The object of this column is to enhance our readers' collections of interesting and novel problems in chemical engineering. Problems of the type that can be used to motivate the student by presenting a particular principle in class, or in a new light, or that can be assigned as a novel home problem, are requested, as well as those that are more traditional in nature and which elucidate difficult concepts. Please submit them to Professor James O. Wilkes (e-mail: wilkes@engin.umich.edu) or Mark A. Burns (e-mail: maburns@engin.umich.edu), Chemical Engineering Department, University of Michigan, Ann Arbor, MI 48109-2136.

PROBLEMS IN MASS TRANSFER AND SEPARATION PROCESSES

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This article is the third in a series of articles. The first two dealt with selected problems and their solutions in heat transfer[1] and fluid flow.[2]

Mass transfer can be defined as the transfer of species as a result of a concentration difference in a mixture. The process can take place in a gas, a vapor, or a liquid. It results from a diffusion of molecules as well as from their bulk motion. The former is analogous to the transfer of heat by conduction and the latter to the transfer of heat by convection.

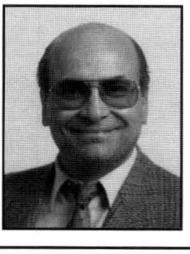
In general, mass transfer tends to be more complex than heat transfer. As stated by Maxwell in 1860, diffusion and convection are always present together because the former always causes the latter.[3] Therefore, mass flux normally includes both diffusion and convection. Thermal conduction, on the other hand, can easily occur without convection and can be studied separately.

The importance of mass transfer and separation operations in chemical engineering cannot be overemphasized. There is scarcely any chemical process that does not require an initial purification of raw materials or a final separation of products using mass-transfer operations. A good classification of separation processes in general and mass-transfer operations in particular is provided by Wankat.[4] Application of mass transfer in biology and medicine is also important and complex.[5] For example, the oxygen/carbon dioxide exchange between blood and tissues and blood and lungs that we take for granted not only involves mass transfer but also numerous chemical reactions.

The following sample problems from mass transfer are meant to invoke curiosity in students and to expand their horizons on the subject matter.

PROBLEMS

1. An open bottle of perfume on a table eventually fills the room with the fragrance of the perfume. Explain.
2. Hanging out the wash in such places as Alaska in winter will enable the wet clothes to become dry below 0°C. Explain.
3. Freeze drying is commonly used in saving water-damaged books when a library has suffered flooding after



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a fire. Why?

4. Consider a saturated sugar solution in contact with sugar crystals. All that can be seen are sugar crystals in a clear solution. Initially, no changes are apparent—but over time the shape of the crystals can be observed to change. Explain what is taking place.

5. What is the fastest way to make a tasty cup of soup from bouillon cubes?

6. Why do deep-sea divers breathe a mixture of oxygen and helium rather than air?

7. Speculate how we can diagnose an illness using a person's breath.

8. Dried prunes swell in water and cucumbers shrink into pickles in brine. Explain the process taking place in each case.

9. Explain how a gas mask prevents poisoning that may occur from various toxic gases.

10. Aspirin (acetylsalicylic acid) is used extensively to relieve pain, reduce fever, and relieve redness and swelling caused by arthritis. It is also used to reduce the formation of blood clots that may cause heart attack and stroke. More recently, aspirin has been found to inhibit the growth of small tumors and to prevent deaths from digestive tract cancers.[7] The major disadvantage of aspirin is that it can cause stomach upsets, especially in those people with sensitive stomachs. Suggest a means of preventing stomach upsets for these people.

11. The basic principle of an artificial kidney is to pass blood through a membrane tube. On the other side of the membrane is a dialyzing saline solution into which unwanted substances in the blood pass by diffusion.[8,9] Increasing the mass-transfer rate across the membrane would greatly improve the clinical value of the system. Suggest at least three ways of improving the mass-transfer rate in an artificial kidney.

