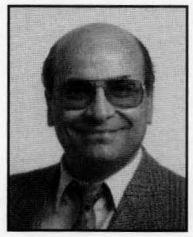
ChE *class and home problems*

The object of this column is to enhance our readers' collections of interesting and novel problems in chemical engineering. Problems of the type that can be used to motivate the student by presenting a particular principle in class, or in a new light, or that can be assigned as a novel home problem, are requested, as well as those that are more traditional in nature and which elucidate difficult concepts. Please submit them to Professor James O. Wilkes (e-mail: wilkes@engin.umich.edu) or Mark A. Burns (e-mail: maburns@engin.umich.edu), Chemical Engineering Department, University of Michigan, Ann Arbor, MI 48109-2136.

PROBLEMS ON FLUIDS IN MOTION AND AT REST

A.R. KONAK
Southern Alberta Institute of Technology • Calgary, Alberta, Canada T2M 0L4

Engineering courses should not be presented as a dry set of facts, experiments, and calculations; they should have a context related to practical applications and daily experiences in order to awaken the students' curiosity and awareness. The following potpourri of classroom and home problems, the second in a series,[1] are short and meant to be thought provoking. Mixed in with standard exercises, they can provide variety and a better understanding of the fundamentals of fluid flow, statics, and related phenomena.



A. Riza Konak *received his BSc and PhD degrees in chemical engineering from the University of Birmingham (England). He started his career as assistant professor, and then spent fifteen years in industry, mostly in applied research and development and engineering with a major oil and gas company, before returning to academia. He currently teaches unit operations, process design, simulation, and control.*

PROBLEMS

1. **a)** Why does a stream of water coming out of a tap get progressively thinner? **b)** What is the relationship between the velocities at the exit of the tap and at a distance, d, below the exit?
2. Explain why bacteria living in the air find life just as "viscous" as people would find if they were living in a viscous fluid—say, honey.
3. Why are golf balls dimpled and tennis balls smooth?
4. It is possible to suspend a table tennis ball or a golf ball in the exhaust of a powerful vacuum cleaner, and the ball will not escape the air stream on its own. Explain why this situation is pretty stable.
5. The heart beats about 72 times a minute at rest. Over two ounces (70 mL) of blood are pumped at an average pressure of 100 mm of mercury per beat. Estimate the power developed by the heart. Is the result surprising?
6. Explain how blood would flow at much higher rates than normal during a strenuous exercise without causing excessive blood pressure.
7. The blood pressure of a giraffe lying down is measured at 120 mm of mercury. The blood pressure doubles when the animal stands up. If the heart of a standing giraffe is 1.8 m above the ground, what would be its height? The specific gravity of mercury is 13.6. State your assumptions.
8. A waiter brings you a cup of coffee. While he is putting it down on the table he turns the cup around so that the handle faces you. Did the coffee move with the cup, or did it remain still?
9. In Florentine (Italy) in the seventeenth century, well diggers observed that, in suction pumps, water would not rise more than about 10 meters. In 1642 they came to the famous Galileo for help, but he did not want to be

bothered with the problem and delegated it to his disciple, Toricelli. How did Toricelli explain the mystery?

10. An owner of a sailboat decides to check out the speed of his boat by using an L-shaped tube. The horizontal leg of the tube is immersed in water facing in the direction of travel, while the four-meter long vertical leg is above the water level. What is the maximum speed the owner can measure? What improvements can you suggest?
11. Why does hydrogen travel twenty-two times faster than carbon dioxide in a straight pipe of uniform cross section at the same temperature, pressure, and mass flow rate? You may assume ideal gas relationship holds for both fluids.
12. A small boat is floating in a swimming pool. Which of the following events raises the water level more in the pool? **a)** A rock is dropped directly into the pool. **b)** The same rock is placed in the boat.
13. In which of the following media is the velocity of sound highest and lowest? Air, water, steel.
14. The velocity of sound is nearly the same in water and mercury, although mercury is 13.6 times as dense as water. Why?
15. Atmospheric air at room temperature (20°C) is allowed to enter an initially evacuated and insulated container. Estimate the temperature of air in the container when it is full.
16. A group of mountaineers reaches the top of a mountain and discovers that water boils at 85°C. What information do they need to estimate their altitude? What is their altitude?

