The object of this column is to enhance our readers' collections of interesting and novel problems in chemical engineering. Problems of the type that can be used to motivate the student by presenting a particular principle in class, or in a new light, or that can be assigned as a novel home problem, are requested, as well as those that are more traditional in nature and that elucidate difficult concepts. Manuscripts should not exceed fourteen double-spaced pages and should be accompanied by the originals of any figures or photographs. Please submit them to Professor James O. Wilkes (e-mail: wilkes@umich.edu), Chemical Engineering Department, University of Michigan, Ann Arbor, MI 48109-2136.

AN OPEN-ENDED MASS BALANCE PROBLEM

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Mass balances, together with energy and momentum balances, are the basis for understanding almost any problem in chemical engineering. When undergraduate students have a clear understanding of these kinds of problems, they are halfway to success in attaining their chemical engineering degree.

A logical way to teach mass balances is to start with the simplest situation (steady state, few streams, no chemical reaction) and to gradually increase the difficulty of the situations, giving examples and asking the students to solve them. Most of these problems are closed, with just one possible solution, so getting the right answer is often mechanical.

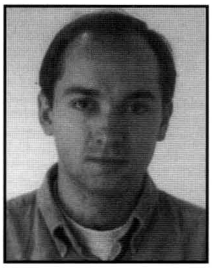
In my first year, I noticed that when I was giving a lecture, many students spent the time simply copying information from the blackboard, instead of thinking about the strategy for solving the problem. I also found that some students had difficulty with unsteady-state situations. As a result, I became interested in making my lessons more practical and closer to reality, as well as more user-friendly. To answer this need for practicality, I devised the following open-ended problem as an additional task that could be useful not only for encouraging students to analyze a real-life situation, but also for discussing different approaches suggested by the students themselves.

BACKGROUND

Fresh water is a key factor for progress and a valuable resource in arid or semi-arid regions, which is the case in most parts of Spain. To address this situation, in July 2001, a hydrological plan for national water management was approved by the Spanish government. One of the most controversial parts of this plan was to take water from the Ebro river in the north of Spain and redirect it to the Mediterranean regions in the south and east (up to 1050 Hm^3 each year). The water would be used to promote development in those areas by creating new agricultural land and developing tourism on the Mediterranean coast (hotels, aquatic parks, golf courses, etc.).

Everyone in the country has an opinion about this plan. Most people in the receiving region are in favor of it because it signals progress and economic development. Ecologists, however, feel that it will contribute to destruction of the coastal areas through unlimited building of hotels and apartments. They also fear that the expectation of vast quantities of water will encourage the cultivation and resulting destruction of virgin land.

People from donor regions in the north do not generally agree with redirecting water to other areas, arguing that they



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need the water for their own industrial and agricultural development. In addition to these economic concerns, ecologists point out that taking away such a vast amount of water from the Ebro river could have a disastrous ecological effect on the mouth of the river. To compound the problem, some farmers agree with the plan because it would involve construction of new reservoirs that would solve their watering needs during the summer dry period—but people living in the mountains, where the reservoirs would be constructed, are not happy with the plan. So, there are many different points of view, often diametrically opposed. The media often fuels the controversy, criticizing or supporting the hydrological plan depending on regional interests.

PROBLEM STATEMENT

The University of Zaragoza is located in a donor region. In February, 2001, a local newspaper began publishing articles with dramatic headlines such as "drought menace set to create critical situation for agriculture," "reservoirs at their lowest level for ten years," "serious concern about the situation," etc. Similar news stories have appeared in the past, but they appeared during the summer, not the winter.

I read the articles and analyzed the numbers given to support this "critical limit situation." Reservoirs that were at 82% capacity last year were only at 58% this year in the same month. The article went on to say that the situation would be even worse (only 38% capacity) if two of the reservoirs were taken out of the calculation (curiously, the two largest).

To emphasize this situation, the newspaper pointed out that the Yesa reservoir (470 Hm^3 total capacity) contained only 73 Hm^3 of water, *i.e.*, 17% of its capacity. This reservoir is in a secondary river of the Ebro and is one of the most controversial parts of the hydrological plan. The proposal is to increase its capacity from 470 Hm^3 to 1000 Hm^3 in order to guarantee the agricultural water requirements within its area of influence. Some people fear, however, that such a large amount of water will simply guarantee diversion of the water to the south and east regions of Spain. The plan would also mean flooding some villages. One journalist stated that 700 Hm^3 of water was required to meet agricultural demands, but there were only 80 Hm^3 available, of which a mere 30 Hm^3 was available for agricultural use, because 10 Hm^3 was required for domestic water supply and 40 Hm^3 (10% of the total capacity) was considered "dead water" that could not be used (presumably for ecological reasons, to preserve fauna). The newspaper also argued that depending on water from early-spring thawing was not a solution since that source is not reliable.

