

CALCULATING MINIMUM LIQUID FLOWRATES

New Method for Rich-Phase Gas Absorption Columns

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The McCabe-Thiele method for analyzing binary absorption and stripping problems is taught as a standard part of most undergraduate chemical engineering degrees. The case of mutually insoluble carrier streams is usually considered first, with the problem set out as illustrated in Figure 1. V and L are the mole (or mass) flow rates of the vapour and liquid streams (or light and dense streams in the case of liquid-liquid extraction), a and b refer to the top and bottom of the column, and x and y are the mole (or mass) fractions of the soluble component in streams L and V respectively.¹¹ The operating line is obtained by performing a material balance around stages 1 to n of the column*

$$y = \left(\frac{L}{V}\right)x + \frac{y_a V_a - x_a L_a}{V} \quad (1)$$

In general this is a curved line, although for *lean-phase* systems (where the concentration of the soluble component is always less than 10%), the magnitude of the stream flowrates, L and V , remain nearly constant and so the operating line can be approximated as a straight line:

$$y = y_a + \left(\frac{L}{V}\right)(x - x_a) \quad (2)$$

Students are often required to determine the theoretical minimum liquid flowrate, $L_{a,\min}$, required in order to achieve a desired separation. This is the liquid flowrate at which an infinite number of stages would be required and it occurs when the operating line just touches the equilibrium line (the "pinch" point).

For lean-phase problems, the solution is trivial. The stu-

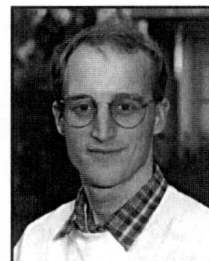
* For staged problems, the compositions and flows entering and leaving the control volume below stage n are usually written as x_2, y_{n+1}, L_n , and V_{n+1} . In this paper we use the symbols x, y, L , and V , which are for packed columns, but the two sets of terms are interchangeable.

dent only needs to draw a straight operating line from the known conditions at the top of the column (x_a, y_a) which just touches the equilibrium line. If the equilibrium line is also straight (e.g., Henry's law, $y = mx$) then this will occur at end b of the column where L_b leaves in equilibrium with the entering gas V_b at the point $x_b = y_b/m$ (see Figure 2a).

For *rich phase* systems (concentrations greater than 10%), however, the operating line has significant curvature because the ratio of L to V varies down the length of the column. In this case, assuming the required operating line passes through the point ($y_b/m, y_b$) may not always be correct. If the operating line is concave up then it is possible that the operating line may cut the equilibrium line at some point between y_a and y_b (see Figure 2c), and so the operating line through ($y_b/m, y_b$) gives too low an estimate of $L_{a,\min}$. What is more, the student only becomes aware of this error if they take the time to plot the operating line for the minimum liquid flowrate case, whereas they often without checking go on to solve the main part of the question which involves calculating the number of stages required when L_a is some multiple of $L_{a,\min}$.

In this case where the operating line is concave up (or concave down for stripping problems), to find the operating line which just touches the equilibrium line the students must either adopt a lengthy trial-and-error approach, or else

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they must solve the problem graphically^[2,3] by first converting the problem into mole (or mass) ratios, X and Y , where $X = x/(1-x)$ is the moles of solute per mole inert carrier fluid and $Y = y/(1-y)$ is the moles of solute per mole of inert carrier vapour ($0 < X, Y < \infty$). When mole ratios are used, the liquid and vapour flow rates are given as $L' = (1-x_a)L_a$ and $V' = (1-y)V_a$, the moles of solute-free liquid and vapour flow rates respectively. For mutually insoluble solute streams, L' and V' remain constant, and so the operating line is a straight line given by the equation

$$V'(Y - Y_a) = L'(X - X_a) \quad (3)$$

Once the equilibrium data has also been converted into mole ratios and plotted, the minimum condition can easily be found graphically, using a ruler to find the straight line starting at (X_a, Y_a) which touches the equilibrium line. The slope of this line is L'/V' , from which $L_{a,\min}$ can be calculated.^[2,3]

This paper presents a new analytical approach for finding the minimum liquid flowrate in rich phase problems that does not require converting the problem into mole ratios. The new method requires that an analytical expression for the equilibrium line be known and that this remains constant through the length of the column, *i.e.*, the column must be operating isothermally. This method involves rearranging Eq. (1) into an explicit expression for y in terms of x and solving to find the point at which it just touches the equilibrium line. This analytical method can be taught to students to complement the traditional graphical approach.