SOLUTIONS

1. **a)** Water coming out of a tap accelerates as it falls. Since the volume of water is constant and is equal to volume times the area, the cross-sectional area gets smaller downstream.
   b) Bernoulli's equation between the two points indicated in Figure 1 yields

$$\frac{P_1}{\rho g}+\frac{v_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{v_2^2}{2g}+z_2$$

   where $P_1 = P_2$ = atmospheric pressure and cancel out each other. Therefore we have

$$v_2^2=v_1^2+2gd$$

   It is interesting to note that this is the famous free fall formula for a body falling a distance $d = z_1 - z_2$ under the influence of gravity, with an initial velocity v_1 and final velocity v_2.

2. Let's take some typical numbers for the average size of bacteria ~ 10^{-5}m, which is moving at ~10^{-3}m/s, say, in air whose kinematic viscosity is ~10^{-5}m^2/s (the bacteria might be floating in air like a bunch of very small dust particles). The Reynolds number is

$$\mathrm{Re}=\frac{\left(10^{-5}\,\mathrm{m}\right)\left(10^{-3}\,\mathrm{m/s}\right)}{10^{-5}\,\mathrm{m^2/s}}=10^{-3}$$

   Thus, although the air has very low viscosity, the bacteria would find life very viscous due to their size and slow movements. As Shapiro points out,[2] it is more meaningful to speak of a very viscous situation than of a very viscous fluid.

3. Golf balls initially started out smooth, but it was discovered, quite accidentally, that scarred balls went further than the smooth ones. This discovery led to dimpled golf balls. The classical explanation involves the boundary layer around the ball that becomes turbulent for a dimpled ball before it does for a smooth one. The turbulent layer remains attached to the ball longer and thus produces a smaller wake and a smaller drag. A table tennis ball, on the other hand, goes slower and roughening increases the friction drag. Shapiro[2] covers this phenomenon in great detail.
4. As soon as the ball strays out of the air stream, the air flows faster on the opposite side where the streamlines are straight; an increase in speed means a decrease in pressure, as would be predicted by the Bernoulli's equation, and so the ball tends to move back into the air stream.
5. The power developed by the heart is equal to the pressure developed times the flow rate. Since 100 mm of mercury = 13330 Pa

$$\mathrm{Power}=\left(13330\,\mathrm{Pa}\right)\left(70\times10^{-6}\times72/60\,\mathrm{m^3/s}\right)=1.12\,\mathrm{W}$$

   or, say, 1 watt. This is a surprisingly low number and is about 1% of the total power generated by the body.
6. During normal body function, the blood flow is,[3] say,

$$\left(70\,\mathrm{mL/beat}\right)\left(72\,\mathrm{beats/min}\right)=5040\,\mathrm{mL/min}=5\,\mathrm{L/min}$$

   During an exercise the blood flow may be

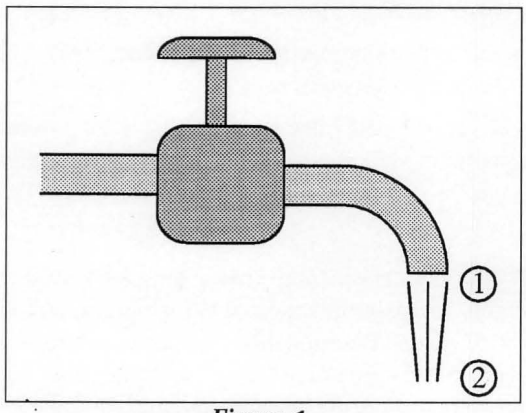


Figure 1

$$(200\,\text{mL/beat})(120\,\text{beats/min}) = 24000\,\text{mL/min} = 24\,\text{L/min}$$

This is almost a five-fold increase. When blood flows at a steady rate through long, smooth vessels, the flow is laminar; turbulent flow is encountered under some conditions, such as in the root of the aorta and at the major arterial branches. The laminar flow is governed by Poiseuille's law, which states that the rate of blood flow is directly proportional to the fourth power of the diameter of the vessels. In the body as a whole, about two-thirds of the flow resistance to circulation is in the small blood vessels with diameters that can range between 8 and 30 micrometers,[4] but they have strong vascular walls that allow the internal diameters to change by as much as four-fold. From Poiseuille's equation we can see that a four-fold increase in vessel diameter theoretically could increase the flow by as much as 256-fold.