I showed this information to my students, stressing the need to carefully evaluate numerical data, especially when presented by nontechnical people. Often the media will misunderstand or erroneously report such data, *i.e.*, the journalist in this case who confused cubic meters and cubic hectometers. I went on to explain that the problem was not how much water currently existed in the reservoirs, but how much we would need in the future. In February, a 58% capacity did not seem so bad since additional water would likely come from spring rains. Although there could be shortages in some reservoirs, such as Yesa, it was likely that the spring rain and melting snow would provide enough additional water.

We cannot simply compare one year with another and conclude that there is a problem. The prior year, for example, could have been an exceptionally rainy year. Even in a dry year, alternatives can be designed to reduce water consumption that will help prevent problems.

I asked students if they thought this situation was as critical as the media wanted us to believe. I posed this concept as the statement of the problem, but provided no additional information. It would be their job to analyze the problem, to find sources of information, and to make assumptions. While their solutions could, depending on the results, be used to support or reject the hydrological plan or the construction of new reservoirs, this was not the main goal of the exercise.

This paper [presents] an open-ended problem that can easily be adapted to many local conditions . . . The problem is beneficial to students in many ways: it can make mass balance classes more realistic, it can facilitate the assimilation of concepts such as unsteady state, and it can help students carefully analyze information provided by daily television, radio, and newspaper reports.

PROPOSED SOLUTIONS TO THE PROBLEM

The problem statement was challenging to the students, who were a bit confused—they did not know how to start. They had not quite grasped the meaning of the problem. I provided some help by stating that a way to solve the problem could be to

- *First, define a "critical situation"—for example, reaching 10% water capacity, which would mean a "dead" reservoir, or 0% water capacity, or would there be enough water to supply the population for a limited period of time or would there be enough water for agricultural, etc.?*
- *Second, predict if this "critical situation" could be reached in the future.*

In the first part, the point selected is a result of personal choice and reasons for or against it can be given. The second part, however, is a quantitative prediction which may or not be valid, depending on how the prediction is made.

There are two approaches to the problem

- *Consider a general case of total water reserves*
- *Consider the evolution of a particular reservoir, such as Yesa, since the first approach does not consider the water levels of a reservoir that are far below the average of the whole river.*

The problem can be solved by an unsteady-state mass balance, *i.e.*

$$\begin{aligned}\text{Accumulation} &= \text{Final water} - \text{Initial water}\\ &= \text{Inputs} - \text{Outputs}\\ &\text{or}\\ A = V(t) - V_0 &= I - O \qquad (1)\end{aligned}$$

Water accumulation in reservoirs is the difference between inputs and outputs. Outputs are defined as water designated for agriculture, industry, and domestic use as well as water that is returned to the river (thus guaranteeing ecological flow). As long as we can quantify initial water reserves, (V_0^*), inputs (I^*), and outputs (O^*), over a future period of time, we can predict the final water reserves at the end of that period, $V^*(t)$, by applying Eq. (1) (in which the asterisk refers to future values).

INSTRUCTOR'S SOLUTION #1

All Reservoirs in the Ebro

Data pertaining to the current situation, *i.e.,* water accumulated in reservoirs, allows us to control or regulate outputs so we can manage water for different needs. We cannot, however, control global inputs, *i.e.,* water that comes from rain and snow. How we quantify the inputs and outputs of accumulated water will be predicted differently in the future.

The whole system of the reservoirs, the main river, and the secondary rivers are represented in Figure 1.

One of the main sources of data about the Ebro river is the official organization "Confederación Hidrografica del Ebro" (CHE). Historical data can be found at the organization's web site <http://www.chebro.es/>. Information such as average consumption, current relation to the river as a whole, and specifics about each reservoir can be found here.