THE NEW METHOD

In most text books, the equation for the operating line is left as shown in Eq. (1). This equation cannot be used immediately to solve for y in terms of x , because L and V are both also functions of x for the rich phase case.

Since the flow of inert carrier fluid remains constant for

mutually insoluble streams, $(1-x)L = (1-x_a)L_a$. Therefore

$$L = \frac{(1-x_a)L_a}{(1-x)} \quad (4)$$

A total material balance around stages 1 to n gives $V = L + V_a - L_a$, which when using Eq. (4) can be re-written as

$$V = \frac{(1-x_a)L_a}{(1-x)} + V_a - L_a \quad (5)$$

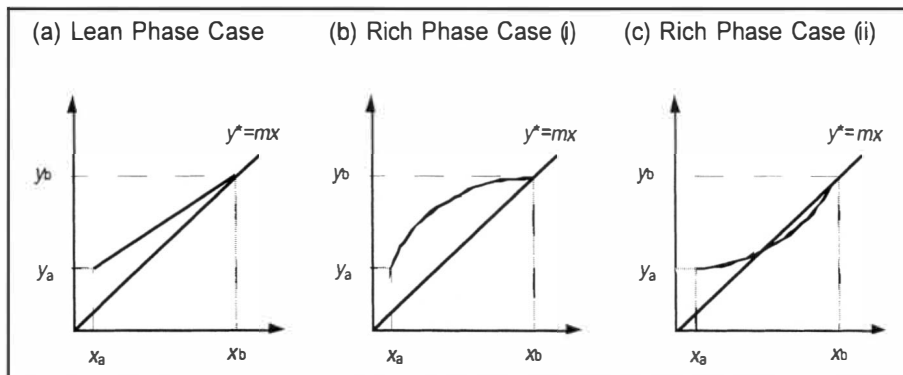
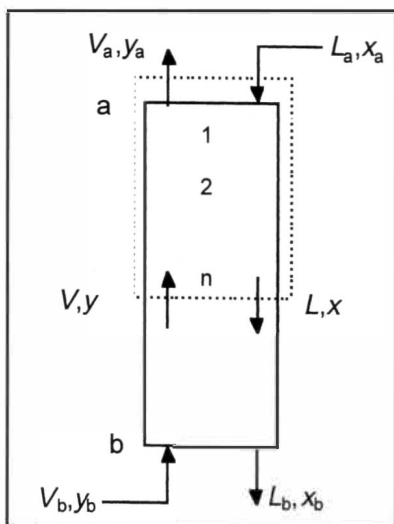
Substituting Eqs. (4) and (5) into Eq. (1) and rearranging gives an explicit equation for y in terms of x as the only variable

$$y = \frac{y_a V_a - x_a L_a + x(L_a - y_a V_a)}{V_a - x_a L_a + x(L_a - V_a)} = \frac{\left(\frac{L_a}{V_a}\right)(x - x_a) + y_a(1-x)}{\left(\frac{L_a}{V_a}\right)(x - x_a) + (1-x)} \quad (6)$$

Eq. (6) can also be differentiated to give the equation for the slope of the operating line at any point

$$\frac{dy}{dx} = \frac{L_a V_a (x_a - 1)(y_a - 1)}{\left[V_a - x_a L_a + x(L_a - V_a)\right]^2} = \frac{\left(\frac{L_a}{V_a}\right)(1-x_a)(1-y_a)}{\left[\left(\frac{L_a}{V_a}\right)(x - x_a) + (1-x)\right]^2} \quad (7)$$

Equation (6) is simple to derive, requiring only a substitution and rearrangement of Eq. (1) or (3). Although trivial to derive, however, it is not presented in this form in any of the standard introductory texts on separation processes.^[1]-5] Its usefulness lies in the fact that as an explicit function for y in terms of x , it is easy to differentiate, giving Eq. (7), which is novel. Equations (6) and (7) are extremely useful because they can be used directly to solve for y and dy/dx at any point down the column in terms of only one variable, x . Choosing end a of the column as the reference point was arbitrary. These equations can equally well be written in



◀ **Figure 1.** Schematic of absorption/stripping column showing definitions of L , V , a , b , x , and y (after McCabe, et al.,^[11])

▲ **Figure 2.** Operating and equilibrium line plots for gas absorption: (a) lean phase with straight operating line; (b) rich phase case with operating line concave down; and (c) rich phase case with operating line concave up.

terms of end b (or in terms of any other known point along the length of the column) by simple substitution of L_b for L_a , x_b for x_a , etc.