7. The additional pressure which the heart must overcome when the animal is standing is 120 mm of mercury, or

$$(120)(13.6/1) = 1632\,\text{mm of water}$$

or 1.6 m of water. Hence, the head of the giraffe is 1.6 m above its heart, assuming that the specific gravity of blood is the same as water and is equal to one. Thus the giraffe is

$$1.6 + 1.8 = 3.4\,\text{m tall}$$

8. The famous "no slip at the wall" principle of hydrodynamics tells us that a very thin layer of coffee will attach to the cup and will move with it; most of the coffee, however, will remain stationary.

9. Toricelli figured out that water was not being pulled up by the vacuum, but rather was being pushed up by the local air pressure. When the pump lowered the air pressure above the column of water, the normal air outside the pump pushed down harder on the ground water, forcing the water in the pipe upward. Toricelli checked out his theory by using a column of mercury and a bowl of mercury, and that led to his discovery of the first barometer. Since the local air pressure is about equal to 10 m of water (760 mm of mercury), a suction pump will not raise water if the water level in the well is more than about 10 m.

10. The device described is basically a Pitot tube. The maximum velocity that can be measured is given by

$$v(\text{max}) = \left[(2)\left(9.81\,\text{m/s}^2\right)(4\,\text{m})\right]^{1/2} = 8.86\,\text{m/s} = 32\,\text{km/h}$$

A Bourdon tube attached to the long leg would considerably increase the range of velocity, also simplifying the device. A Bourdon tube consists of a flattened tube, shaped as a segment of a circle or spiral. The increased pressure inside the tube straightens the tube a little. The movement at the end of the tube is transmitted through a linkage which causes a pointer to rotate, thus giving an indication of pressure.

11. The mass flow rate of gas is

$$G = Au\rho$$

where

A = pipe cross-sectional area
u = average velocity of the gas
ρ = density of the gas

The ideal gas law is

$$P = \frac{\rho RT}{M}$$

where P, R, T, and M are pressure, gas constant, temperature, and molar mass, respectively. Combining these two equations gives

$$u = \left(\frac{GRT}{AP}\right)\left(\frac{1}{M}\right)$$

Thus, the average velocity is inversely proportional to the molar mass. Since respective molar masses of carbon dioxide and hydrogen are 44 and 2, hydrogen travels 22 times faster.

12. Archimedes' principle states that an object that is totally or partially immersed in a fluid is acted upon by a buoyant force equal to the weight of fluid displaced. When the rock falls to the bottom of the pool, the water level rises in proportion to the volume of the rock. When the same rock is placed in the boat, however, the level rises higher because the buoyant force is greater; that is, the amount of water displaced is larger than before, causing the boat and the rock to float. Using Archimedes' principle, it can be shown that the volume of water displaced when the rock is in the boat is equal to the volume of the rock times the relative density of the rock (relative density being the density of rock divided by the density of water).

13. The velocity of sound is highest in steel and lowest in air. Generally, the more incompressible the medium, the higher the sonic velocity. At room temperature the approximate sonic velocities in air, water, and steel are 340, 1480, and 5100 m/s, respectively.

14. The sonic velocity is given by

$$c = \left(\frac{E}{\rho}\right)^{1/2}$$

where E is the bulk modulus and ρ is the density of the material. The E values of water and mercury are 2.13 and 28 GPa, respectively. Since mercury is 13.6 times denser than water, the E/ρ ratios are about equal.