Part of the site information involves the evolution of water reserves, which is presented by comparing the current year's evolution with that of the previous year, along with average evolution over the past five years. A prediction of final water at the end of a period of time (t) could be made (considering the initial situation, with reservoirs at 58% capacity, and knowing inputs and outputs) by simply applying Eq. (1). Although there is no data for individual inputs and outputs, we can assume that inputs (I*) and outputs (O*) in the future will be the average of those of previous years (I and O). This simplifies the problem since all we need to know is the difference between the two—that is, the difference between the final and the initial accumulated water over several weeks. This can be expressed mathematically by

Mass balance in the future (Eq. 1): $V^*(t) - V_0^* = I^* - O^*$

Mass balance in the past during the equivalent period of time (average last five years) (Eq. 1): $I - O = V(t) - V_0$

Assumption: $I^* = I$ and $O^* = O$

Combining mass balances with our assumptions, we get

$$V^*(t) = V_0^* + \left[V(t) - V_0\right] \qquad (2)$$

The average difference can be used to calculate the final water situation every week over a whole year by using Eq. (2), *i.e.* the evolution of water reserves. Such results can predict if a critical situation, for instance 10% of total capacity, will be reached. The results are shown in Figure 2.

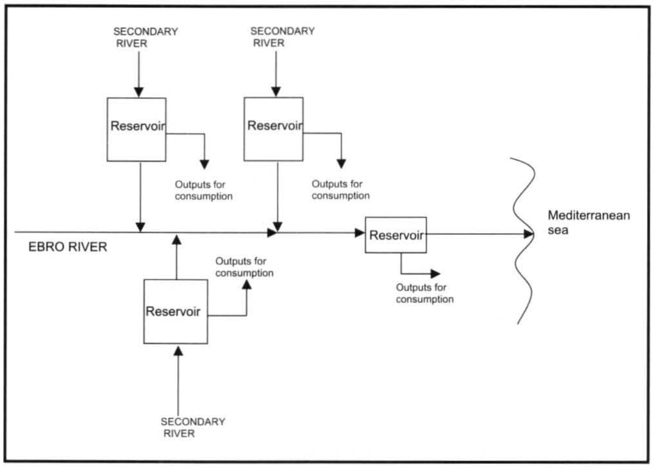

Figure 1. *Schematic representation of Ebro's basin.*

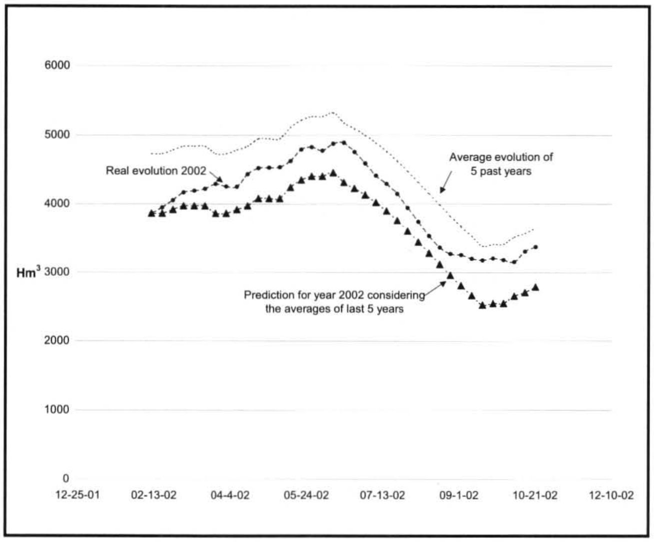

Figure 2. *Predicted and real evolution of water reserves in the year 2002.*

The water accumulated in the reservoirs is far from dropping to 650 Hm^3, which would be 10% of the total capacity (6504 Hm^3). The minimum capacity, during February through October, was approximately 2500 Hm^3 (38% of total capacity.) We can therefore conclude that the situation is not really critical, or at least, not as much as the media want us to believe.

In order to accurately support this prediction, real quantities of water, collected during 2002, are presented in Figure 2, but show a better situation than predicted. This strengthens the argument that the situation, in February 2002, is not critical.