12. Heart/lung machines have been developed to take the place of the heart and lungs during the course of certain heart operations. The system mainly consists of: 1) an oxygenator that supplies oxygen to the red blood cells and removes carbon dioxide from the blood, 2) a pump that takes over the pumping action of the heart, 3) a filter that removes small particles that may have been introduced into the blood from the machine, 4) a heat exchanger that cools the blood before it is returned to the patient, lowering the body temperature and allowing more time to perform the operation (afterward, the heat exchanger warms the blood and thus the patient), and 5) a bubble trap to prevent gas bubbles from returning to the patient's circulation and thus from blockage of a blood vessel.[8,9] Suggest effective means of oxygenating blood, assuming the process is mainly mass-transfer controlled.

13. Leaching, also known as solid-liquid extraction, is a separation technique employed to remove a solute from a solid mixture using a liquid solvent. Brewing tea (where tea leaves are soaked in hot water) and percolating coffee (where boiling water is circulated repeatedly through ground coffee beans held in a perforated drum) are well-known examples of leaching processes. What major variables should be considered in a leaching operation?

[These} sample problems from mass transfer are meant to invoke curiosity in students and to expand their horizons on the subject matter.

14. As part of his investigation, Inspector Callaghan paid a visit to a missing stockbroker's private office where he was last seen. In looking around, he noticed a half-empty coffee mug on the missing stockbroker's desk. Based on his experience and the stain left in the mug, the inspector estimated that the stockbroker had been missing for almost a month. Was he right? You may assume the following information: Vapor pressure of water (coffee) at room temperature (20°C) is 2.33 kPa, and the 10-cm high coffee mug was full initially except the first 1 cm from the brim.

15. A mixture of four liquids is to be separated by fractional distillation. The average volatilities for the individual components are 2.5, 1.8, 1.0, and 0.6. The recovery of the second-lightest component in the distillate is 99 mole percent and that of the third-lightest component in the bottoms product is 98 mole percent. What would be the approximate number of theoretical stages.

16. A dilute pollutant gas in a mixture of gases is being absorbed by a pure solvent under isothermal conditions. *Approximately,* how many theoretical equilibrium stages are required for (a) 99% pollutant recovery, and (b) 99.9% pollutant recovery. State your assumptions.

SOLUTIONS

► 1. The perfume has a high vapor pressure and its molecules diffuse through the air, eventually filling the room. Air freshener and moth flakes also have high vapor pressures; it is the vapor from these compounds diffusing through the air that makes them effective.

► 2. Clothes will stay dry despite the very cold temperatures because ice sublimes at low temperatures; that is, there is enough vapor pressure associated with ice. The moisture freezes first and then the clothes remain dry. The slow disappearance of snow in winter, although the temperature remains below the melting point of ice, visibly demonstrates the sublimation of ice as well.

► 3. Again, the process is one of sublimation. Since there is no application of heat involved, as there would be in conventional drying, the damage to books is kept to a minimum.

► 4. This is evidence that sugar molecules are dissolving from the crystals into the solution, and at the same time dissolved sugar molecules are being incorporated into the crystals. When equilibrium is reached, the rates of these two processes (*i.e.,* dissolution and crystallization) are equal.

► 5. The fastest way to make a cup of soup involves adding hot water to the bouillon cubes while stirring the solution. Stirring increases the breaking process, and together with heat increases the rate of dissolution, which is a mass-transfer controlled process. A similar situation occurs with cubed sugar in tea and coffee. Crushing the cubes first also achieves faster dissolution rates.

► 6. The reason is that nitrogen is more soluble in blood than helium. If deep-sea divers breathe in air, nitrogen dissolved in their blood effervesces when the divers return to the surface, resulting in the formation of numerous small bubbles in the blood stream. These bubbles can block the small blood vessels and starve the tissues of oxygen, causing a painful condition known as the "bends." In serious cases, the bends can cause death. The risk of bends is reduced if helium is used instead of nitrogen to dilute the diver's oxygen supply.

