Statistics for Engineers,* Prentice-Hall, Inc., New Jersey (1977)
23. Murphy, D. C., and Hasenjaeger, J. T., "Entropy as a Measure of Decentralization," Proc. of the Amer. Inst. for Decision Sci., Boston, MA, 67-69, Nov (1973)
24. Nicholas, G., and I. Prigogine, *Self-Organization in Non-Equilibrium Systems,* John Wiley and Sons, New York (1977)
25. Onsager, L., "Reciprocal Relations in Irreversible Processes," *Phys. Rev.,* **37**, (1931)
26. Patel, V., "Entropy Analysis of the Structure of a Corporation," MS Thesis, Univ. of Cincinnati (1982)
27. Philippatos, G. C., and C. J. Wilson, "Entropy, Market Risk, and the Selection of Efficient Portfolios," *Apl. Economics,* **6**, 77-81 (1974)
28. Prigogine, I., "Entropy, Time, and Kinetic Description," in G. Nicolis *et al* (eds) *Order and Fluctuations in Equilibrium and Nonequilibrium Statistical Mechanics,* John Wiley, New York (1981)
29. Prigogine, I., "Time, Structure, and Fluctuations," *Science,* **201**, 777-785 (1978)
30. Prigogine, I., and J. M. Wiame, "Biologie et Thermodynamique des Phenomenes Irreversibles," *Experimentia,* **2**, 451-453 (1940)
31. Quastler, H., *The Emergence of Biological Organization,* Yale Univ. Press, New Haven, CT (1964)
32. Schrodinger, E., *What is Life? The Physical Aspects of the Living Cell,* Cambridge Univ. Press, Cambridge (1955)
33. Shannon, C. E., and W. Weaver, *The Mathematical Theory of Communication,* Univ. of Ill. Press, Urbana (1949)
34. Thiel, H., *Economics and Information Theory,* Rand-McNally and Company, New York (1967)
35. Von Bertalanffy, L., "The Theory of Open Systems in Physics and Biology," *Science,* **111**, 23-29 (1950)
36. Warnold, I., K. Lundholm, and T. Schersten, "Energy Balance and Body Composition in Cancer Patients," *Cancer Res.,* **38**, 1801-1807 (1978)
37. Zotin, A. E., "The Second Law, Negentropy, Thermodynamics of Linear Irreversible Processes," in I. Lamprecht and A. I. Zotin (eds) *Thermodynamics of Biological Processes,* Watler de Gruyter, Berlin, 21 (1978) ❐