HEAT TRANSFER IN GLASS, ALUMINUM, AND PLASTIC BEVERAGE BOTTLES

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In August 2004, the Pittsburgh Brewing Company began packaging its Iron City Lager in bottles made of aluminum rather than glass. Advertisements stated that the contents of an aluminum bottle not only got colder faster, but stayed colder longer than the contents of a traditional glass bottle.[1] Despite the fact that the thermal conductivity of aluminum is much higher than that of glass, it was claimed that an aluminum bottle would keep the contents cold for up to 50 minutes longer than a glass bottle would.[2] This claim appears to have originated from a study done in February 2004 by an un-named independent laboratory for Danzka, a Danish vodka producer that began using aluminum bottles at that time.[3] Aluminum bottles are now used by several beverage companies who maintain the "gets colder faster" claim but have dropped the illogical "stays colder longer" claim.[4,5] The myth of the insulating ability of aluminum beverage bottles persists, however, on the Web and elsewhere.

Part of the motivation for substituting aluminum for glass appears to be less product loss due to bottle breakage and lower shipping cost due to lower weight.[6] Some beverage companies have also begun using plastic bottles for these reasons as well as for safety at beaches and public events.[7] The claim has been made that plastic bottles stay cold as long as glass and longer than aluminum.[8]

There have been few published scientific studies on the thermal performance of bottles. Researchers at Bucknell University reported at the 2005 Annual AIChE meeting that the contents of aluminum bottles cooled down much faster but also heated up slightly faster than those in glass bottles.[9] Conversely, researchers at Loyolla College found that on heating in air, an aluminum bottle kept it contents colder slightly

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Ryan Shevlin *and* ***Tanya Soffen*** *contributed to this study as seniors in chemical engineering at WPI. This work is based in part on their Major Qualifying Project (senior thesis) completed in May 2009. Although they have now graduated and begun their professional careers, they are continuing their bottle studies with a focus on taste and enjoyment.*

TABLE 1 Properties of Materials Used							
	Inside Diameter d (m)	Effective Height z (m)	Thickness t (m)	Density ϱ (kg/m^3)	Heat Capacity C_p (J/kg K)	Thermal Conductivity k (W/m K)	k / t (W/m^{2}K)
aluminum	0.059	0.174	3.81e-4	2700	900	160	420000
glass	0.060	0.168	2.03e-3	2203	703	1.38	680
plastic	0.062	0.157	3.56e-4	1350	1300	0.2	560
water				1000	4180	0.6	
air				1.205	1006	0.025	

longer than a glass bottle but explained that the two bottles behaved essentially identically because the heat transfer is controlled by natural convection and radiation at the outer surface rather than conduction through the bottle wall in this situation.[10] Calculations they made indicated that thermal conductivity of the bottle material should have little effect upon heating in air but should have a significant effect upon cooling in ice water or heating while hand-held.

In this paper we report experiments and calculations that quantify the thermal performance of glass, aluminum, and plastic bottles under various conditions and provide an interesting way to teach heat transfer principles. We measured the temperature of water in 16 oz bottles upon cooling in a refrigerator, cooling in ice water, heating in air, and heating while hand-held, and used COMSOL Multiphysics software to illustrate the appropriate heat transfer mechanisms and calculations. Although we undertook this investigation as a senior project we believe our methods and results can be readily applied to teaching heat transfer fundamentals via a course project, a laboratory exercise, or a class demonstration. While this problem has particular appeal to some students, it should be noted that students should be at least 21 years of age to appreciate it fully.

EXPERIMENTAL

Readily available 16 oz beverage bottles (Budweiser, aluminum and glass; Miller Lite, plastic) were drained, rinsed with water, and air dried. Number 3 rubber stoppers were sliced longitudinally halfway through to accommodate thermocouple wires that were extended into the bottles to a height of 4 inches from the bottom of each. Type J thermocouples were used with National Instruments interfaces connected to Labview software for continuously monitoring and recording the temperature of each bottle. The thermocouples were found to give the same readings to within 0.05 ˚C in ice water. Bottles were filled with 475 ml of deionized water. Weight as well as volume of each fill was carefully checked to ensure the bottles contained the same amount.

