THE IMPORTANCE OF SYSTEM SELECTION ON COMPRESSIBLE FLOW ANALYSIS

Filling Vessels

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This paper describes an undergraduate laboratory experiment that was developed to illustrate some of the important concepts of compressible flow. A compressible flow can be classified as one that demonstrates a significant change in fluid density with a change in either the pressure or the temperature, and is therefore usually observed for gases. Indeed, compressible gas flows are encountered in many chemical engineering situations, *e.g.,* the transport and storage of high-pressure gases representing feed or product streams from a chemical reactor, hence the importance of this subject in the undergraduate curriculum.

A schematic of the experimental apparatus is shown in Figure 1. It consists of an insulated pressure vessel connected to the air mains via a globe valve. The first stage of the experiment involves pressurizing the vessel (via the mains) from initial atmospheric conditions up to a predetermined elevated pressure (of maximum value 750 kPa) and measuring the corresponding temperature change within the vessel. The second stage of the experiment involves discharging the vessel to the atmosphere via a nozzle and recording the pressure versus time relationship for this process.

This paper considers only the first, or pressurizing, stage of the experiment. The principal objectives are

- *To measure the temperature change within the vessel for a number of different final pressures.*
- *To develop a theory for this temperature change for comparison with the experimental data.*

Of particular significance are the concerns associated with development of the theory for the vessel temperature rise during pressurization. Theoretical analysis of the compressible flow can be divided into three stages: 1) definition of the "system" and the "surroundings" for the problem geometry; 2) application of an energy balance (the first law of thermodynamics) for the defined system; and 3) algebraic manipulation of the energy balance using the appropriate simplifying assumptions and established thermodynamic relationships. This paper illustrates the importance of the first step (that of defining the correct system and surroundings) in the model development, and in particular how a slightly incorrect definition can produce significant errors in the resulting model predictions.

THEORY FOR PRESSURIZING THE VESSEL

The experiment described in this paper involves pressurizing an insulated vessel (as shown in Figure 1) from a starting pressure, P_0, up to a final equilibrium pressure, P_E, before discharging the vessel to the atmosphere through a nozzle. In this section, the theory for the first, or pressurizing, stage is considered, with the objective of determining an expression for the change in temperature of the system, $(T_E - T_0)$, which accompanies the pressurizing process.

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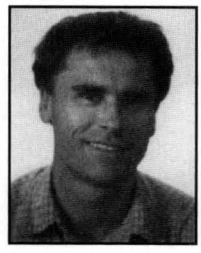

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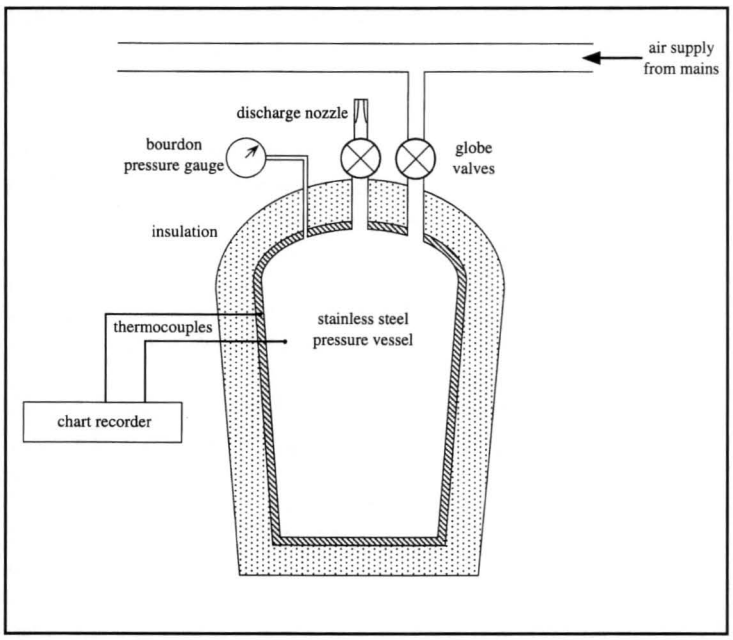