Provided that we have an analytical expression for the equilibrium relationship which is constant through the length of the column, these two equations can now be used to analytically find the minimum liquid flowrate, $L_{a, \min}$, required to achieve a given separation. The simplest case where the equilibrium line is given by Henry's law ($y^* = mx$), will now be considered as an example.

$L_{a, \min}$ occurs when the operating line and equilibrium line touch at a single point between y_a and y_b . This intersection point can be found analytically and is given by (see Appendix A)

$$\frac{L_{a, \min}}{V_a} = \frac{-\beta - \sqrt{\beta^2 - 4\alpha\delta}}{2\alpha} \quad (8)$$

where $\alpha = (1 - mx_a)^2$, $\beta = 4m(y_a + x_a) - 2(m + y_a)(mx_a + 1)$, and $\delta = (m - y_a)^2$. For stripping problems where the operating line is below the equilibrium line, $V_{b, \min}$ is found by

$$\frac{L_b}{V_{b, \min}} = \frac{-\beta + \sqrt{\beta^2 - 4\alpha\delta}}{2\alpha} \quad (9)$$

where $\alpha = (1 - mx_b)^2$, $\beta = 4m(y_b + x_b) - 2(m + y_b)(mx_b + 1)$, and $\delta = (m - y_b)^2$.

To decide whether using Eq. (8) or (9) is necessary, it must be first determined if the pinch point lies between y_a and y_b , or at y_b . The general solution strategy for finding $L_{a, \min}$ in any rich-phase gas absorption problems is as follows.

Step 1

Begin by assuming that the pinch point where the operating line just touches the equilibrium line is at the base of the

column. Therefore $x_b = y_b/m$ and L_a is found by an overall material balance around the column:

$$\text{In} = \text{Out (at Steady State)}$$

$$\text{Solute: } y_b V_b + x_a L_a = x_b L_b + y_a V_a$$

$$\text{Inert Liquid: } (1 - x_a) L_a = (1 - x_b) L_b$$

Re-arranging and solving for L_a gives:

$$L_a = \frac{(1 - x_b)(y_b V_b - y_a V_a)}{(x_b - x_a)} \quad (10)$$

Step 2

Calculate the slope of the operating line at $x_b = y_b/m$ using Eq. (7).

Step 3a

If the slope of the operating line at $x_b = y_b/m$ is less than the slope of the equilibrium line (*i.e.*, $dy/dx < m$), this indicates that the operating line has crossed the equilibrium line from above, as shown in Figure 2b, so the L_a found in Step 2 is the correct $L_{a, \min}$.

Step 3b

If the slope of the operating line is greater than the slope of the equilibrium line (*i.e.*, $dy/dx > m$), then this indicates that the operating line is intersecting the equilibrium line from below, as shown in Figure 2c. In this case, Eq. (8) can then be used to find the correct minimum liquid flowrate.

For *stripping* problems, the operating line lies below the equilibrium line and the full conditions are known at end b, but not end a. The aim is to find $V_{b, \min}$ and the requirements for the slope of the operating line at the point of intersection are reversed. The solution procedures for both absorption and stripping problems are summarized in Table 1. A worked example problem illustrating both this solution procedure and the traditional approach is given in Appendix B.

The above solution procedure can be easily adjusted to consider other analytical expressions for the equilibrium line, $y^* = f(x)$. If the equilibrium line is given by equilibrium data which does not readily fit any simple analytical expression, then the student has no choice but to convert the problem into mole ratios and solve graphically.

DISCUSSION

The new method proposed is fully analytical. However, the intuitive understanding behind the derivation that students need to appreciate is based on a graphical understanding of the problem. Hence, it cannot replace the traditional graphical approach using solute-free coordinates. It is, however, complimentary and provides students with a different set of tools for tackling such problems. In addition, the derivation of this method serves to remind students that the basic tools of analytical geometry they learned at school can be applied to apparently unrelated engineering problems.