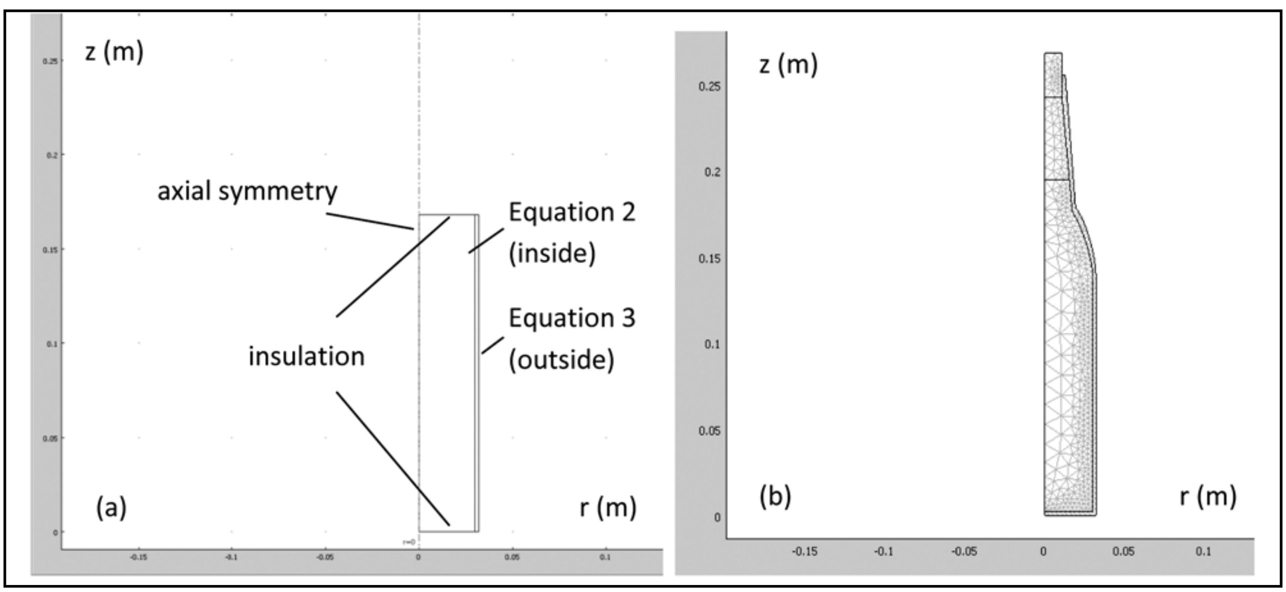

Figure 1. *2-D axially symmetric geometry of glass bottle modeled as (a) equivalent cylinder filled with water, and (b) more realistic bottle shape with air between water and rubber stopper. Boundary conditions are indicated on the equivalent cylinder model.*

A small (2 cubic ft) refrigerator that was otherwise empty was used for cooling experiments. Bottles were placed on a plastic rack and were not touching the walls or each other to minimize conduction. Duplicate measurements were made with bottles in different positions within the refrigerator to determine if bottle placement affected the results. Measurements were also made with two of the same type of bottle in the refrigerator and with two thermocouples in the same bottle. Heating in air was studied by removing the bottles from the refrigerator and placing them on a plastic rack in the room. Cooling in ice water experiments were conducted by simply submerging each bottle up to the neck in an ice water bath. Heating while hand-held experiments were conducted with one person holding one bottle in each hand after the bottles were removed from the ice water bath. Over the course of the experiment, the subject placed the bottles on a shelf intermittently, rubbed the hands together to warm them, and alternated which hand held which bottle, but care was taken to ensure that each bottle was held for the same length of time.

ANALYSIS

We assume that the bottles can be considered as cylinders of the appropriate diameters with the heights adjusted to yield volumes of 475 ml. The measured inside diameters and the effective heights of the bottles are shown in Table 1. Heat transfer through the bottle material can be described by

$$\rho C_P \frac{\partial T}{\partial t} = \nabla \cdot k \nabla T \tag{1}$$

with boundary conditions at the inside and outside walls given by:

$$-k \left. \frac{\partial T}{\partial r} \right|_{r=R_i} = h_i \left(T - T_i \right) \tag{2}$$

$$-k \left. \frac{\partial T}{\partial r} \right|_{r=R_o} = h_o \left(T_o - T \right) \tag{3}$$

The heat transfer coefficient at the inside wall, h_i, accounts for conduction and natural convection in the fluid (water) in the bottle. The heat transfer coefficient at the outside wall, h_o, accounts for convection in an air (or water) layer surrounding the bottle and for heat transfer via radiation. We have simplified our analysis by calculating the heat transfer in the radial direction only, assuming insulation boundary conditions at the top and bottom of our cylinders as shown in Figure 1a. This assumption renders our 2-D axially symmetric model to be equivalent to a 1-D model since the temperature will be uniform in the axial direction. We prefer the visual representation of the 2-D model, however, and believe it provides a better physical feel for the problem. For example, it is easier to visualize the area available for heat transfer in 2-D axial symmetry than in 1-D. While not technically precise, with reasonable approximations for heat transfer coefficients, this simple analysis explains the observed trends in our data sufficiently well. To provide a physical interpretation of the heat transfer coefficient, we have also developed models of our bottles that include an equivalent stagnant air or water layer at the outside surface of the bottle. Physical properties used for this layer, the water in the bottles and the ice water bath, and the bottle materials are included in Table 1.

COMSOL Multiphysics finite element software was used to solve Eq. (1) and give a visual representation of the calculated temperature in the bottles as a function of time and position. We also drew the glass and aluminum bottles more accurately as shown in Figure 1b and considered heat transfer through the rubber stoppers and the air above the water in the bottles. We found that modeling results were substantially the same with those geometries as with the equivalent cylinder models.

RESULTS AND DISCUSSION

Cooling a glass bottle in a refrigerator. The solid line in Figure 2 shows the measured temperature as a function of time inside the glass bottle upon cooling in the refrigerator. It took about 8 hours for the beverage to reach the control temperature of the refrigerator (about 1 °C). Note that the inflection in the experimental results was always present and appears to be due to the density maximum for water at 4 °C.

