THE INTERFACE BETWEEN ChE AND MATHEMATICS

What Do Students Really Need?

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The Mathematical Association of America (MAA), through its Committee on the Undergraduate Program in Mathematics (CUPM), is conducting a Curriculum Foundations Project, a major analysis of the undergraduate mathematics curriculum. The goal of the project is to develop a curriculum document that will assist college mathematics departments as they plan their programs for the next decade. Historically, CUPM curriculum recommendations have had a significant influence on the design of undergraduate mathematics programs. These important and influential guidelines were last revised in 1981. Therefore, the CUPM curriculum guidelines need to be reconsidered; such a review and the resulting recommendations are likely to have widespread impact on the teaching of undergraduate mathematics.

Given the impact of mathematics instruction on engineering, the sciences, and the quantitative social sciences (especially instruction during the first two years), significant input from these partner disciplines is needed to inform the MAA curriculum document. The CUPM subcommittee on Calculus Reform and the First Two Years (CRAFTY) gathered much of this necessary information between Fall 1999 and Spring 2001 through a series of invitational disciplinary workshops funded and hosted by a wide variety of institutions (see Table 1).

Each workshop is focused on a particular partner discipline or on a group of related disciplines, the objective being a clear, concise statement of what students in that area need to learn in their first two years of college mathematics. The workshops are not intended to be dialogues between mathematics and the partner disciplines, but rather a dialogue among representatives of the discipline under consideration, with mathematicians there only to listen to the discussions and to provide clarification on questions about the mathematics curriculum. For this reason, almost all of the individuals invited to participate in each workshop are from the partner disciplines.

The major product of each workshop is a report or group of reports summarizing the recommendations and conclusions of the workshop. These are written by the representatives from the partner disciplines, with the mathematics community as the primary audience, and they address a series of questions formulated by CRAFTY (see Table 2). Uniformity of style is achieved across the reports by using the same basic questions for each workshop. Having a common list of questions also aids in comparing the reports of different workshops. The questions are simply designed to guide the workshop discussions, however, and therefore are

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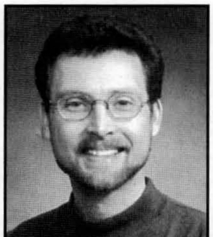

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intentionally vague. In addition, workshop participants are asked to focus primarily on the first question category, "Understanding and Contents," with the other questions being of secondary importance.

The reports from each workshop are then widely circulated within the specific disciplines, as well as in the mathematics community, in order to solicit a broad range of comments. At the completion of this process in the spring of 2001, the reports will be published and used in the formulation of the MAA curriculum document. A curriculum conference that includes invitees from all disciplines will be convened in Fall 2001 to synthesize the workshop findings and begin writing the MAA curriculum document, scheduled to be published in 2002.

In addition to providing input into the larger CUPM review, the reports serve as valuable resources for initiating discussions at individual institutions between mathematics departments and partner disciplines. Some mathematics departments have already begun using the reports to stimulate interdepartmental discussions. Such discussions, as well as those at the CRAFTY workshops, generate good will between mathematicians and colleagues in partner disciplines. In general, colleagues from partner disciplines value mathematics and welcome the opportunity to state their views about mathematics education, provided their opinions are taken seriously. Promoting and supporting informed discussions with the partner disciplines may ultimately be the most important outcome of the MAA Curriculum Foundations Project.

TABLE 1

MAA Curriculum Foundations Workshops

Physics and Computer Science
Bowdoin College • Maine • Oct. 28-31, 1999
William Barker: barker@bowdoin.edu

Interdisciplinary (Math, Physics, Engineering)
USMA • West Point • Nov. 4-7, 1999
Don Small: ad5712@usma.edu

Engineering
Clemson University • South Carolina • May 4-7, 2000
Susan Ganter: sganter@clemson.edu

Health-Related Life Sciences
Virginia Commonwealth University • May 18-20, 2000
William Haver: whaver@atlas.vcu.edu

Technical Mathematics (at two sites)
Los Angeles Pierce College • California • Oct. 5-8, 2000
Bruce Yoshiwara: byoshiwara@hotmail.com
J. Sargeant Reynolds Community Col. • Virginia • Oct. 12-15, 2000
Susan Wood: swood@jsr.cc.va.us
Mary Ann Hovis: hovisma@ltc.tec.oh.us

Statistics
Grinnell College • Oct. 12-15, 2000
Thomas Moore: mooret@math.grin.edu