► 7. Analysis of normal exhaled breath using chromatography reveals over one hundred different organic compounds. Eventually, it may be possible to diagnose certain illnesses by analyzing a person's breath and comparing the results with those obtained from healthy individuals. Chromatography is a powerful analytical technique for separating and analyzing mixtures where a gas or liquid, called a carrier fluid, flows through a packed bed of particles. A pulse of feed mixture to be analyzed is injected into the carrier fluid. The components of the feed separate as they travel through the column with different velocities.

► 8. The skins around the fruits and vegetables act as osmotic membranes. Prunes swell in water because water enters the prune to dilute the concentrated sugar solution within. Cucumbers shrink into pickles in brine because water leaves the comparatively dilute solution within the cucumber. The movement of water in each case is through the cell membranes. In agriculture, osmosis helps us understand how water flows from the ground-root system to the concentrated sap cells in the upper regions of a tree or plant.

► 9. Finely divided charcoal (activated carbon) used in gas-mask canisters readily adsorbs organic molecules. The oxygen and nitrogen of air pass through the activated carbon bed unaffected, but relatively large molecules of toxic gases are adsorbed—a mass-transfer operation. Various chemicals are added to the activated carbon to render specific poisons harmless. Some materials, such as tear gas, are a mixture of aerosol and vapor, which necessitates using a filter preceding the charcoal or mixed with it.[6]

► 10. Dissolution of any drug in the stomach is normally a mass-transfer-limited operation.[3] To decrease the mass-transfer rate, we can design slow-release or coated tablets. The latter allows the release of aspirin in the intestinal tract after it has gone through the stomach. We can, of course, take smaller doses of aspirin or take aspirin suppositories. Researchers are also working on aspirin substitutes that do not upset the stomach.

► 11. The following methods come to mind: 1) Stir the saline solution rapidly to decrease the resistance to mass transfer on the saline side; 2) Make the membrane as thin as is practical; 3) Increase the length of time the fluids on both sides of the membrane remain in contact; 4) Change the nature and characteristics of the membrane.

► 12. The following methods are practiced:[8] 1) Bubble oxygen through a bath of blood and then remove the bubbles; 2) Drip the blood downward over the surfaces of large areas of plastic sheet in the presence of oxygen; 3) Pass the blood over the surfaces of rotating discs in an atmosphere of oxygen (a quantitative description of this is given by Lightfoot[5]); 4) Pass the blood between thin membranes or through thin tubes that are porous to oxygen and carbon dioxide. Other methods that are familiar to chemical engineers are wetted-wall columns and packed columns. Lightfoot describes and analyzes a packed column in which the blood is poured down over packing particles in the shape of double cones. Such systems, however, may damage the blood, which is a concentrated suspension of red cells, white cells, and platelets in plasma. Plasma itself is a colloidal suspension of plasma proteins in an electrolyte solution.

► 13. Based on the consideration of making a pot of tea or coffee, we may consider the following major variables:

1) *Physical characteristics of the carrier solids* • For the hot water to easily access the tea in the tea leaves and soluble coffee in the solid coffee matrix, we may consider crushing or grinding the solids prior to extraction. This increases the surface area through which hot water can contact the soluble solute. On the other hand, in industrial operations (such as extracting oil from crushed seeds) grinding the solids too finely may render the extraction process uneconomic and may result in packing of solids during extraction.

2) *Solvent selection* • Physical properties of the sol-

vent (such as selectivity for the solute, density, viscosity, and boiling point) should be considered as well as its cost, toxicity, and availability.

3) *Temperature of leaching* • Generally speaking, higher temperatures are beneficial due to increased rates of leaching (lower viscosities and larger diffusivities on the solute and solvent). Care needs to be taken when extracting natural materials, however, since undesirable solutes may also leach out.

4) *Duration of leaching* • Longer times of contact may be both beneficial and harmful. For example, brewing tea for short times may not fully bring out the flavor, but overdoing it will make the tea taste bitter.