To understand the heat transfer process, we began by attempting to model it with conduction-only, with the temperature at the outside of the bottle surface fixed at $T_{refrigerator}$ (imagining that there is enough cold air and cooling power

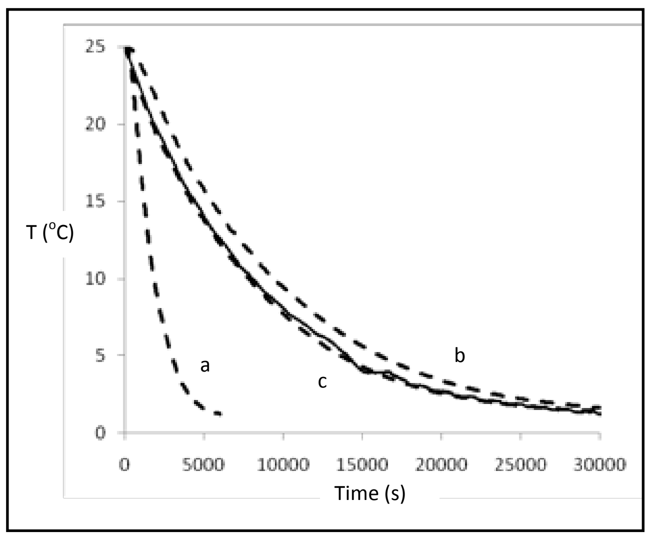

Figure 2. *Experimental results and model predictions for temperature at the center of a glass bottle upon cooling in a refrigerator. Solid curve, experimental results, Dashed curves: (a) conduction only model with T_o = 1 °C; (b) outside heat transfer coefficient, h_o = 9 W/m² K, to account for radiation and natural convection in an air layer surrounding the bottle, conduction through stagnant water inside; (c) h_o = 9 W/m² K and h_i = 400 W/m² K to account for natural convection inside the bottle.*

TABLE 2
Outside Heat Transfer Coefficients and Biot Numbers Upon Cooling in Refrigerator and Ice Water

	Cooling in refrigerator		Cooling in ice water	
	h_o (W/m^2K)	Bi	h_o (W/m^2K)	Bi
aluminum	10	0.000024	200	0.00048
glass	9	0.013	200	0.294
plastic	9	0.016	200	0.356

within the refrigerator that the temperature at the bottle surface is constant). That is, we used a constant temperature boundary condition of $T = T_{refrigerator}$ instead of Eq. (3) and considered conduction through water inside the bottle with a continuity boundary condition at the inside wall instead of the boundary condition of Eq. (2). This is clearly incorrect, but some students think this way initially and it is instructive to illustrate the fallacy and correct it incrementally. As shown in curve a of Figure 2, this severely under predicted the time required to come to thermal equilibrium with the refrigerator temperature.

The fixed T boundary condition at the bottle surface is incorrect because the air around the bottle heats up when the warm bottle is placed in the refrigerator. The air some distance away from the bottle will be at $T_{refrigerator}$, but not the air at the bottle surface. It should be clear that introducing a stagnant air layer around the bottle, where T decreases from $T_{surface}$ to $T_{refrigerator}$, would significantly increase the predicted time to reach thermal equilibrium. But how thick should the air layer be? And would it really be stagnant? In reality, density differences brought about by temperature differences in the air near the bottle will result in natural convection-circulation of the air near the bottle. This flow of air near the bottle will result in improved heat transfer over that of a truly stagnant air layer. Complicating matters even further is the fact that heat transfer by thermal radiation provides a significant contribution for objects being cooled in a refrigerator.[11] Rather than deal with all the complexities of this process in detail, Eq. (3) is used as a boundary condition at the bottle surface with the outside heat transfer coefficient, h_o, accounting for contributions from resistance to heat transfer across an outer air layer, natural convection in the air layer, and thermal radiation. By methods described in the Appendix, we estimated h_o to be about 9 W/m^2 K. Using this in the boundary condition of Eq. (3) instead of a constant temperature boundary condition, but still assuming conduction only inside the bottle resulted in curve b of Figure 2.

Natural convection also occurs inside the bottle and is better modeled using the boundary condition of Eq. (2) than by conduction-only with a continuity boundary condition. As explained in the Appendix we estimated the inside heat transfer coefficient, h_i, to be about 400 W/m^2 K and including that along with h_o, as described above, we obtained curve c in good agreement with the experimental data. In this case, the thermal conductivity of the water in the bottle was increased 1000 fold in the model ensuring that all the resistance to heat transfer on the inside of the bottle was lumped into h_i.

Analysis of the results from Figure 2 indicates that the largest resistance to heat transfer is from the air layer surrounding the bottle. The Biot number, defined for this case by,

$$\mathrm{Bi} = \left(h_o t\right) / k \tag{4}$$

gives a measure of the relative importance of convective and conductive heat transfer. A common rule of thumb is that a Biot number less than 0.1 indicates that the thermal resistance due to convection dominates the heat transfer process to the extent that resistance to heat transfer due to conduction is negligible. When applied to the bottle wall, as shown in Table 2, the Biot number for our glass bottle cooling in a refrigerator is small enough that the bottle wall should not affect the process and the wall temperature will be essentially uniform.