A similar situation arises with aluminum and steel. The E values for aluminum and carbon steel at room temperature are 69 and 207 GPa, respectively, while their

densities are 2710 and 7860 kg/m^3.[5] Thus, the sonic velocity in steel is

$$\left(\frac{207\times10^9\,\text{Pa}}{7860\,\text{kg/m}^3}\right)^{1/2}=5132\,\text{m/s}$$

and in aluminum is

$$\left(\frac{69\times10^9\,\text{Pa}}{2710\,\text{kg/m}^3}\right)^{1/2}=5046\,\text{m/s}$$

As mentioned in the solution to Problem 13 above, the more incompressible the medium (the larger the E value), the higher the sonic velocity. But the higher the density of the medium, the lower the sonic velocity—due, presumably, to the closely packed atomic structure of the medium. The examples cited above indicate that one factor might work against the other for two completely different materials.

15. The air inside the container is not in motion and hence its total energy is in the form of internal energy. Once the container is full, this internal energy is supplied by the atmospheric air filling in the container. Therefore, the internal energy of air in the container at the final temperature T_f equals the enthalpy of atmospheric air at 20°C. Or, in equation form

$$c_v T_f = c_p T_{air}$$

$$\frac{R}{k-1}T_f=\frac{Rk}{k-1}T_{air}$$

or

$$T_f = k\,T_{air} = 1.4(20+273)=410.2\,\text{K}=137.2^\circ\text{C}$$

Here, R is the gas constant and k is the ratio of specific heats c_v and c_p at constant volume and constant pressure, respectively. Thus, there is a considerable increase in temperature under adiabatic conditions.

16. They need to know the vapor pressure of water at 85°C and the properties of standard atmosphere at different heights. The vapor pressure of water at 85°C is 57.81 kPa from the steam tables. This corresponds to about 4440 m height in the table which gives the properties of the atmosphere (*e.g.,* see Table A.3, The Properties of U.S. Standard Atmosphere, in reference 6).

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ChE letter to the editor

To the Editor:

I appreciate the inclusion of appropriate quotations from my book *The Interpretation and Use of Rate Data. The Rate Process Concept,* revised printing, Hemisphere Publishing Coprporation, Washington, DC (1979), in the excellent article by Shacham and Brauner in *Chemical Engineering Education,* **29**(1), p. 22. However, I wish to point out that the ungrammatical phrase in the last sentence of their article, namely, "data justifies," appeared correctly as "data justify" in my book, page 309.

Stuart W. Churchill
Carl V.S. Patterson Prof. Emeritus
University of Pennsylvania

REVIEW: Polymer Molecules

Continued from page 93.

tions of one of the authors suggest some personal bias regarding the inclusion of this chapter. In fact, I found no references to this material in the last two chapters. Although I have no argument with the importance of the topics covered in Chapter 7, I suspect that most faculty members using the book for a course will skip over this material unless their research interests lie in this area. To the authors' credit, they do state in the Preface that this chapter "can be studied or not depending on the interests of the reader."

In the final two chapters, the authors apply the principles established in Chapters 5 and 6 to two very important areas: rubber elasticity in Chapter 8 and polymer solutions in Chapter 9. Both of these areas typically receive a few pages each in traditional undergraduate polymer science textbooks. In these treatments, a simplistic physical description is followed by the end result, an equation for students to apply in problems at the end of the chapter. It is refreshing to see more complete descriptions and detailed derivations in both areas. For example, many textbooks are content to give the expression for solvent activity from Flory-Huggins theory as gospel for polymer solutions. In Chapter 9 the authors derive the configurational entropy of mixing for Flory-Huggins theory, discuss the limitations of the theory, and develop newer treatments such as equations of state.

In summary, I recommend *The Science of Polymer Molecules* for consideration as a textbook for a graduate course in polymer science or polymer physical chemistry. Problems are included at the end of each chapter, evidence that the authors were serious about writing a textbook for graduate students in this area. With minor exceptions, *The Science of Polymer Molecules* is presented in a reader-friendly manner that will further motivate students with an interest in macromolecules and their behavior. ❐