Figure 1. *Schematic representation of the experimental apparatus.*

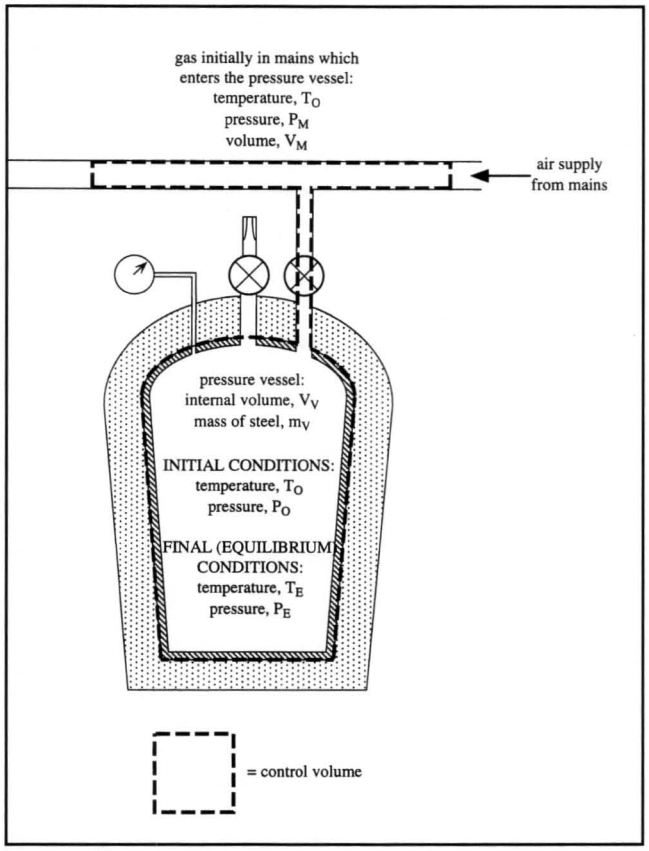

Figure 2. *Definition of the system and surroundings, the initial conditions, and the final conditions.*

In developing a theory for the temperature change, the first stage is to correctly define the "system," or control volume, on which to perform the thermodynamic analysis. Referring to Figure 2, the control volume is bounded by the outer surface of the stainless steel vessel and the air in the mains pipe that enters the pressure vessel during pressurization. Thus, the system consists of the air initially inside the vessel, the air in the mains that enters the vessel, and the stainless steel wall of the pressure vessel.

It is important to consider the air in the mains as part of the system because the system can be treated as a closed system and Eq. (1) applies. Otherwise, when the globe valve is opened, mass will cross the system boundary and an open system would apply, and Eq. (1) would need to be modified.

The initial and final conditions for this system are also shown in Figure 2, while the principal assumptions required in the analysis presented below are: 1) the insulation on the pressure vessel allows the vessel to be regarded as adiabatic, and 2) the air behaves as a perfect gas. Applying the first law of thermodynamics to the closed system outlined above gives[1]

$$Q_S - W_S = \Delta U_S \tag{1}$$

where W_S is the work done by the system on the surroundings, Q_S is the heat transfer between the system and the surroundings, and U_S is the internal energy of the system. As discussed above, the system is assumed to be adiabatic, *i.e.*, $Q_S = 0$. The work done by the system on the surroundings is given by

$$W_S = P_M \Delta V_S \tag{2}$$

where P_M is the mains pressure (the pressure at which the volume change is taking place), and ΔV_S is the change in volume of the system, given by

$$\Delta V_S = V_V - (V_V + V_M) = -V_M \tag{3}$$

where V_V is the volume of the vessel and V_M is the volume of gas in the mains that enters the vessel.

Combining Eqs. (2) and (3) and applying the perfect gas law to gas initially inside the mains gives

$$W_S = -P_M \left(\frac{m_M R T_0}{P_M} \right) = -m_M R T_0 \tag{4}$$

where T_0 is the initial temperature of the system, R is the specific gas constant, and m_M is the mass of gas in the mains that enters the pressure vessel (see Figure 2).