Business, Finance and Economics
University of Arizona • Arizona • Oct. 28-29, 2000
Deborah Hughes Hallett: dhh@math.arizona.edu
William McCallum: wmc@math.arizona.edu

Mathematics Education
Michigan State University • Michigan • Nov. 1-3, 2000
Sharon Senk: senk@pilot.msu.edu

Biology and Chemistry
Macalester College • Nov. 2-5, 2000
David Bressoud: bressoud@macalester.edu

Mathematics Preparation for the Major
Mathematical Sciences Research Institute • Feb. 9-11, 2001
William McCallum: wmc@math.arizona.edu

THE CRAFTY ENGINEERING WORKSHOP AT CLEMSON UNIVERSITY

One of the CRAFTY workshops was sponsored and hosted by Clemson University on May 4-7, 2000. It focused on the needs of engineering from the first two years of college

TABLE 2

MAA Curriculum Foundations Workshop Questions

Understanding and Content

- What *conceptual mathematical principles* must students master in the first two years?
- What *mathematical problem-solving skills* must students master in the first two years?
- What broad *mathematical topics* must students master in the first two years?
  What *priorities* exist between these topics?
- What is the desired *balance* between *theoretical understanding* and *computational skill*?
  How is this balance achieved?
- What are the *mathematical needs of different student populations* and how can they be fulfilled?

Technology

- How does *technology affect* what mathematics should be learned in the first two years?
- What *mathematical technology skills* should students master in the first two years?
- What different mathematical technology skills are required of *different student populations*?

Instructional Interconnections

- What impact does *mathematics education reform* have on instruction in your discipline?
- How should *education reform in your discipline* affect mathematics instruction?
- How can *dialogue* on educational issues between your discipline and mathematics best be maintained?

Instructional Techniques

- What are the *effects of different instructional methods* in mathematics on students in your discipline?
- What instructional methods best *develop the mathematical comprehension* needed for your discipline?
- What guidance does *educational research* provide concerning mathematical training in your discipline?

mathematics instruction. The workshop had thirty-eight invited participants, with roughly equal representation from each of four areas in engineering (chemical, civil, electrical, mechanical) and mathematics. The workshop resulted in four documents, one for each of the four engineering areas, addressing the MAA questions specified at the outset of the workshop.

This paper focuses on the recommendations of the chemical engineering group. It is not intended to be a definitive document, but rather a working paper that generates discussion among chemical engineers in order to provide additional feedback for the mathematics community. Therefore, the authors welcome comments and additional ideas.

REPORT OF THE CHEMICAL ENGINEERING GROUP

The Chemical Engineering group members are listed in Table 3.

What Chemical Engineers Do

Since this report was originally written for mathematicians, an appropriate introduction is to discuss what chemical engineers do, why mathematics is needed, and how it is used. A reasonably broad definition is that *chemical engineers design materials and the processes by which materials are made.*

Traditionally, chemical engineers have been associated with the petroleum and large-scale chemical industries, but (especially in recent years) chemical engineers have also been involved in pharmaceuticals, foods, polymers and materials, microelectronics, and biotechnology. The core subjects that underlie and unify this broad field are thermodynamics, chemical reaction processes, transport processes (*i.e.,* the spatial and temporal distribution of mass, momentum, and energy) and process dynamics, design, and control.

On top of this fundamental framework, a central emphasis of chemical engineering education is *model building and analysis.* A good chemical engineer brings together the fundamentals to build and refine a mathematical model of a process that will help him or her understand and optimize its performance. To be good at model building and analysis, students must have at hand the mathematical background to understand and work with the core scientific areas, as well as to find solutions to the final model that they build. In this context, the "solution" to a mathematical problem is often in the *understanding* of the behavior of the process described by the mathematics, rather than the specific closed form (or numerical) result.

TABLE 3
Chemical Engineering Group Members

- **Jenna P. Carpenter** • Interim Academic Director, Chemical Engineering, Civil Engineering and Geosciences • Louisiana Technological University
- **Michael B. Cutlip** • Professor of Chemical Engineering and Director of the Honors Program • University of Connecticut
- **Michael D. Graham** • Associate Professor of Chemical Engineering • University of Wisconsin-Madison (discussion leader/recorder)
- **Anton J. Pintar** • Associate Professor of Chemical Engineering • Michigan Technological University
- **Jan A. Puszynski** • Professor of Chemistry and Chemical Engineering • South Dakota School of Mines and Technology

Here is an example: A starting point for understanding any process is writing down the conservation laws that the system or process satisfies...for conserved quantities, accumulation = input - output. Depending on the level of detail of the model, this equation might be, for example, a large set of linear algebraic equations that determine the relationships between fluxes of chemical species throughout the process (a species balance), or it might be a set of parabolic partial differential equations governing the temperature and composition of the fluid in a chemical reactor. In the thermodynamics of multiphase systems, energy is conserved but takes on a variety of forms; a good knowledge of multivariable differential calculus is essential here to keep track of everything.