TABLE 1

Solution Procedure for Finding $L_{a, \min}$ or $V_{b, \min}$ for Rich-Phase Absorption or Stripping Problems Where the Equilibrium Line is Given by Henry's Law and the Two Carrier Phases are Mutually Insoluble

	<u>Absorption Problem</u> (x_a, y_a, V_a, y_b , all known)	<u>Stripping Problem</u> (y_b, x_b, L_b, x_a, L_a , all known)
Step 1	Assume $x_b = y_b/m$. Calculate $L_a = \frac{(1 - x_b)(y_b V_b - y_a V_a)}{(x_b - x_a)}$	Assume $y_a = mx_a$. Calculate $V_b = \frac{(1 - y_a)(x_b L_b - x_a L_a)}{(y_b - y_a)}$
Step 2	Calculate dy/dx at (x_b, y_b) using Eq. (6)	Calculate dy/dx at (x_a, y_a) using Eq. (6)
Step 3a	If $dy/dx < m$, then $L_{a, \min}$ is the L_a found in Step 1.	If $dy/dx > m$, then $V_{b, \min}$ is the V_b found in Step 1.
Step 3b	If $dy/dx > m$, the $L_{a, \min}$ is found using Eq. (7).	If $dy/dx > m$, then $V_{b, \min}$ is found using Eq. (8)

Although no formal survey of students was performed, the author's informal impression after lecturing one class has been that many of them preferred to use analytical expressions, rather than having to convert the problems into mole ratio units and then use graphical methods.

Equation (6) is not only useful for finding the point at which the operating line touches the equilibrium line, but it can also be given to students to help them plot the curved operating lines that occur in any rich-phase problem. This is required in order to be able to step off the number of stages via the McCabe-Thiele method, or to perform the numerical integration required to find the number of transfer units in a packed column.

Even if the full analytical method is not used, the equation for the slope of the operating line, Eq. (7), is valuable because it enables students to test whether the end point (y_b , y_b/m) is the correct pinch point without having to plot the full operating line.

Equations (8) and (9) are also potentially useful for software packages for computer-based learning packages where each student in a class can be given different computer-generated problems to solve independently and then enter their answers into the computer for checking.

CONCLUSION

Explicit equations for the operating line and its slope in rich-phase gas absorption and stripping problems have been derived with x as the only variable. These expressions, although trivial to derive, are not presented in any of the standard introductory texts on separation processes. They have been used to develop a new analytical method for finding the minimum liquid flowrate in rich-phase problems without needing to convert the problem into solute-free coordinates and then use graphical methods.

The method presented here is restricted to cases where there is an analytical expression for the equilibrium line, which remains constant along the length of the column (*i.e.*, isothermal operations). It is also required that the two carrier phases are mutually insoluble. The method is ideally suited for use in computer packages for teaching students how to solve these problems. The explicit equation for the operating line is also useful for plotting the curved operating line in order to step off the number of equilibrium stages via the McCabe-Thiele method, or for numerical integration to find the number of transfer units in packed columns.

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Appendix A

The Analytical Solution for L_{\min} or V_{\min}

The minimum liquid or vapor flowrate occurs when the operating line just touches the equilibrium line $y = mx$ between y_a and y_b . This point occurs when the two lines intersect, which from Eq. (6) is at

$$y = mx = \frac{\left(\frac{L_a}{V_a}\right)(x - x_a) + y_a(1 - x)}{\left(\frac{L_a}{V_a}\right)(x - x_a) + (1 - x)} \quad (\text{A1})$$

Rearranging in terms of x gives:

$$x^2 \left[m \left(\frac{L_a}{V_a} - 1 \right) \right] + x \left[m + y_a - \left(\frac{L_a}{V_a} \right) (mx_a + 1) \right] + \left[\left(\frac{L_a}{V_a} \right) x_a - Y_a \right] = 0 \quad (\text{A2})$$

This is a quadratic equation of the general form $ax^2 + bx + c = 0$. We want the case where the operating line only just touches the equilibrium line. This occurs when there is only one point of intersection which is at $b^2 - 4ac = 0$. This gives

$$\left(\frac{L_a}{V_a} \right)^2 (1 - mx_a)^2 + \left(\frac{L_a}{V_a} \right) [4m(y_a + x_a) - 2(m + y_a)(mx_a + 1)] + (m - y_a)^2 = 0 \quad (\text{A3})$$

This is a quadratic equation of the form

$$\alpha(L_a/V_a)^2 + \beta(L_a/V_a) + \delta = 0$$

which can be solved for L_a/V_a to give

$$\frac{L_a}{V_a} = \frac{-\beta \pm \sqrt{\beta^2 - 4\alpha\delta}}{2\alpha} \quad (\text{A4})$$

where $\alpha = (1 - mx_a)^2$, $\beta = 4m(y_a + x_a) - 2(m + y_a)(mx_a + 1)$, and $\delta = (m - y_a)^2$. Unfortunately the square root term does not simplify, so equation (A4) is best left in terms of α , β , and δ .