The change in internal energy of the system, ΔU_S, is given by

$$\Delta U_S = \Delta U_G + \Delta U_V = -W_S = m_M R T_0 \tag{5}$$

where ΔU_G is the change in the internal energy of the gas

within the system, and ΔU_V is the change in internal energy of the stainless steel within the system.

The change in internal energy for the gas is given by

$$\Delta U_G = m_G C_{V_G} (T_E - T_0) \tag{6}$$

where m_G is the mass of gas in the system and C_{V_G} is the heat capacity at constant volume. Equation (6) indicates that ΔU_G is related to the change in system temperature only (and not to the change in system pressure).

The change in internal energy for the stainless steel vessel wall is given by

$$\Delta U_V \approx m_V \Delta h_V = m_V C_{P_V} (T_E - T_0) \tag{7}$$

where Δh_V is the change in enthalpy of the stainless steel vessel, m_V is the mass of the stainless steel vessel, and C_{P_V} is its specific heat capacity.

Substituting Eqs. (6) and (7) into Eq. (5) yields

$$\Delta U_S = m_G C_{V_G} (T_E - T_0) + m_V C_{P_V} (T_E - T_0) = m_M R T_0 \tag{8}$$

Equation (8) can be modified using the perfect gas law,

$$m_G = \frac{P_E V_V}{R T_E} \tag{9}$$

and

$$m_M = \frac{P_E V_V}{R T_E} - \frac{P_0 V_V}{R T_0} \tag{10}$$

Substituting Eqs. (9) and (10) into Eq. (8) and rearranging results in the following quadratic equation for the final equilibrium temperature, T_E:

$$\alpha_2 T_E^2 + \alpha_1 T_E + \alpha_0 = 0 \tag{11}$$

where

$$\alpha_2 = m_V C_{P_V} T_0 \tag{12}$$

$$\alpha_1 = \left(\frac{P_E V_V C_{V_G} T_0}{R} \right) + (P_0 V_V T_0) - (m_V C_{P_V} T_0^2) \tag{13}$$

$$\alpha_0 = -(P_E V_V T_0^2) - \left(\frac{P_E V_V C_{V_G} T_0^2}{R} \right) \tag{14}$$

Equations (11) through (14) can be solved to give the final equilibrium temperature of the system, T_E, for a specified value of the final equilibrium pressure, P_E.

EXPERIMENTAL

Equipment Description • A schematic representation of the experimental apparatus is shown in Figure 1. It consists of a 102-liter insulated stainless steel pressure vessel, the inlet of which is connected to the high-pressure air mains via a globe valve, while the vessel is discharged to the atmosphere through a nozzle attached via a second globe valve. The temperature of the stainless steel pressure vessel wall and that of the air within the pressure vessel are measured using Chromel-Alumel thermocouples connected to a chart recorder for continuous recording, while the vessel pressure is displayed on a bourdon pressure gauge.

Experimental Procedure • The preliminary stages in conducting this experiment are to check that both the inlet and exit valves to the pressure vessel are closed and to record the ambient temperature and pressure conditions for the vessel. The chart recorder can then be switched on, set to the correct scale and speed (0.1 mV cm^{-1} and 20 cm hr^{-1}, respectively, in our case), and the recording pens zeroed. Temperature recording is commenced and the inlet valve connecting the vessel to the mains air supply is opened. The pressure within the vessel is visually observed to increase and when it has reached the desired value, the inlet valve is closed and the system left to reach equilibrium, *i.e.,* the chart recorder indicates a constant temperature for both the vessel wall and internals, which typically takes around five minutes. The final equilibrium conditions of vessel pressure and temperature are also recorded.

The vessel is then discharged by placing a muffler over the exit nozzle (to reduce noise pollution) and opening the outlet valve. The pressure is recorded at regular intervals during the discharging process, which typically takes around two to three minutes, and ends when the vessel has reached atmospheric pressure. The vessel wall and internal temperatures are allowed to reach steady state, taking approximately five minutes, before the process is repeated for a different final pressure.

