Random Thoughts . . .

IT GOES WITHOUT SAYING

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I never liked lectures as a student. Regardless of the subject or the lecturer I could never keep my attention from wandering, and even when I thought I was learning something I usually discovered later that I really hadn't gotten it. I like lectures even less as a teacher; I consider myself a pretty good lecturer, but the inevitable sea of glazed eyes in class and the subsequent questions in my office about things I taught explicitly have convinced me that I'm not accomplishing that much when I stand up and talk at students for fifty minutes.

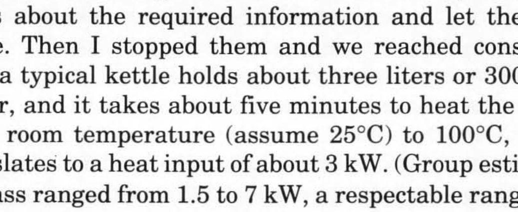
The fact is that what routinely goes on in most college classes is not teaching and learning, but stenography: professor transcribes notes from notebook to chalkboard, students transcribe from chalkboard back to notebook. Even if the notes are supplemented with all sorts of insightful commentary, research shows that students in lectures generally retain a reasonable percentage only of what they hear in the first ten minutes and relatively little of anything that happens thereafter. They really only learn by thinking and doing, not watching and listening. And so I've been spending a growing amount of my time lately seeking ways to shift the focus from me to them during class.

For example, here is an in-class exercise I used in our second-semester sophomore course on chemical process analysis, just after we derived the transient open-system energy balance equation. (The exercise could equally well be used in the junior transport course.) I had the class divide themselves into groups of three at their seats and presented a series of problems. After I posed a problem I would give the groups some time to work on it (rarely enough to get a complete solution, often only enough to get started), then stop them and either present my solution or call on one or two of the groups to present as much as they had gotten. Here's how it went—my questions and comments to the class are in italics.

- ***I'm going to ask you several questions about a teakettle filled with water. In answering them, you'll need the heat capacity of liquid water (J/g·°C) and the heat of vaporization (J/g). Take a moment and come up with round-number estimates of these quantities.*** (They did, and we then agreed to use 4 J/g·°C and 2000 J/g in our calculations.)
- ***OK. Now, suppose we put the kettle on the stove and crank the burner up to maximum heat. Get me a rough estimate of the rate of heat input (kW) to the water in the kettle. Work in your groups—three people talking, one writing. Go.***

Initially there was bafflement, as this was anything but a well-defined problem. Some groups figured out that they would have to come up with estimates of how much water a typical teakettle holds and how long it takes to bring a full one to a boil, and others just scratched their heads. I let them go at it for a few minutes, then gave hints about the required information and let them resume. Then I stopped them and we reached consensus that a typical kettle holds about three liters or 3000 g of water, and it takes about five minutes to heat the water from room temperature (assume 25°C) to 100°C, which translates to a heat input of about 3 kW. (Group estimates in class ranged from 1.5 to 7 kW, a respectable range.)

- ***So that means I've got to pay the electric company for 3 kW, right?*** (Wrong! Only a fraction of the heat output from the burner goes into the water—I'm using considerably more than 3 kW.)
- ***Where does the additional heat go?*** (Into the kettle itself, the stove, and the room air.)
- ***All right—let's agree that our system initially consists of 3 kg of water at 25°C and we are adding heat to it at a constant rate of 3 kW. My plan is to leave the kettle on the burner until there's no more water left in it. The next question is, if the system is the water in the kettle, which system variables change with time?*** (T and M, the temperature and mass of the water.)
- ***Take about 30 seconds and sketch plots of T vs. t and M vs. t.***



The class and I agreed that we couldn't be sure without more analysis that the ramps would be straight lines but that the curves would certainly look something like those two. I then asked them if they were quite sure that the M vs. T plot would be horizontal up to t_b, and after a short time it occurred to several of them that pre-boiling evaporation would lead to a slight decrease in M. We agreed to neglect this effect in our analysis, and then reached consensus that the mass-time variation would be described by the transient mass balance

$$\frac{dM}{dt} = -\dot{m}_{out}$$

and the temperature-time variation by the transient energy balance equation we had just derived in the last class

$$\frac{dU}{dt} = Q - \dot{m}_{out}\,\hat{H}_{out}$$

(input, kinetic and potential energy, and shaft work terms having been dropped). In these equations $\dot{m}_{out}\,(g/s)$ is the rate of evaporation, U(J) is the total internal energy of the water in the kettle, and $\hat{H}_{out}\,(J/g)$ is the specific enthalpy of the vapor.