A more conservative approach is shown in Eq. (3) which illustrates modification by including a coefficient $(\alpha<1)$.

$$V^*(t)=V_0^*+\alpha\left[V(t)-V_0\right] \qquad (3)$$

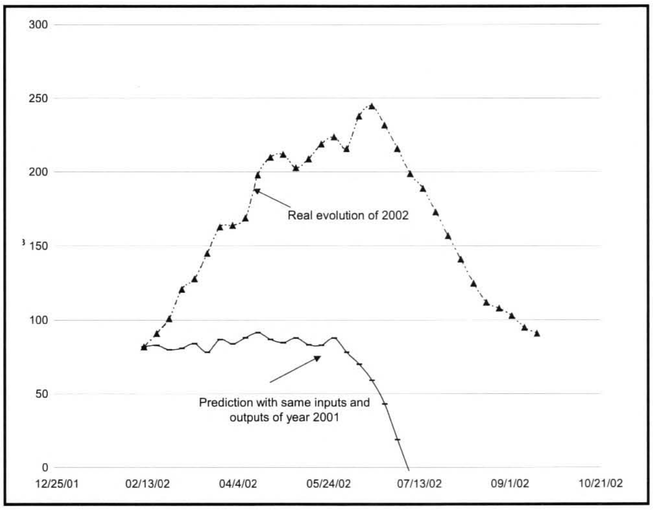

Figure 3. *Predicted and real evolution of water reserves for Yesa reservoir in the year 2002 without regulation of outputs.*

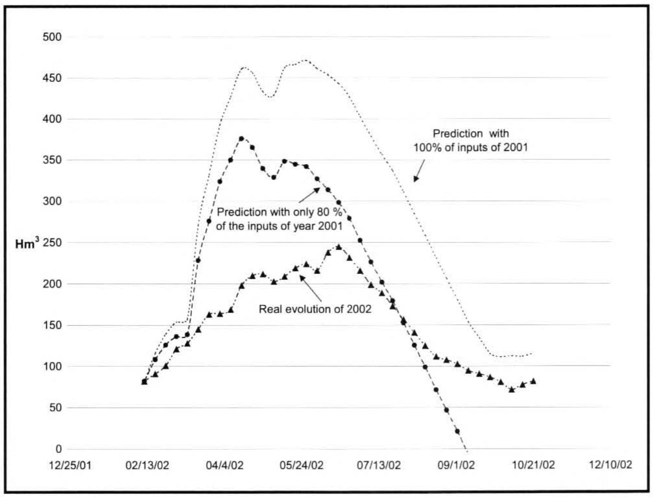

Figure 4. *Predicted and real evolution of water reserves for Yesa reservoir in the year 2002 with regulation of outputs.*

INSTRUCTOR'S SOLUTION #2

Yesa Reservoir Only

A main criticism of the first approach is the lack of consideration for specific situations of reservoirs where the water levels are far below the 58% average of the whole river. To increase the accuracy of this approach, it is necessary to analyze individual cases such as the Yesa reservoir which was only at 17% capacity (82 Hm^3).

The same approach can be made (considering inputs and outputs) to predict the evolution of accumulated water in the future. Data from the previous five years was difficult to obtain, so we substituted it with data from 2001. By applying Eq. (2) to this data we get the result shown in Figure 3, where the real evolution of water reserves has been plotted.

At the beginning of 2001, the Yesa reservoir was almost full. Considering that it was a rainy year and that no water was accumulated during the following winter and spring (of 2002), we see that the outputs almost equal the inputs, giving the results shown in Figure 3. This result is unrealistic because it does not take into consideration the regulation of outputs, which is the only option that can be taken when extremely low water reserves (as they were at the beginning of 2002) verge on becoming dead reservoirs.

A different approach to consider involves the regulation of outputs. First, we can assume that 2001 and 2002 inputs will be the same. Second, we can also assume that outputs will remain at 5 Hm^3/week, the minimum, until April when agricultural demand for water increases. We arrive at the 5 Hm^3/week minimum by deducing that outputs rarely drop lower according to historical data. Third, we assume that outputs from the current year and the previous year are the same from April

Assumptions: $I^* = I$
$O^* = 5$ Hm^3/week until April
$O^* = O$ from April

Taking all these assumptions into account, the results obtained through Eq. (1) are shown in Figure 4, as well as the real evolution of water reserves. The main inconvenience of using input data from 2001 is that it was a rainy year and the inputs could be overestimated. A more reliable prediction could be made by reducing inputs—for example, only 80% of the inputs from 2001 will be achieved in the year 2002. The choice of 80% is arbitrary; any other amount could be chosen. These results can also be seen in Figure 4.

The predicted evolution is quite different depending on the assumed inputs (100% or 80% of the year 2001.) In the first case, accumulated water levels do not fall below the initial 17% level (73 Hm^3); it always exceeds 100 Hm^3 (21%) even at the end of the summer dry period, which is well over the critical minimum of 47 Hm^3 (10%). In the second case, however, accumulated water descends to the 10% level by the end of August, and reaches zero in September. A minimum decline in water inputs, with respect to the previous year, can therefore lead to a critical situation if outputs are not regulated and restricted.