Mathematics for Chemical Engineering

The purpose of the original report was not to prescribe the mathematics curriculum—chemical engineers do not want mathematics instruction to provide only what students can "get by" with knowing. Nor is it appropriate to come down on either side of the "traditional" vs. "reform" debate—it is likely that both sides are right, to an extent. Instead, some general thoughts on subject matter and emphasis are presented here.

Precalculus Foundations

By foundations, we mean

- *Basic knowledge of families of functions (polynomical, exponential,...) in terms of data, graphs, words and equations, basic trigonometric identities and geometry, properties of logarithms, etc.*
- *Equations, inequalities*
- *Basic logic and algorithms*
- *Small linear systems of equations*
- *Coordinate systems*
- *Basic arithmetic and manipulation skills*

Mastery of the above areas is crucial. Probably the most important thing the mathematics community can do here is to actively investigate the pedagogy of K-12 education—to help sort out which "reforms" are productive from those that are merely "fads" and to encourage schools not to neglect the education of the more mathematically inclined students by focusing the curriculum too narrowly on the average performer. Another important role here is to provide programs that help K-12 mathematics teachers understand some applications of the mathematics that they teach (engineering faculty should do much more here).

Linear Mathematics

Chemical engineering students would benefit from earlier exposure to the basics of linear systems in R^N, particularly

- *The geometry of linear spaces*
- *Vector algebra (especially in 3D)*
- $\mathbf{Ax} = \mathbf{b}$ *(existence and uniqueness, Gausian elimination, geometric interpretation, over- and underdetermined systems, and least squares problems)*
- $\mathbf{Ax} = \lambda\mathbf{x}$ *(characteristic polynomial and diagonalization, Jordan form, range and nullspace of* $\mathbf{A}$*, geometry)*

At the University of Wisconsin-Madison, for example, there is a course on "linear mathematics" that introduces these notions and applies them to systems of ordinary differential equations (see next section). Many chemical engineering students take this in lieu of the traditional differential equations class.

Calculus and Differential Equations

The importance of visualization in calculus cannot be overemphasized, especially as a guide to differential and vector calculus in multiple dimensions, plotting (*e.g.*, what function is linear on a log-log plot?), working in cylindrical and spherical coordinate systems, and converting between coordinate systems. Somewhat less time could be spent on techniques for evaluating complicated integrals, with the time spent instead on, for example, visualizing the application of the chain rule in multiple dimensions. Understanding of truncated Taylor series for local approximation of functions is very important and should be seen early and often. In differential equations, a thorough knowledge of linear constant coefficient systems (initial value problems and boundary value problems; see previous section) is preferable to emphasis on existence theory and series solutions for nonconstant coefficient problems. Some qualitative theory for nonlinear systems is also desirable.

Probability and Statistics

Alumni surveys typically show that this is the most common application of mathematics for the practicing chemical engineer with a bachelor's degree, in addition to the extensive use of spreadsheets. Key issues here include parameter estimation, experimental design, sampling, and the origins and properties of various distribution functions.

Students interested in graduate school should be encouraged by their mathematics professors, as well as their engineering advisors, to take additional mathematics courses. A final general comment: students should have some idea of the power of a theorem, but for engineers, concepts are more important than proofs. In other words, it is appropriate for chemical engineering students to learn mathematical facts without always seeing the associated proofs.

. . . [the] discussions . . . generate good will between mathematicians and colleagues in partner disciplines. In general, colleagues from partner disciplines value mathematics and welcome the opportunity to state their views about mathematics education, provided their opinions are taken seriously.

Technique and Technology

A fair amount of the discussion at the MAA engineering workshop, within the chemical engineering group and others, centered around the use of technology in the mathematics courses for engineers. In the discussions, "technology" meant a number of different things, from numerical methods to graphing calculators to symbolic manipulation packages. We'd like to emphasize here some points to be kept in mind when thinking of the introduction of these tools into mathematics courses. We do this in the form of responses to two questions, representing both sides of the issue (admittedly, these questions are straw men):

"My laptop can do that. Why should I learn to do it by hand?"