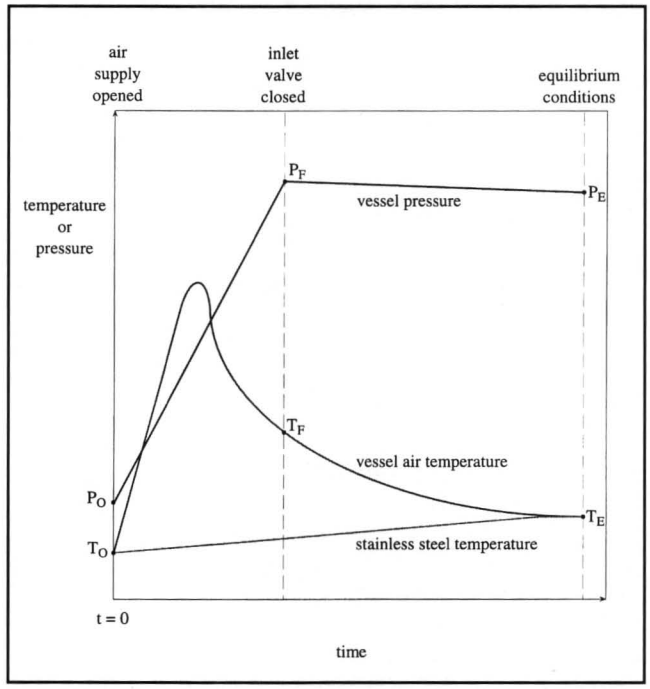

Figure 3. *Schematic representation of the variation in the vessel temperature and pressure during pressurization.*

Important Note: There are a number of inherent safety implications associated with this experiment that should be noted. First, although air is a benign material under atmospheric conditions, it can become hazardous at elevated pressures (4-10 atmospheres in this case). At 10 atmospheres, the pressurized air has enough force to shoot a water jet 100 meters into the air! Second, the experiment should only be attempted using a properly certified pressure vessel fitted with the appropriate pressure relief system. This is absolutely essential when undertaking any experiment under pressurized conditions, but is especially important when gases are involved since any rupture could result in a catastrophic explosion of the vessel.

RESULTS AND DISCUSSION

Variation in the Vessel Temperature and Pressure During Pressurization • The first important stage in discussing the experimental results is a qualitative understanding of the variation in vessel wall temperature, internal air temperature, and vessel pressure as a function of time during pressurization. A schematic plot showing the typical time dependency of these three parameters is given in Figure 3. Considering initially the vessel pressure, at t=0 the inlet valve to the vessel is opened and the air from the mains starts to enter the vessel, causing the pressure to rise steadily with time. When the desired final vessel pressure, P_F, is attained, the inlet valve is closed, thereby isolating the vessel. The vessel is then left until all three parameters have reached equilibrium; during this 'equilibrating' period, the vessel pressure drops slightly to a final equilibrium pressure P_E. This drop in pressure is a result of the drop in the internal air temperature during the equilibrating period, for reasons discussed below, as can be illustrated by the perfect gas law at constant volume

$$P_E = P_F\left(\frac{T_E}{T_F}\right) \tag{15}$$

The variation in internal air temperature with time shows an initial rapid rise, resulting in a peak temperature shortly after the inlet valve is opened, followed by an exponential decay to the final equilibrium value, T_E. In contrast, the temperature of the stainless steel vessel wall shows a steady increase from the initial temperature, T_0, to the final equilibrium temperature, T_E (the same as that of the internal air). During the pressurization process, the work done on the system is converted into heat, as defined by Eq. (8), which produces the initially rapid rise in the internal air temperature. As the air temperature rises, then the temperature difference between the air and the stainless steel wall increases, resulting in an increasing rate of heat transfer from the air to the wall. The maximum in the air temperature represents the point where the rate of heat transfer is sufficiently high that the air temperature must start to fall. The air temperature continues to drop and the wall temperature continues to rise until they both reach the same equilibrium temperature, T_E, and no more heat transfer can take place.