This point can be illustrated clearly by modifying our model to include conduction through an equivalent stagnant air layer appended to the outer edge of the bottle wall. Note that air outside the bottle will be in motion due to natural convection, but there will always be a boundary layer near the bottle surface where the dominant heat transfer mechanism is conduction. The thickness, δ_e, of the equivalent stagnant layer (effective thermal resistance layer) that we envision is not necessarily a physically measurable length out from the bottle surface to a point where $T = T_{refrigerator}$, but instead is given as the length required to match the observed heat transfer coefficient,

$$\delta_e = \frac{k}{h_o} \tag{5}$$

Rearranging this equation as $h_o = k / \delta_e$ provides a physical interpretation of the heat transfer coefficient. Considering the thermal conductivity, k, of air as 0.025 W / m K, an effective air thickness of 0.00278 m is required to match our h_o value of 9 W / m^2 K. Including an air layer of this thickness in an equivalent conduction-only model with a fixed $T = T_{refrigerator}$ boundary condition at the outer edge of the air layer reproduced curve b in Figure 2.

To obtain curve c in Figure 2 using our equivalent conduction-only model, we used Eq. (5) to determine that conduction through a thermally resistant water layer with thermal conductivity of 0.6 W / m and thickness of 0.0015 m is equivalent to using h_i = 400 W/m^2 K. It appears that convective mixing inside the bottle results in an effective thermal resistance layer only 1/20th as thick as the bottle inside radius. The rest of the water in the bottle is considered to have a very high thermal conductivity so that it will have uniform temperature and pose no further resistance to heat transfer in this model. An alternative approach that perhaps gives a better feel for the convective mixing going on inside the bottle and provides the same temperature vs. time result (curve c) is to use a moderately high thermal conductivity for all the water in the bottle. An effective thermal conductivity of 12 W/m K (increasing the water thermal conductivity 20 fold) was required for this approach.

The heat transfer rate by conduction through a composite material is given by

$$q = \frac{\Delta T}{\sum R_j} \tag{6}$$

where the resistance to heat transfer due to each material j is given by

$$R_j = \frac{t_j}{k_j A_j} \tag{7}$$

and A_j is the area available for heat transfer into material j. Our equivalent conduction-only model provides a visual representation of the resistance to heat transfer given by the effective outside air layer (representing the outside heat transfer coefficient), glass wall, and the effective inside thermally resistant water layer (representing the inside heat transfer coefficient) as shown in Figure 3a where the predicted temperature is plotted as a function of position in the radial direction for three different times: 60, 600, and 6000 s. It can be seen that the