- *Sense of form of mathematical expressions, understanding of what manipulations are available, facility with these manipulations*
- *Fluency in the language of mathematical concepts*
- *Appreciation and recognition of mathematical rigor*
- *Discipline, maturity, confidence of mastery*
- *Closed form results are best, if available*
- *Recognition of limitations of closed form results, where things get difficult*
- *Knowledge of what computers do*

"Use of computers dumbs down the mathematics course—why use them?"

- *Solution of realistic (complex) problems, many of which involve numerical solutions. In upper-level courses, extensive use is made of programs such as MATLAB, Octave (available at <www.che.wisc.edu/octave>), MathCad, Mathematica, and Polymath*

- *Efficient exploration of solution and design space*
  - *Visualization, especially in multidimensional and vector calculus*
- *Relief from tedium*
- *Confidence in results derived by hand*

Ultimately, the technology should take a back seat in mathematics courses until it becomes necessary for solving interesting problems. For example, in a linear algebra course, students should be able to do LU decomposition of a 3x3 system by hand before they are shown that a computer algebra system can complete the process with one command. At the same time, it is useful to point out the relationship between numerical techniques and exact ones (*e.g.*, a Riemann sum can be evaluated numerically to approximate an integral). Students should have a solid understanding regarding limitations of numerical methods and their accuracy. They should clearly see the power of analytical solutions when such solutions can be found.

A Suggestion for Coupling Mathematics and Engineering Education

One set of issues that arose repeatedly in the MAA engineering workshop discussions was the concern that students do not see connections between mathematical tools and concepts and the wide utility of these in engineering. A related concern was the time lag between exposure to mathematics and its application to the solution of real engineering problems. The notion of "just-in-time" learning was discussed, and the suggestion was made that mathematics courses be more application- or example-driven and be more evenly spread through the curriculum, rather than "front loaded" into the first two years. The chemical engineering group shared these concerns, but also thought that

1) *Part of the beauty and power of mathematics is that it is example-independent—calculus applies to economics just as it does to mechanics*
2) *The time spent developing the background for engineering applications is time not spent on mathematical principles and tools*
3) *A straightforward "just-in-time" approach will not satisfy all engineering majors (e.g., electrical engineers do not need Laplace transforms at the same time as chemical engineers).*

An alternative structure can be considered for addressing these concerns, which are essentially about how to connect mathematics and engineering in the students' minds. Specifically, the college mathematics curriculum could include *discipline specific supplements,* especially in the calculus sequence. These could be workbooks or web pages containing, for example,

- *Engineering background material, e.g., some basic thermodynamics, and how specific mathematical principles and/or tools (such as total differentials and partial derivatives in several dimensions) are used*
- *Exercises or projects integrating mathematics and engineering*
- *Additional discipline-specific emphases, e.g., trigonometric identities and manipulations for electrical engineering students.*

These could be used independently by the students, or used in a one-credit course running in parallel with the calculus courses, or simply be resources for mathematics instructors wishing to gain perspective on engineering applications or bring engineering applications into the mathematics classroom. This is perhaps overambitious, but certainly worth considering. It was suggested that, within chemical engineering, CACHE (Computer Aids for Chemical Engineering <www.CACHE.org>) could play a role in studying this possibility in conjunction with MAA.

CONCLUDING REMARKS

It is clear that the application of mathematical concepts and the generation of mathematical solutions to engineering problems are essential to the educational programs of all undergraduate engineering students. Enhanced cooperation between mathematics faculty and engineering faculty can lead to a better experience for our students. Without exception, the participants felt that the workshop was a very productive way to promote dialogue between the mathematics and engineering education communities and encouraged the organization of more workshops of this type. Another venue that mathematicians can explore is the American Society for Engineering Education <www.asee.org.>, which has a mathematics division. On the other hand, it may be productive for engineering educators to attend MAA meetings.

Perhaps most importantly, mechanisms need to be implemented to promote interaction between engineering and mathematics faculty within individual universities—good relationships at this level will enable mathematics faculty to understand what material the engineering faculty would like to see reinforced and emphasized, as well as enabling engineering faculty to gain a better understanding of the issues surrounding mathematical preparation of entering freshman engineering majors.

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