The fact is, 2002 was not as rainy as 2001. Inputs were down, which led to water restrictions and tighter regulation of water outputs for agriculture. This resulted in a more rational usage of the limited water resources by implementing better efficiency of watering techniques or lowering the demand for growth. No indications have been reported by local media about catastrophic damage to agriculture or significantly lowered production as compared to previous years. We can thus conclude that a largely available resource can be used up inefficiently, even if it is as valuable as fresh water.

STUDENTS' SOLUTIONS

The proposed solutions here represent just one approach to the problem; there are several different approaches that could be made. Apart from merely qualitative solutions or simple compilations of past data without predictions, various solutions were proposed by the students, which are summarized as follows:

Solution 1

Data Source:	CHE	
Estimations:	Current amount of water accumulated in reservoirs is	4486 Hm^3
	58.8% of total capacity of	7630 Hm^3
Local Water Consumption:	domestic drinking water	313 Hm^3/year
	agriculture	6310 Hm^3/year
	livestock	66 Hm^3/year
	industry	414 Hm^3/year
	water transfer to other areas	246 Hm^3/year
	ecological flow	3536 Hm^3/year
	TOTAL CONSUMPTION	10885 Hm^3/year
Critical Situation:	no water in reservoirs	
Assumptions:	Flow in the mainstream of the Ebro river is the average of the average flows measured at different points in the mainstream, since all the secondary streams go into this mainstream.	
	average flow	7073 Hm^3/year
Conclusion:	Variation of accumulated water at the end of the year will be	
	A = 7073 - 10885 = -3812 Hm^3/year	
Final Amount of Water in Reservoirs:	4486 + (-3812) = 674 Hm^3/year	

Despite a negative accumulation term, at the end of the year there was water in the reservoirs. In conclusion, with 58.8% capacity, there was no critical limit situation.

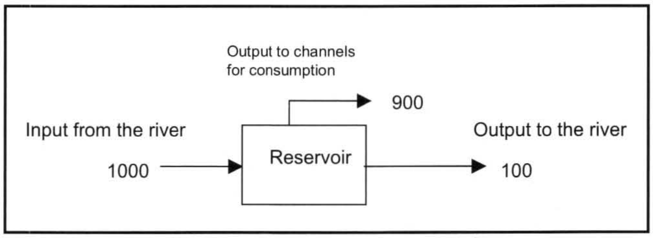

Figure 5. *Schematic representation of a reservoir*

Instructor's Comments • The assumption that the flow in the mainstream of the Ebro river is the average of the average flows measured at different points in the mainstream involves at least two mistakes. If the average is made upstream and downstream of a reservoir, we are failing to take into account water that is taken out of the river for consumption, as can be seen in Figure 5.

Measured average flow is (1000 + 100) / 2 = 550 Hm^3/year, while in fact it is 1000 Hm^3/year.

In addition to this effect, not all the water from secondary rivers goes into the main river (Ebro), since part of it could be used for consumption, as can be seen in Figure 6.

The calculated average flow is (200 + 300) / 2 = 250 Hm^3/year, while the real available amount of water is 1200 Hm^3/year.

Solution 2

Data Source:	local newspapers	
Estimations:	current amount of water accumulated in reservoirs is	3950 Hm^3
	(60% of total capacity of	6583 Hm^3)
Water Accumulated one year ago:		5394 Hm^3
Critical Situation:	in reservoirs, less than	1029 Hm^3
	(for emergency situations	700 Hm^3)
	(5% of total capacity that cannot be used because of its low purity	329 Hm^3)
Assumptions:	Since we are in a dry period, there will be no pre-	

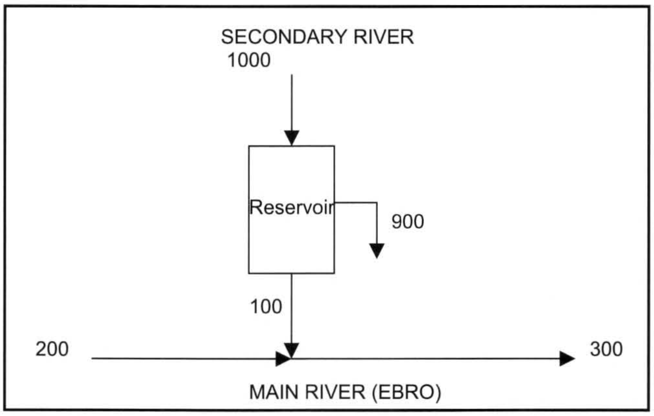

Figure 6. *Schematic representation of a secondary river*

cipitation at all in the future. The only inputs of water to the Ebro will come from melting of current snow into three secondary rivers: the Aragón, Gallego, and Cinca estimated as a total amount of 202 Hm^3