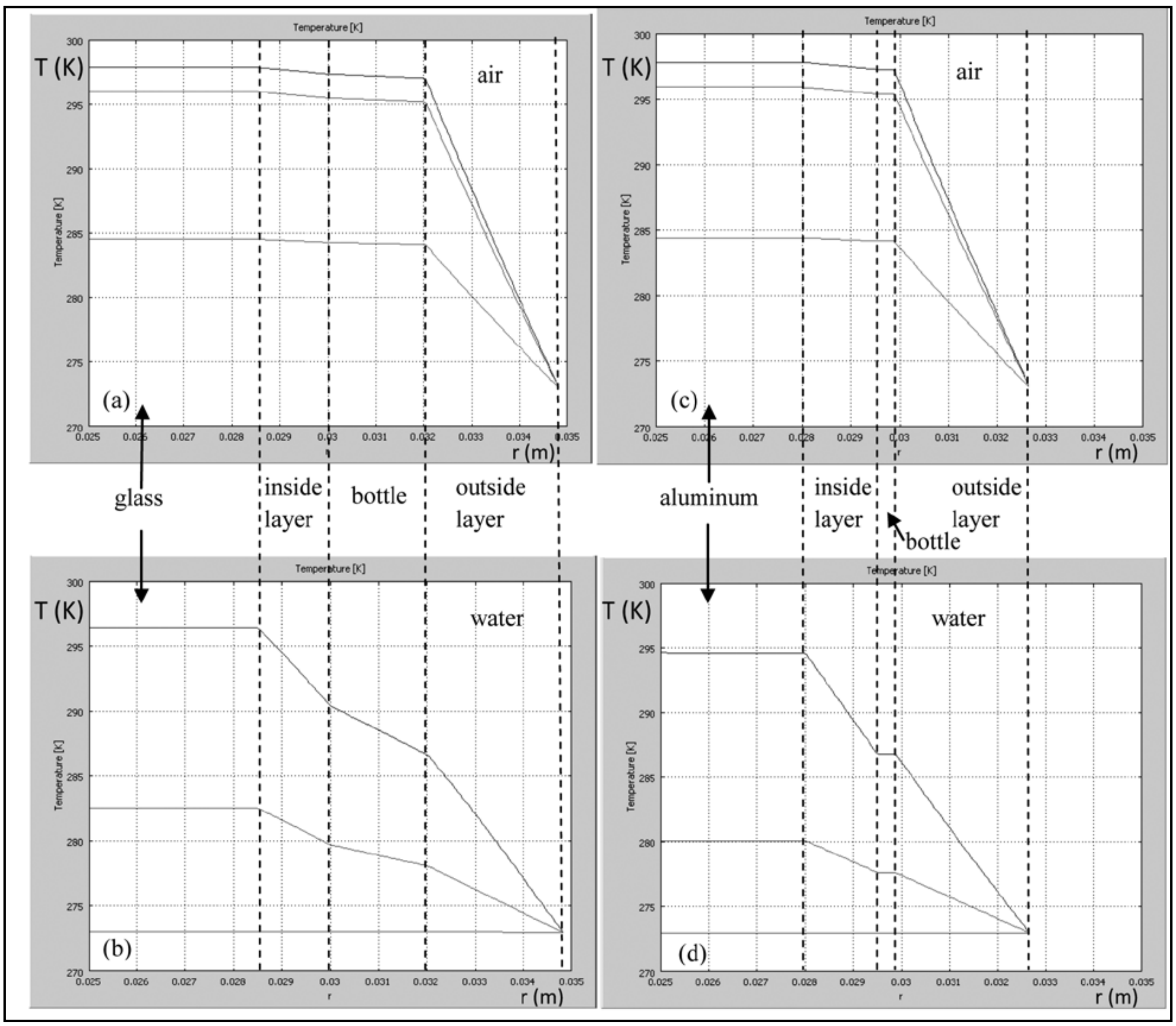

Figure 3. Predicted temperature profiles in the radial direction for our conduction-only model with effective outside thermal resistance layer of 0.00278 m and effective inside thermal resistance layer of 0.0015 m at three times (60, 600, and 6000 s) for cooling with outside T = 0 ℃ and initial inside T = 25 ℃ for four cases: (a) glass bottle, air outside; (b) glass bottle, water outside; (c) aluminum bottle, air outside; (d) aluminum bottle, water outside. Note that the r-axis begins at r = 0.025 m in these figures.

effective outside air layer gives the largest resistance (largest temperature drop) and that conduction through the bottle wall has little effect. Note that combining Eqs. (5) and (7) indicates that the resistance outside the bottle equals $1 / (h_o A_o)$. We can also use the built-in post-processing features of COMSOL Multiphysics to evaluate the heat flux across the bottle wall and use Newton's law of cooling,

$$q = h_o A_o \left(T_o - T_a\right) \tag{8}$$

to evaluate the value of h_o represented by the air layer. For

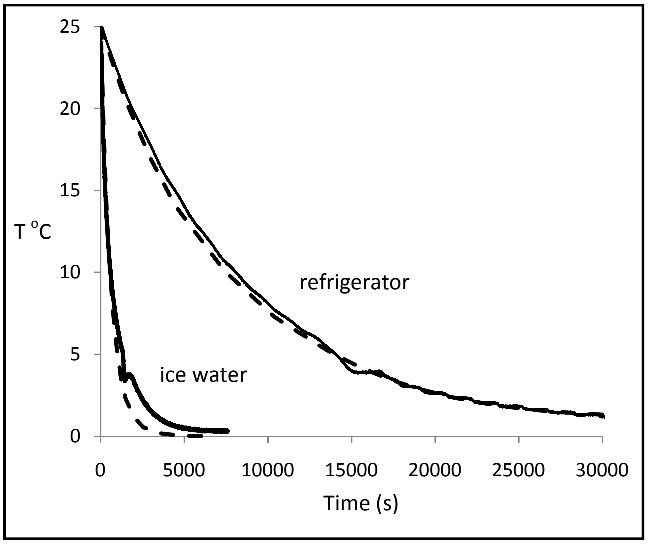

Figure 4. *Comparison of cooling rates for glass bottle in refrigerator or ice water. Solid lines, experimental data. Dashed lines, equivalent stagnant layer conduction models.*

example, at 60 s, the heat flow across the outside wall was 7.39985 W and ΔT was approximately 23.35 K. Taking into account the bottle outside surface area of 0.03381 m^2 yields a value of h_o near 9 as expected.

Cooling a glass bottle in ice water. Figure 4 shows the dramatic difference between cooling methods for a glass bottle. An ice water bath is much more efficient than a refrigerator since it takes less than 1.5 hours to make the contents ice cold. The increase in efficiency can be explained as arising mostly from the higher thermal conductivity of water compared to air. Our equivalent conduction-only model was used to illustrate this point by assuming, as an approximation, that a 0.00278 m layer of water rather than air was controlling the heat transfer at the outside of the bottle. Simply using the properties of water instead of those of air in the outer layer of our previous model yielded the predictions shown in Figure 3b and the dashed line of Figure 4. While this does not fit the experimental data exactly, it does show that the difference in thermal conductivity of water and air accounts for most of the difference between the two cooling processes. Comparing Figures 3a and 3b we can see that the resistance to heat transfer offered by the ice water layer is much less than that of the air layer. We can also see that the resistance in the bottle wall is significant in the ice water case. This fact is also reflected in the Biot number since the value for a glass bottle cooling in ice water is no longer less than 0.1 as shown in Table 2. An experienced heat transfer teacher might argue that knowing the Biot numbers is more useful than Figure 3, but we found that Figure 3 clarifies the meaning of the Biot number for the uninitiated.