Temperature Increase During Pressurization Versus Final Pressure • Typical experimental and theoretical results for the system temperature change, $(T_E - T_0)$, as a function of the final vessel pressure, P_E, are shown in Figure 4 (in which the theoretical results are based on the input parameters given in Table 1). For this case the lowest pressure was selected first and the consecutively higher pressures were chosen. It can be seen that the experimental and theoretical data both show an approximately linear trend of increasing temperature difference with increasing final pressure. The good overall agreement between the two sets of data for the first few pressures confirms that the theory provides an

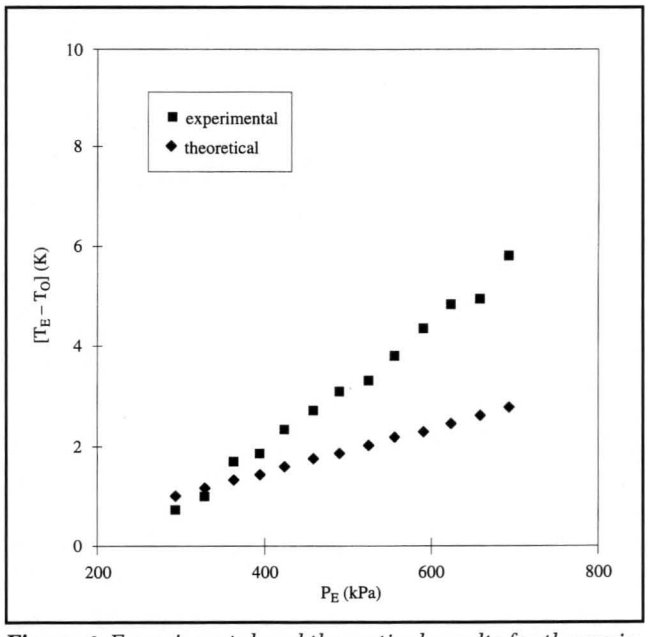

Figure 4. *Experimental and theoretical results for the variation in the system temperature difference as a function of the final vessel pressure (data points collected from lower to higher pressure).*

TABLE 1
Input Parameters for the Theoretical Calculations

Parameter	*Value*	*Notes*
V_V	0.102 m^3	
m_V	43.0 kg	
C_{V_G}	718 J kg^{-1} K^{-1}	for air[2]
R	287 J kg^{-1} K^{-1}	
C_{P_V}	503 J kg^{-1} K^{-1}	for stainless steel[2]
P_0	101 kPa	in Figure 4
T_0	295 - 299 K	in Figure 4
P_0	101 kPa	in Figure 5
T_0	295 K	in Figure 5

acceptable description of the experimental process. Although the theory predicts the correct order of magnitude and variation in the results, however, there are differences between the absolute experimental and theoretical results. The reason for the discrepancy is outlined below.

Considering initially the results in Figure 4, clearly there is good agreement between the experimental and theoretical data at the lower final equilibrium pressure ($[T_E - T_0] \approx 1$ K at $P_E = 300$ kPa), but the experimental rate of increase in $[T_E - T_0]$ with increasing P_E is significantly higher than predicted by the theory such that by the highest pressure employed, $P_E = 700$ kPa, the experimental temperature rise is about 6 K compared to a theoretical value of about 3 K. This discrepancy between the experimental and theoretical values is a direct result of the limitations in the experimental procedure. For the first experimental data point at $P_E = 300$ kPa, the pressure vessel has been unused for several hours or days and is therefore in true equilibrium with the ambient surroundings. Time constraints during a typical experimental run are such that following discharge of the pressure vessel to the atmosphere it is only feasible to allow of the order of five to ten minutes for the system to return to ambient equilibrium conditions before the next pressurization stage is commenced. Inevitably, some residual heat remains in the vessel wall for the subsequent pressurizing run, which results in a slightly higher final equilibrium temperature compared to that which would have been achieved had this been the first point to be measured. This trend continues as the successive data points are collected, resulting in a progressively higher experimental temperature rise, compared to that which would have been obtained under ideal conditions. This represents the principal reason for the discrepancy between the experimental and theoretical data points shown in Figure 4.