In the next series of exercises, the groups concluded or were led to conclude that the periods before and after boiling commences must be analyzed separately, and that for the first phase of the process, (1) $\dot{m}_{out} = 0$, (2) M is constant, and (3) provided that the heat capacity C_v is constant, $U = MC_v(T - T_{ref})$. The last result harked back to material in the stoichiometry course that they had not seen for months, and we spent a little time reviewing it.

- ***Now use all that to simplify the energy balance.***

I expected them to jump immediately to

$$\frac{dU}{dt} = MC_v\frac{dT}{dt} = Q$$

Instead, I got blank stares, which puzzled me but should not have. This transition from dU/dt to dT/dt is a trivial application of the chain rule for differentiation, which I've used so much I no longer think about it. They had never seen it outside of last year's calculus class, however, where it was taught abstractly and didn't mean anything to them. Once I figured out what was going on (after some unproductive knee-jerk chastising on the order of "Haven't any of you seen this stuff before?"), I backtracked and gave them a two-minute calculus refresher that might have been the most valuable thing they got in the hour. Then they went back, derived the equation, substituted for MC_v and Q, integrated to solve for T(t), and confirmed that it takes 300 seconds for the water to reach the boiling point.

We then looked at the period t > 300 s. I wrote the energy balance equation again

$$\frac{dU}{dt} = \frac{d}{dt}\left(M\hat{U}\right) = M\frac{d\hat{U}}{dt} = -\dot{m}_{out}\,\hat{H}_{out} + Q$$

- ***Right?***

All of them bought it, but being used to my tricks they weren't too surprised when I announced "Wrong!" I gave them a moment to figure out the error, and it finally occurred to several of them that M is also a variable and the long-forgotten product rule for differentiation was required. I then wrote the correct formula:

$$\frac{dU}{dt} = \frac{d}{dt}\left(M\hat{U}\right) = M\frac{d\hat{U}}{dt} + \hat{\mathbf{U}}\frac{dM}{dt} = -\dot{m}_{out}\,\hat{H}_{out} + Q$$

This equation baffled them completely—they had not previously encountered one with two derivatives in it. I asked if anyone could figure out how to get rid of one of them; no one could, so I pointed to the material balance equation still up on the board and substituted $-\dot{m}_{out}$ for dM/dt to arrive at

$$M\frac{d\hat{U}}{dt} = -\dot{m}_{out}\left(\hat{H}_{out} - \hat{U}\right) + Q$$

It only remained to lead them to the conclusions that (1) $\hat{U}$, the specific internal energy of liquid water at 100°C, is constant, so that the derivative drops out, and (2) provided that

$$\hat{U} = \left(\hat{H} - P\hat{V}\right) \approx \hat{H}$$

for liquid water at 100°C (which I convinced them is the case by pulling values of $\hat{U}$ and $\hat{H}$ from the steam table), the final result for the energy balance is the intuitive one that

$$Q \approx \dot{m}_{out}\left[\hat{H}_{H_2O(v,100^\circ C)} - \hat{H}_{H_2O(\ell,100^\circ C)}\right] = \dot{m}_{out}\,\Delta\hat{H}_v$$

Thus, we could finally calculate the rate of evaporation as

$$\dot{m}_{out} \approx Q/\Delta\hat{H}_v \approx [3000\ J/s]/[2000\ J/g] = 1.5\ g/s$$

and the time for all the water to evaporate as (3000 g)/(1.5 g/s) = 2000 s = 33.3 minutes. All of the values on the plots of T vs. t and M vs. t could now be filled in, which I did. I ended with a short review of everything we had done.

This exercise covered several important concepts in a variety of topics, including transient material and energy balances, thermophysics, thermodynamics, applied calculus and differential equations, and order-of-magnitude estimation, and showed how to put the concepts together to analyze a familiar system. It took me a little over an hour to get through it—all of one class period and about a third of the next one.

Could I have covered the same material in less time by simply lecturing? Sure, but I don't think the students would have gotten much out of it. Many (perhaps most) would have tuned out early in the lecture; others would have dutifully copied down whatever I wrote on the board but few would have understood enough of it to be able to use it on a slightly different problem. As it was, though, most of them stayed actively involved throughout the presentation (it's hard to hide in a group of three); they worried about the problems I wanted them to worry about, and after trying and sometimes failing to solve them, listened intently to hear what they should have done. When I later gave homework problems that required the use of similar analyses they did extremely well on them, and they also did much better on related test questions than I believe a normally taught class would have done. In short, they learned the material.

It isn't necessary to do something like this every class period—in fact, I'm not sure it would be desirable or even possible to do that. However, as a break from the usual straight lecture format, it's worked well for me every time I've tried it. Check it out for yourself. ❐