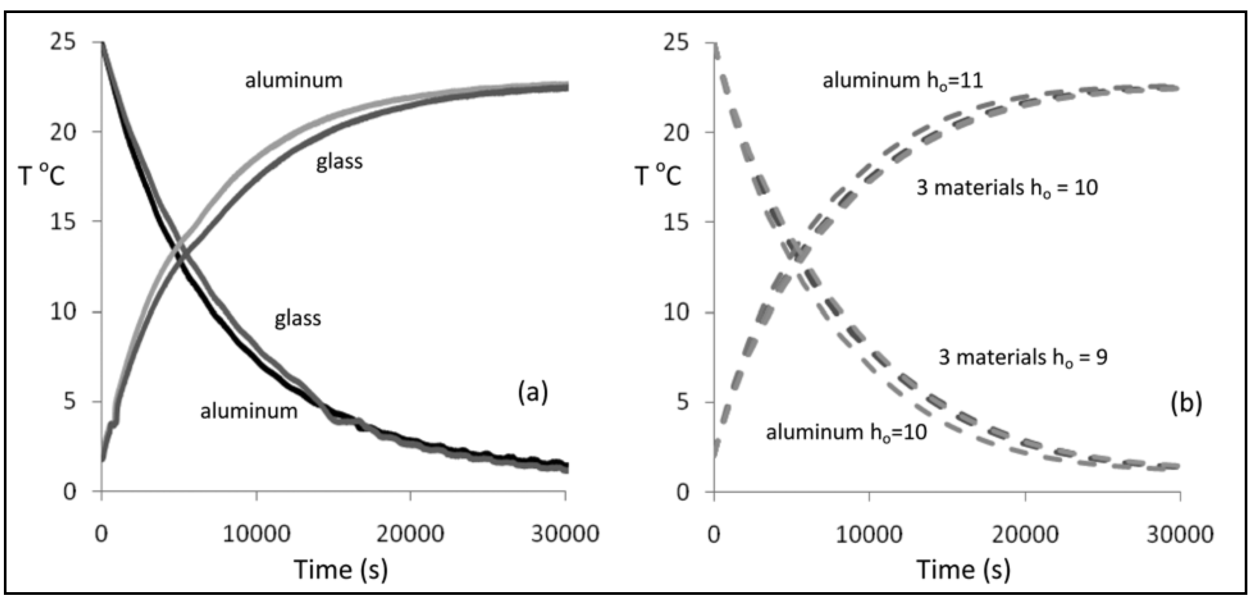

Figure 5. *Comparison of aluminum, glass, and plastic bottles upon cooling in a refrigerator and heating in air. (a) experimental results; (b) predicted results with h_i = 400 W/m²K, h_o = 9 W/m²K (10 W/m²K for aluminum) on cooling and h_o = 10 W/m²K (11 W/m²K for aluminum) on heating. Note that experimental results for plastic bottles were nearly identical to those of glass bottles in both situations.*

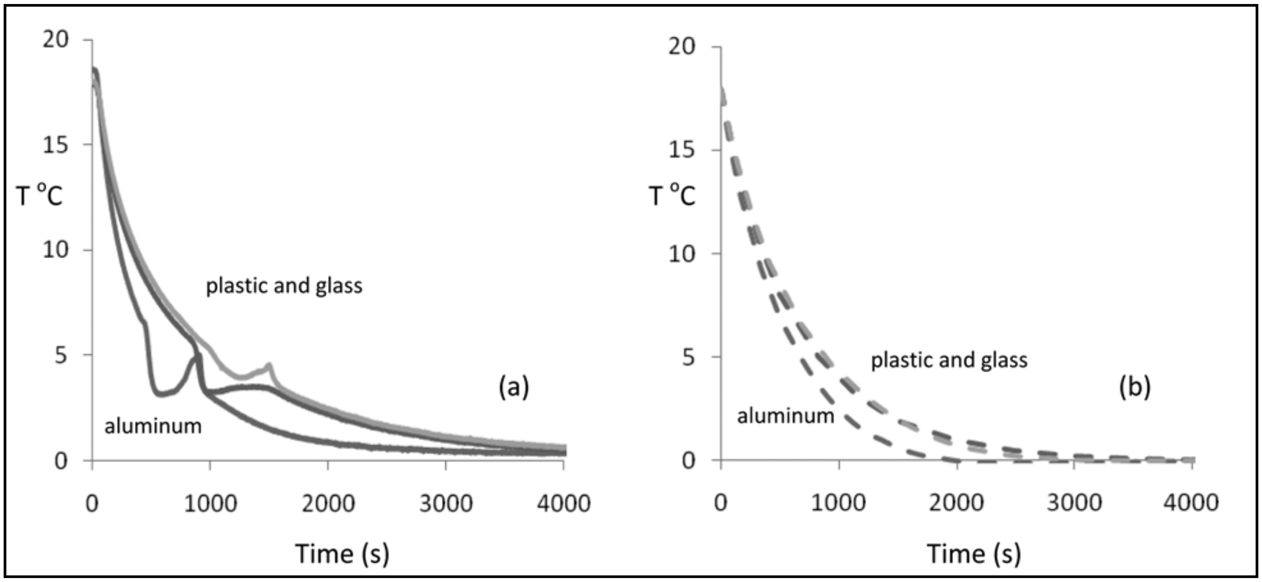

Figure 6. *Comparison of aluminum, glass, and plastic bottles upon cooling in ice water. (a) experimental results; (b) predicted results with h_o = 200 W/m²K, h_i = 400 W/m²K.*