► 14. This is a case of evaporation of a single component (assumed pure water) into a stagnant gas (air) at room temperature and one atmospheric pressure. Combining Stefan's law with the evaporation rate results in the following equation[10]

$$L^2 - L_0^2 = 2\,\frac{MD}{\rho}\,\frac{C_w C_t}{C_a}\,t$$

where

- L Effective distance through which diffusion is taking place = 5 cm on the day of inspector's observation
- L L_0 at time t = 0, *i.e.*, 1 cm
- M Molar mass of water = 18 kg/kmol
- C_t Total molar concentration of water vapor and air = $(1/22.4\ \text{kmol/m}^3)(273\ \text{K}/293\ \text{K}) = 0.04159\ \text{kmol/m}^3$
- C_w Saturation molar concentration of water vapor at the interface = $(2.33\ \text{kPa}/101.3\ \text{kPa})(0.04159\ \text{kmol/m}^3) = 0.000956\ \text{kmol/m}^3$
- C_a Log-mean concentration of air = $(C_{a1} - C_{a2})/\ln(C_{a1}/C_{a2})$ where $C_{a1} = C_t = 0.04159\ \text{kmol/m}^3$ and $C_{a2} = [(101.3 - 2.33)/101.3]\ (0.04159\ \text{kmol/m}^3) = 0.04063\ \text{kmol/m}^3$ (air concentration near the coffee surface)
- D Diffusion coefficient of water at room temperature = $25.6 \times 10^{-6}\ \text{m}^2/\text{s}$
- ρ 1000 kg/m^3

The result for t is about 2.7 million seconds, or 31 days, so the inspector was right.

► 15. It appears that there is not enough information to arrive at an answer. For example, there is no information on the composition of the feed or its thermal condition. The reflux ratio is also unknown. But we are asked to find an approximate answer, which we can do. In many cases, actual to minimum reflux ratio is around 1.2, for which Gilliland (see Douglas[11] for example) noted that the number of theoretical stages is about double the minimum number of theoretical stages as calculated from Fenske's equation. Thus

$$N = 2N_m = 2\,\frac{\ell n\left[\dfrac{x_{D_{LK}}}{x_{D_{HK}}}\right]\left[\dfrac{x_{W_{HK}}}{x_{W_{LK}}}\right]}{\ell n\,\dfrac{\alpha_{LK}}{\alpha_{HK}}}$$

where x is the mole fraction and the subscripts D, W, LK, and HK refer to the distillate, bottoms, light key, and heavy key components. We can compile the following table:

Component	Feed (moles)	Distillate (moles)	Bottoms (moles)	Relative volatility
A	a	a	0	2.5
B(LK)	b	0.99b	0.01b	1.8
C(HK)	c	0.02c	0.98c	1.0
D	d	0	d	0.6

$$N = 2N_m = 2\,\frac{\ell n\left[\dfrac{0.99b}{0.02c}\,\dfrac{0.98c}{0.01b}\right]}{\ell n\,\dfrac{1.8}{1.0}} = 29$$

Thus, we need approximately 28 theoretical plates and a reboiler.

► 16. Again, there does not appear to be enough information to solve this problem. We can, however, make some assumptions that will lead to an approximate answer. Since the pollutant gas is dilute and the absorption is isothermal, we may use the Kremser equation and assume L/mG ~ 1.4 where the L, G, and m are the liquid flow rate, gas flow rate, and slope of the equilibrium line, respectively. This leads to[11]

$$N + 2 = 6\log\left[\frac{y_{in}}{y_{out}}\right]$$

for most cases. Here N is the number of theoretical equilibrium stages, and y is the pollutant gas concentration in and out of the tower. Thus, for 99% recovery we have

$$(G\,y_{in})(0.01) = (G\,y_{out})$$

$$\frac{y_{in}}{y_{out}} = 100$$

yielding approximately ten theoretical stages. Similarly, for 99.9% recovery, we get sixteen theoretical stages.

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