Equation (A4) gives two solutions for L_a/V_a when the operating line just touches the equilibrium line at one point. However, only one of these is the correct point of touching between y_a and y_b . For absorption problems, the negative root should be taken, which will give the solution for $(L_a)_{\min}/V_a$. For *stripping* problems, α , β , and δ should be written in terms of x_b and y_b and the positive root taken to give the solution for $L_b/(V_b)_{\min}$.

Appendix B

Worked Example Comparing the New Method and the Traditional Graphical Method

Problem

A dry cleaning plant produces an air stream containing 2 mol% acetone. Regulations require that the concentration of acetone be reduced below 0.1 mol% before this stream is released to the environment. This is to be done by absorption with water in a countercurrent packed column. The water enters the column already containing 0.5 mol% acetone. The equilibrium relationship for acetone in air and water is given by $y = 0.1246x$ where x and y are the mole fractions of acetone in the aqueous and vapour phase respectively. Find the theoretical minimum liquid flowrate required to achieve this separation (these figures are given as examples only).

Solution

First it is necessary to find the flowrate of the gas stream exiting the column. We will assume that the water and air are mutually insoluble and take a basis of $V_b = 100$ moles and use $y_a = 0.001$. At steady state, a material balance around the column on the air gives $(1-y_a)V_a = (1-y_b)V_b$ from which

$$V_a = (1-0.02)(100)/(1-0.001) = 98.10$$

The New Analytical Method

Step 1

We will initially assume that the minimum liquid flowrate occurs when the aqueous stream leaves in equilibrium with the entering gas stream, which gives

$$x_b = y_b/m = 0.02/0.1246 = 0.1605.$$

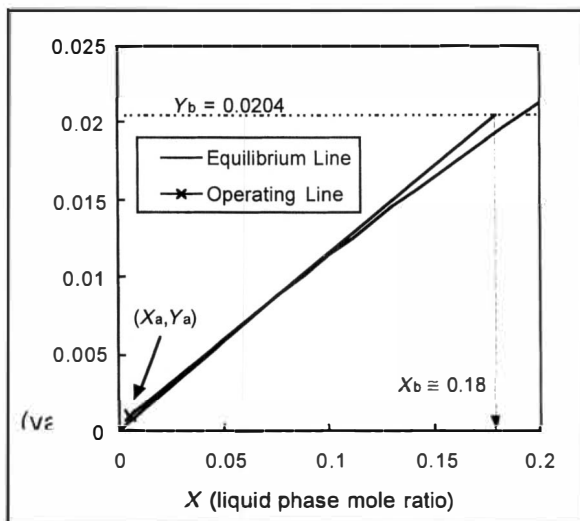


Figure 3. Plot of mole ratios Y vs. X showing the traditional graphical approach to find the operating line, which just touches the equilibrium line.

Note that since $x_b > 0.10$, this is a rich-phase problem and the operating line will be significantly curved. Equation (9) can now be used to find L_a

$$L_a = \frac{(1-x_b)(y_b V_b - y_a V_a)}{(x_b - x_a)} = \frac{(1-0.1605)(0.02 \times 100 - 0.001 \times 98.10)}{0.1605 - 0.005} = 10.27$$

Step 2

Equation (6) is used to find the operating line slope at the point (x_b, y_b)

$$\left. \frac{dx}{dy} \right|_b = \frac{\left(\frac{L_a}{V_a} \right) (1-x_a)(1-y_a)}{\left[\left(\frac{L_a}{V_a} \right) (x_b - x_a) + (1-x_b) \right]^2} = \frac{\left(\frac{10.27}{98.1} \right) (1-0.005)(1-0.001)}{\left[\left(\frac{10.27}{98.1} \right) (0.1605 - 0.005) + (1-0.1605) \right]^2} = 0.142$$

Since $dy/dx|_b > m$, this indicates that the operating line is crossing the equilibrium line from below, so we have underestimated $L_{a,\min}$. Go to step 3b.

Step 3b

Solve for $L_{a,\min}$ using Eq. (7)

$$\begin{aligned} \alpha &= (1 - mx_a)^2 = [1 - 0.1246(0.005)]^2 = 0.9988 \\ \beta &= 4m(y_a + x_a) - 2(m + y_a)(mx_a + 1) \\ &= 4(0.1246)(0.001 + 0.005) \\ &\quad - 2(0.1246 + 0.001)[0.1246(0.005) + 1] \\ &= -0.2484 \\ \delta &= (m - y_a)^2 = (0.1246 - 0.001)^2 = 0.01528 \end{aligned}$$

$$L_{a,\min} = V_a \left(\frac{-\beta - \sqrt{\beta^2 - 4\alpha\delta}}{2\alpha} \right) = 98.10 \left(\frac{0.2484 - \sqrt{(0.2484)^2 - 4(0.9988)(0.01528)}}{2(0.9988)} \right) = 10.94$$

Therefore, the theoretical minimum liquid flowrate is 10.94 moles of water per 100 moles of entering gas. Note that this is 6% higher than the original estimate found by assuming that the liquid exits in equilibrium with the entering gas. The magnitude of this error depends on the degree of curvature of the operating line.