Comparison of various bottle materials. At this point some students might be thinking: "All this theory is fine, but which bottle is better?" The measured and predicted results for cooling in a refrigerator and heating in air are shown in Figure 5. Comparing Figure 3a for glass and Figure 3c for aluminum indicates that the resistance to heat transfer in the air outside of the bottles represented by h_o dominates the process in both cases. Therefore, it is no surprise that Figure 5b shows no significant difference between predicted results if we use the same values of h_i and h_o for all bottles. Our experimental results, in Figure 5a, show that the aluminum bottle always cooled and heated slightly faster than the other two, however. It appears that other factors, like differences in condensation on the bottles and differences in emissivity, that we have not taken into account are needed to explain why the aluminum bottle cools and heats slightly faster. Increasing the values for h_o for the aluminum bottle allowed us to more closely model the observed results. The low Biot numbers in Table 2 for cooling in the refrigerator (and similar results that would be obtained for heating in air) indicate that wall material should have minimal effect on these heat transfer processes.

Experimental and calculated results for cooling the three bottles in ice water are shown in Figure 6. The ice water cooling process appears to be influenced by the density of water maximum at 4 °C and is not accurately modeled with our simple model using constant heat transfer coefficients. Nevertheless, our simple model results, shown in Figure 6b, indicate that the difference in bottle materials does account for some of the observed difference in cooling rates. The resistance to heat transfer illustrated in Figures 3 and the Biot numbers shown in Table 2 help explain why the bottle material has a more significant effect when cooled in ice water than when cooled in air. Figures 3c and 3d show our equivalent resistance-layer conduction-only model results for the aluminum bottle in air and water, respectively. The small thickness and high thermal conductivity of the aluminum bottle yield little resistance to heat transfer (and small Biot number), even when compared with the relatively small resistance offered by the water layer in Figure 3d. The thermal conductivity of plastic is less than that of glass, but the wall thickness of the plastic bottle is also less, resulting in similar thermal properties (and Biot numbers) for the plastic and glass bottles. For glass and plastic, but not for aluminum, the resistance due to the wall does slow down the cooling process in the ice water case.

The fact that the aluminum cools faster will only have practical significance if the process is stopped before equilibrium is reached. The temperature when "the mountains turn blue" is about 6 °C, for example.[12] In ice water, the aluminum bottle will reach that temperature faster than the glass bottle will; how much faster will depend on the starting temperature. For example, for the starting temperature of 18.5 °C, shown in Figure 6, it was about 5 minutes faster, but for a starting temperature of 24 °C (measured but not shown), it was about 8 minutes faster. This advantage for the aluminum bottle is counteracted by the disadvantage that the aluminum bottle heats up significantly faster than the others when hand-held as shown in Figure 7, on the next page (plastic results were similar to those for glass).

To give students a physical feel for the heat transfer process, we found it effective to fill an aluminum bottle and a glass bottle with ice water and have them hold each one. The aluminum bottle feels colder because it conducts heat away from the hand more readily. This explains why aluminum cools faster in ice water and heats faster when hand-held. It might also explain why the myth persists that aluminum keeps beverages colder longer since aluminum feels colder even when the

If time is of the essence, an aluminum bottle in an ice water bath will reach a satisfying temperature several minutes faster than the other bottles due to the high thermal conductivity and thin wall of the aluminum bottle. The aluminum bottle will warm faster than the others when hand-held, but the practical significance of this fact will depend on how rapidly the beverage is consumed. We suspect that the thermal performance of the bottle will not have a major effect on beverage enjoyment, but our studies on this aspect are ongoing.

contents are the same temperature. Having students place their hands next to the two bottles without touching them allows the students to note that the air gap between the hand and the bottle prevents the faster heat transfer to the aluminum that was observed when the hands touched the bottles. Students will also recognize that room-temperature water feels colder than room-temperature air because the water conducts heat away from the body faster.

CONCLUSIONS

The experiments and calculations presented here were both fun and informative. They were an excellent way to reinforce our understanding of heat transfer processes. When cooled in a refrigerator, bottle material has little effect on the cooling rate and about 8 hours is required to cool a 16 oz bottled beverage. We recommend an ice water bath for 1 to 1.5 hours (depending on the starting temperature) if rapid cooling is desired. In this case, plastic and glass bottles behave similarly because the lower thermal conductivity of plastic is offset by a thinner wall. If time is of the essence, an aluminum bottle in an ice water bath will reach a satisfying temperature several minutes faster than the other bottles due to the high thermal conductivity and thin wall of the aluminum bottle. The aluminum bottle will warm faster than the others when hand-held, but the practical significance of this fact will depend on how rapidly the beverage is consumed. We suspect that the thermal performance of the bottle will not have a major effect on beverage enjoyment, but our studies on this aspect are ongoing.