The argument presented above to describe the observed difference between experimental and theoretical results for $[T_E - T_0]$ in Figure 4 is further reinforced by examining the data in Figure 5 for a different experimental run. In this separate experiment, the first data point to be collected was that for the highest pressure, $P_E = 750$ kPa, and the value of the final equilibrium pressure was progressively reduced as the experiment proceeded. As a result, the best agreement between theory and experiment in the Figure occurs at the highest value of P_E, *i.e.*, for the first data points collected, and as the experiment proceeds, the experimental temperature difference becomes progressively higher than that predicted by the theory. The largest discrepancy between the experimental and theoretical values of $[T_E - T_0]$ is observed to occur at the lowest value of P_E, *i.e.*, for the final data point collected.

In summary, the variation between theoretical prediction and experimental measurement is determined by the amount of time allowed for the system to equilibrate between each experiment. Although not included in this paper, a much closer agreement can be obtained by accounting for the residual heat remaining in the steel shell. This is achieved by assigning different initial (measured) temperatures, T_0, to the steel shell and the mains air in the theoretical analysis.

Experimental Error • The possible sources of error in this experiment are: 1) the accuracy of the thermocouples used to monitor the temperature of the vessel wall and internal air; 2) the accuracy with which the vessel pressure can be read from the bourdon pressure gauge; 3) the accuracy with which the chart recorder output can be read and converted to a temperature. Typical errors associated with converting the thermocouple signal to the chart recorder (or any other form of datalogger) and pressure gauge readings are negligibly small and do not affect the observed trend in the experimental and theoretical data.

By far the greatest source of error lies in the measurement of a small temperature difference of a few degrees Celsius between two relatively large numbers. In practice, the accuracy of the thermocouple system is very much a function of the construction of the thermocouple and the method of installation. The absolute error could be as high as 1°C, which represents a percentage error of order 100%. There is also the added difficulty of obtaining or using a single point to obtain a "representative measurement" of the average temperature over the entire stainless steel shell. These are all points that students should consider when discussing their results or experimental methodology. But the main point of

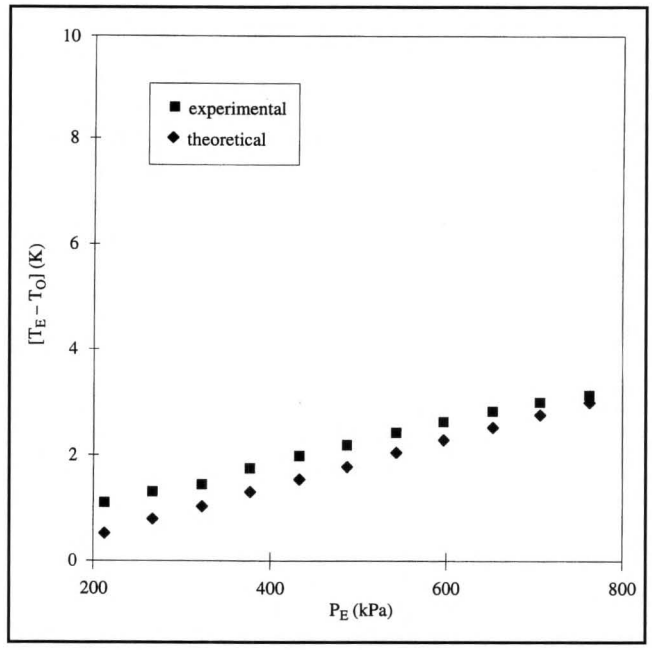

Figure 5. *Experimental and theoretical results for the variation in the system temperature difference as a function of the final vessel pressure (data points collected from higher to lower pressure).*

the experiment is to highlight the difference in temperature predictions when the steel shell is not included in the analysis, and to show that this assumption can lead to incorrect temperature predictions of up to a hundred degrees!