Traditional Graphical Approach

The equilibrium line is plotted on X-Y coordinates, Figure

3, by expressing it as

$$\frac{Y}{1+Y} = 0.1246 \frac{X}{1+X}$$

or

$$Y = \frac{0.1246X}{1+(1-0.1246)X}$$

The bottom end of the operating line (X_a, Y_a) is found by converting $x_a = 0.005$ to $X_a = x_a/(1-x_a) = 0.00503$ and $y_a = 0.001$ to $Y_a = 0.00100$. The top end of the line is at $Y_b = y_b/(1-y_b) = 0.0204$. The line through (X_a, Y_a) that just touches the equilibrium line is then found graphically (see Figure 3). From Figure 3, the operating line which just touches the equilibrium line passes through $X_b \cong 0.18$ which gives $x_b = 0.152$.

The minimum liquid flowrate is now found by an overall

material balance.

In = Out

$$\begin{aligned} \text{Acetone} \quad y_b V_b + x_a L_a &= y_a V_a + x_b L_b \\ 0.02(100) + 0.005L_a &= 0.001(98.10) + 0.152(L_b) \\ 1.9019 &= 0.152L_b - 0.005L_a \end{aligned} \quad (B1)$$

$$\begin{aligned} \text{Water} \quad (1-x_a)L_a &= (1-x_b)L_b \\ 0.995L_a &= 0.848L_b \end{aligned} \quad (B2)$$

Solving the simultaneous Eqs. B1 and B2 gives the minimum liquid flowrate of $L_a = 10.97$ moles per 100 moles feed gas. This compares well with the exact analytical solution of 10.94. It should be pointed out that the traditional solution method takes more time to perform because of the requirement to plot the data. □

che book review

Engineering Flow and Heat Exchange

Revised Edition

by Octave Levenspiel

Plenum Press, New York and London (1998)

Reviewed by

Gabriel I. Tardos

CCNY

This is the first revised edition of this book, first published in 1984. Professor Levenspiel should be commended for producing such an excellent text, written specifically for engineering students. The book is a pleasure to read and offers several amusing problems, all stated in the language of students, with explanations and examples they can easily understand. Very few texts in engineering can make such a claim. I have used this text exclusively since 1992 in my teaching of unit operations to chemical engineering students. The material is broad enough, however, to also be used in mechanical engineering, and perhaps in civil engineering courses as well, to teach flow and heat transfer.

Students (especially undergraduates) tend to sell used textbooks once they finish a subject and pass their final examination. I found, with great pleasure, that *Engineering Flow and Heat Exchange* was not one of those books; seniors use it in their design courses and many graduates keep the book as a reference. This is obviously due to the wealth of information in the book and the ease with which the information can be retrieved and used. Inclusion of compressible

and non-Newtonian fluid flow in the fluid-mechanics section and direct-contact heat exchangers in the heat-exchangers section is a substantial achievement and significantly adds to the usefulness of the text.

One example of the book's unique approach to explaining a complex concept through humor and straightforward, easy-to-understand language is illustrated by how Professor Levenspiel explains the concept of equivalent average slurry density in the problem "Counting Canaries Italian Style." The "slurry" consists of canaries flying in the air inside a closed container. Measuring the pressure before and after the canaries are airborne, and using the Bernoulli equation, gives the change in density and therefore the number of "particles" (birds). Ingenious!

As already mentioned, the book is divided into a section on fluid mechanics and a section on heat transfer. The first part includes basic equations for isothermal flowing systems in Chapter 1, and as an example, flow of incompressible Newtonian fluids in pipes and around solid immersed objects in Chapters 2 and 8, respectively. Unlike other similar texts, the theory is kept short and the assumption is that the student has taken a prior course in fluid mechanics. It is assumed, for example, that the student is familiar with the concept of the Fanning friction factor.

Chapters 3 and 4 address compressible flow of gases (through material taken mostly from thermodynamics) and

Continued on page 355.