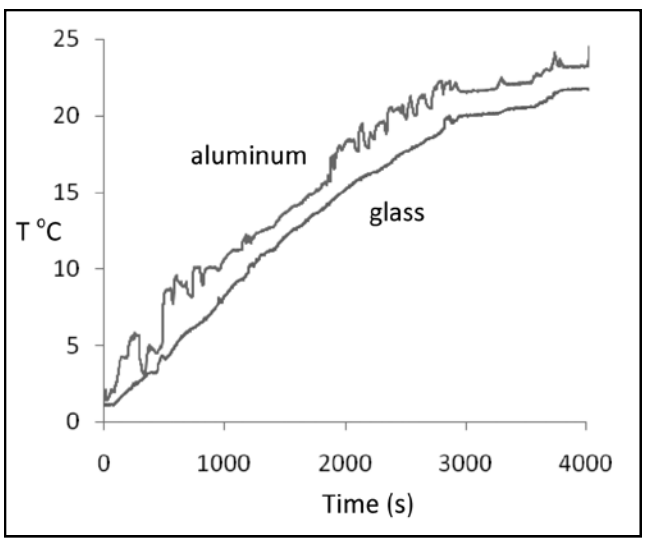

Figure 7. *Hand-held heating experimental results for glass and aluminum bottles.*

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APPENDIX: JUSTIFICATION FOR HEAT TRANSFER COEFFICIENTS USED

The heat transfer coefficients in this paper can be considered to be constant average values fit to our experimental results. That is, they are the values that gave the "best" calculated results when inserted into COMSOL Multiphysics as constants in the boundary coefficients defined by Eqs. (2) and (3). In this appendix, standard methods for estimating heat transfer coefficients are used to justify that the fitted values are reasonable and consistent with known correlations. As indicated by the correlations below, heat transfer coefficients are temperature-dependent and will therefore vary with time. Calculations below show the initial values for a glass bottle. Coefficients evaluated for the other bottle materials were similar to those evaluated for glass. Although COMSOL Multiphysics can easily incorporate (and even estimate for you) temperature-dependent heat transfer coefficients, this capability has not been used.

Outside convective heat transfer coefficient – cooling in refrigerator. The heat transfer coefficient describing natural convection at the outer surface of a vertical cylinder can be estimated by[13]

$$h_o = \frac{k}{H}\left[\frac{4}{3}\left[\frac{7\,Gr\,Pr^2}{5\left(20+21\,Pr\right)}\right]^{1/4} + \frac{4\left(272+315\,Pr\right)H}{35\left(64+63\,Pr\right)d_o}\right] \qquad (A1)$$

where

$$Pr = \frac{\nu}{\alpha} \qquad (A2)$$

$$Gr = \frac{Ra}{Pr} \qquad (A3)$$

$$Ra = \frac{g\beta\left(T_s - T_\infty\right)H^3}{\nu\alpha} \qquad (A4)$$

Pr, Gr, and Ra are the Prandlt, Grashof, and Rayleigh numbers, respectively. Using the height, H, and outside diameter, d_o, of the glass bottle given in Table 1, properties for air at $T_\infty = 1$ °C given in Table A1, and an initial bottle surface temperature, T_s, of 25 °C, the value of h_o given by Eq. (A1) is 4.98 W / m^2 K.

Radiative heat transfer coefficient – cooling in refrigerator. The radiation heat transfer coefficient can be estimated by[14]

$$h_r = \varepsilon\sigma\left(T_\infty + T_s\right)\left[T_\infty^2 + T_s^2\right] \qquad (A5)$$

Using an emissivity, ε, for glass of 0.93 and Boltzmann constant, σ, of 5.67×10^{-8} W / (m^2 K^4) yields a value of h_r = 4.94 W / m^2 K.

Combined outside heat transfer coefficient – cooling in refrigerator. Since we have included thermal radiation in our lumped-parameter outside heat transfer coefficient, an estimate of its initial value is h_o = 4.98 + 4.94 = 9.92 W / m^2 K. Since this value will decrease with time an average value of 9 W / m^2 K seems reasonable.

Outside convective heat transfer coefficient – cooling in ice water. Using the values for water at 0 °C, given in Table A1 yields an outside heat transfer coefficient for water via Eq. (A1) of h_o = 304 W / m^2 K. The presence of crushed ice in the water near the bottle and the fact that the volume expansivity, β, goes from negative to positive and equals zero at 4 °C, caused us to question the accuracy of Eq. (A1) in this situation. With that in mind and realizing that h_o will decrease as T_s decreases indicates that our average value of 200 W / m^2 K is not unreasonable.

Inside heat transfer coefficient. The heat transfer coefficient on the inside of a vertical cylinder can be estimated by[15]

$$h_i = \frac{k}{H}\left[0.55\,Ra^{0.25}\right] \qquad (A6)$$

Using the properties of water at 25 °C shown in Table A1, T_∞ = 25 °C, and T_s = 1 °C in this equation yields h_i = 422 W / m^2 K in agreement with the fitted average value of 400 W / m^2 K that we used. ❐

TABLE A1
Thermophysical Properties for Air at 1 °C and Water at 0 and 25 °C

	kinematic viscosity $\nu \times 10^6$ m^2/s	thermal diffusivity $\alpha \times 10^6$ m^2/s	volume expansivity $\beta \times 10^3$ 1/K	thermal conductivity k W / (mK)
Air at 1 °C	13.357	18.682	3.717	0.023
Water at 0 °C	1.795	0.132	- 0.068	0.558
Water at 25 °C	0.912	0.146	0.255	0.606