Effect of Control Volume Selection on the Theoretical Temperature Results • To highlight the importance of correctly defining the system in the theoretical development, it is of interest to examine the values of the temperature difference, $[T_E - T_0]$, based on the common error of neglecting the stainless steel pressure vessel wall from the control volume in Figure 2. This can be achieved by simply substituting $m_V=0$ in Eqs. (11) through (14), which yields the following expression for the temperature difference:

$$T_E - T_0 = T_0\left(1 - \frac{P_0}{P_E}\right) \Bigg/ \left(\frac{C_{V_G}}{R} + \frac{P_0}{P_E}\right) \qquad (16)$$

Using the input parameters given in Table 1 with $P_0 = 101$ kPa and $T_0 = 293$K, Eq. (16) predicts that at $P_E = 300$ kPa, $[T_E - T_0] \approx 70$K, while at $P_E = 700$ kPa, $[T_E - T_0] \approx 95$K. Comparison of this data with those presented in Figures 4 and 5 illustrates that such a system selection results in almost two orders of magnitude over prediction in the system temperature rise. This clearly shows how a simple and common error in the theoretical analysis of the compressible flow can produce highly significant errors in the resulting model predictions.

CONCLUDING REMARKS

We have discussed an undergraduate experiment on compressible flow, based on the pressurization of an adiabatic pressure vessel. In particular, the vessel is pressurized from ambient conditions up to an elevated value and the corresponding temperature change of the vessel wall and internals is measured; this process is then repeated for a number of different final pressures of between 250 and 750 kPa. Discussion of the experimental results has involved: 1) a qualitative description of the variation of internal and wall temperatures and of the vessel pressure as a function of time, and 2) the development of a theoretical model for the process and a comparison of the resulting model predictions with the experimental data. We found that the time dependency of the internal temperature and pressure can be successfully explained based on simple thermodynamic considerations.

The equilibrium temperature rise versus final vessel pressure was measured experimentally over the range $P_E = 250$ kPa to 750 kPa and showed an approximately linear increase in $[T_E - T_0]$, from about 1 K up to about 6 K, with increasing P_E. The corresponding theory predicts a similar linear trend in the data and good agreement with experiment over the first few data points collected, thereby confirming its applicability. But the theory progressively underpredicts the temperature rise as the experiment proceeds. This observation has been well explained by considering the residual heat that remains in the stainless steel wall of the pressure vessel as the experiment progresses.

The importance of defining the correct system and surroundings in the development of the theoretical model has been identified. For the adiabatic pressure vessel connected to the air mains, the control volume must include the air initially in the vessel, the air in the mains that enters the vessel, and the vessel wall. The common error of neglecting the stainless steel vessel wall from this system control volume produces an almost two order of magnitude overestimate in the temperature rise, which could obviously produce significant industrial consequences.

NOMENCLATURE

C_{P_V}	specific heat capacity for the stainless steel vessel [J kg^{-1} K^{-1}]
C_{V_G}	heat capacity of the gas at constant volume [J kg^{-1} K^{-1}]
Δh_V	change in enthalpy of the stainless steel vessel [J kg^{-1}]
m_G	mass of gas in the system [kg]
m_M	mass of gas in the mains that enters the vessel [kg]
m_V	mass of the stainless steel comprising the pressure vessel [kg]
P_E	final (equilibrium) pressure in the vessel [N m^{-2}]
P_F	vessel pressure when the inlet valve is closed [N m^{-2}]
P_M	mains supply pressure [N m^{-2}]
P_0	initial pressure in the vessel [N m^{-2}]
Q_S	heat transfer between the system and surroundings [J]
R	specific gas constant [J kg^{-1} K^{-1}]
T_E	final (equilibrium) temperature of the system [K]
T_F	vessel internal temperature when the inlet valve is closed [K]
T_0	initial temperature of the system, [K]
U_G	internal energy of the gas within the system [J]
U_S	internal energy of the system [J]
U_V	internal energy of the stainless steel wall of the pressure vessel [J]
V_M	volume of gas in the mains that enters the pressure vessel [m^3]
V_S	volume of the system [m^3]
V_V	volume of the pressure vessel [m^3]
W_S	work done by the system on the surroundings [J]
α_0	constant given by Eq. (11) [J K^2]
α_1	constant given by Eq. (11) [J K]
α_2	constant given by Eq. (11) [J]

REFERENCES

1. Kay, J.M., and R.M. Nedderman, *Fluid Mechanics and Transfer Processes,* Cambridge University Press, Cambridge, U.K. (1985)
2. Perry, R.H., and D. Green, *Chemical Engineers' Handbook,* 6th ed., McGraw-Hill, Singapore (1984) ❐