An Introduction to Probabilistic Graphical Models

Luigi Portinale
Computer Science Institute, DiSIT
Research Center AI@UPO
Università del Piemonte Orientale
Alessandria (Italy)

Abstract

This tutorial covers an introduction to Probabilistic Graphical Models (PGM), such as Bayesian Networks and Markov Random Fields, for reasoning under uncertainty in intelligent systems. Basic terminology, formal concepts, representational and inference issues will be discussed, starting from basic notions about probability theory, in such a way that the novice and the less skilled in the field will be able to follow the details. Further reading and software packages and frameworks will also be discussed

Overview

The tutorial aims at introducing notions and algorithms for Probabilistic Graphical Models (PGM) in Artificial Intelligence (AI). PGMs are the main AI formalism for dealing with uncertain knowledge and reasoning, which is one of the most important aspects to be addressed in the construction of an intelligent agent, especially in the context of decision-support intelligent systems. PGMs are grounded on both probability calculus and graph theory and represent an effective tool for the construction of intelligent decision support systems. After a short review of probability calculus and of the interpretation of probability, we discuss different types of PGMs, namely directed models like Bayesian Belief Networks and undirected models such as Random Markov Fields. Both representational and algorithmic issues are discussed, pointing out potential applications and extensions.

The target audience consists in graduate students, researchers or professionals who want to be acquaintend with the formalism, in order to be able to apply it in the construction of intelligent systems, able to reason with uncertain knowledge and evidence. Familiarity with probability calculus and with graph theory may be useful, but basic concept needed to capture the essential notions of PGMs are reviewed during the tutorial.

Tutorial's content

The tutorial is divided into two main parts.

Part 1

It is the introductory part, devoted to the recap of fundamental notions of uncertain reasoning, probability calculus and to the idea of joining graph theoretic notions with probability, leading to the creation of Probabilistic Graphical Models. This part is detailed as follows:

- Introduction to uncertain reasoning in AI:
  - why dealing with uncertainty is needed for intelligent systems;
  - the role of probability in reasoning under uncertainty: pros and cons;
  - Cox's justification of probability calculus;
- The different interpretations of probability and their relevance to AI and reasoning under uncertainty:
  - classical interpretation (Laplace);
  - frequentist interpretation (Von Mises);
  - subjectivist interpretation (de Finetti) and its role in AI;
  - Bayesian inference as a consequence of subjective probability;
- Recap on basic notions of probability calculus:
  - probability space;
  - random variables and their distributions;
  - the importance of the joint distribution;
  - independent random variables and their role in probabilistic modeling;
- Probabilistic Graphical Models:
  - Bayesian networks (directed models): definition and examples;
  - Markov Random Fields (undirected models): definition and examples;
  - Comparison between pairwise Markov Random Fields and Markov Random Fields based on Gibbs distribution ;
- Conditional dependencies and distribution factorization:
  - comparison of directed and undirected models;
  - Separation and d-separation as notions of conditional independence on graphs;
  - local and global semantics induced by separation;

Part 2

The second part is devoted to the algorithmic issues concerning the inference task on PGMs; both exact approaches and approximate algorithms are introduced and discussed. Hints on possible extensions and to the use of specific development tools are also provided. This part is detailed as follows:

- Factor graphs as a unifying model for inference in directed and undirected PGM:
  - definitions;
  - sum-product algorithm for inference on factor graphs;
- Exact inference in PGMs:
  - belief propagation and junction tree algorithm;
  - Shafer-Shenoy architecture vs Hugin architecture (hints);
- Approximate inference in PGMs:
  - stochastic simulation methods and Markov Chain Monte Carlo;
  - Logical sampling, likelihood weighting and Gibbs sampling for inference on PGMs;
- Hints on possible extensions:
  - Dynamic Bayesian Networks;
  - Decision Networks;
- Development tools and frameworks:
  - Genie/Smile[1];
  - BNT Matlab/Octave toolbox[2];

Useful References

Different textbooks and papers can be used to get an in-depth understanding of the tutorial's topics, among them:

- D. Koller, N. Friedman. *Probabilistic Graphical Models: principles and techniques*, MIT Press, 2009. It is the most comprehensive guide to the world of Probabilistic Graphical Models, covering all the topics introduced in the tutorial and much more.
- K.P. Murphy. *Probabilistic Machine Learning: Advanced Topics*, MIT Press, 2023. A book on machine learning approaches based on probability, where PGMs are discussed also in the context of automatically learning the models.
- M.J. Kochenderfer, T.A. Wheeler, K.H. Wray. *Algorithms for Decision Making*, 2022. PGMs viewed as models for decision making under a decision theory approach (Decision Networks).
- A. Darwiche. *Modeling and Reasoning with Bayesian Networks*, J Wiley, 2008. A very good introduction to modeling and reasoning issues for directed models (Bayesian Networks).
- L. Portinale, D. Codetta Raiteri. *Modeling and Analysis of dependable Systems: a probabilistic Graphical Model Perspective*, World Scientific Publ., 2015. A book devoted to the application of different types of PGMs (Bayesian Networks, Dynamic Bayesian Networks, Decision Networks) to the reliability and dependability analysis of physical systems.
- J. Pearl. *Probabilistic Reasoning in Intelligent Systems: Networks of Plausible Inference*, Morgan Kaufmann 1989. The bible.

[1] https://www.bayesfusion.com/

[2] https://github.com/bayesnet/bnt

Speaker's Bio

Luigi Portinale is full professor of Computer Science at the University of Piemonte Orientale (Italy) since 2004, where he teaches Artificial Intelligence and Machine Learning classes. He is currently director of the Research Center on Artifcial Intelligence of the University of Piemonte Orientale, AI@UPO. He earned a MSc in Computer Science on 1988 (summa cum laude) from the University of Torino and a PhD in Computer Science on 1994 (University of Torino and Milano consortium).

His research interests focus on knowledge representation and reasoning in Artificial Intelligence systems, with particular attention devoted to Case-Based Reasoning (CBR) methodologies, Probabilistic Graphical Models, Machine Learning and Deep Learning.

He received the Sollac prize for the best paper (a joint work with L. Console and D. Theseider Duprè) on 1991 at the *11th International Workshop On Expert Systems and their Applications (Avignon)*, and the D.J. Groen Prize 2015 of the Safety and Reliability Group of the Institution of Mechanical Engineers (UK) for the paper *"Approaching Dynamic Reliability with Predictive and Diagnostic Purposes by exploiting Dynamic Bayesian Networks"*, published on *Journal of Risk and Reliability, 228(5)*, 2014 (with Daniele Codetta Raiteri).

He is the author of more than 150 papers on international journals and proceedings (all refereed works). He is the author (with D. Codetta Raiteri) of the book *"Modeling and Analysis of Dependable Systems: a Probabilistic Graphical Model Perspective"*; World Scientific Publ. (2015).

He is also CTO of Inferendo, an innovative start-up spin-off of University of Piemonte Orientale developing innovative recommendation services using AI and ML.

Prof. Portinale has already organized and presented tutorials on PGMs in several conferences (RAMS 2011, RAMS 2016, AIxIA 2019) as well as in several PhD schools. The tutorial is based on the material used for a class Prof. Portinale teaches in the Italian National PhD program in Artificial Intelligence.