Outputs or consumption in one year are the difference between water one year ago and accumulated water now

5394 - 3950 = 1444 Hm^3/year

Conclusion: Since accumulation (A) = Inputs (I) - Outputs (O), then time to reach a critical situation
A = final water - initial water
= 1029 - 3950 = - 2921 Hm^3
I = water from rain (=0) + water from melted snow = 202 Hm^3
O = consumption in one year x time
1444 Hm^3/year x time (years)
time is therefore 2921 / 1444 = 2.02 years to reach a critical situation, so the current situation is not critical.

<u>*Instructor's Comments*</u> • The assumption of no water precipitation in the future is highly improbable, and the assumption that inputs to the system will come exclusively from snow via three secondary rivers is unrealistic. These assumptions represent, at best, a conservative scenario. The assumption that water consumption in years is the difference between water accumulated now and one year ago is invalid, however. Consumption is not 1444 Hm^3/year; actually it is estimated at 7500 Hm^3/year plus 3400 Hm^3/year ecological flow at the mouth of the Ebro. To know the water consumption figures, inputs and outputs during this period need to be known. According to this incorrect assumption, if this year is like the past year, the water accumulated in reservoirs would have been the same and we would not have consumed any water, and if this year had been more rainy that the past year, instead of consuming water, we would have produced water, showing negative consumption. This makes no sense.

<u>Solution 3</u>

Data Source:	CHE	
Estimations:	current amount of water accumulated in reservoirs is	4033 Hm^3
	62% of total capacity of	6504 Hm^3
Local Water consumption:	human consumption	7405 Hm^3/year
	ecological flow	3154 Hm^3/year
	TOTAL CONSUMPTION	10559 Hm^3/year
Oct. 2000 - Oct. 2001	REAL TOTAL CONSUMPTION	20035 Hm^3
	human consumption	7405 Hm^3
	measured at Ebro's mouth	12630 Hm^3
	average precipitations whole basin	658 mm of water/m^2
	average historical precipitations (1940-1995)	682 mm/m^2
	with a mimum of	526 mm/m^2
Critical Situation:	no water in reservoirs	

Assumptions: since we are in a dry period, there will be no precipitations at all in the future.

Conclusion: since accumulation (A) = Inputs (I) - Outputs (O), then time to reach a critical situation

A = final water - initial water

= 0 - 4033 = - 4033 Hm^3

I = water from precipitations 0 Hm^3

O = consumption in one year x time

10559 Hm^3/year x time (years)

time is therefore 0.382 years

i.e., 4 months and 17 days is the time to reach a critical situation, so the current situation could be critical but this is unlikely to occur because it would require annual precipitations of

10559 / 20035 x 658 = 397 mm water/m^2

which is well below the average annual precipitations of 682 mm/m^2 during the period 1940-1995, and less than the minimum in this period of 526 mm/m^2.

<u>*Instructor's Comments*</u> • The implicit assumption is that water demands will be distributed homogeneously throughout the year, but this is not true. Water demand increases in spring and summer, due mainly to agricultural needs. Thus in February we are close to reaching the highest point of water consumption that begins in spring and 4 months and 17 days to consume all the water is very optimistic. In addition to this problem, water reserves are not equal in every reservoir and 62% total capacity could mean that some reservoirs are full and others are almost empty. A "case by case" study should therefore be carried out.

CONCLUSION

This paper has presented an open-ended problem that can easily be adapted to many local conditions, since use of a limited and valuable resource such as fresh water is a problem almost everywhere. The problem is beneficial to students in many ways: it can make mass balance classes more realistic, it can facilitate the assimilation of concepts such as unsteady state, and it can help students carefully analyze information provided by daily television, radio, and newspaper reports. It can also be helpful for finding possible solutions as well as encouraging class discussions on the validity of assumptions made by different solutions proposed by the students themselves. ❐