# Towards a Temporal Probabilistic Argumentation Framework*

**Stefano Bistarelli, Victor David, Francesco Santini, Carlo Taticchi**
Department of Mathematics and Computer Science, University of Perugia, Italy
name.surname@unipg.it

**Abstract**

In recent years, the notion of time has been studied in different ways in Dung-style Argumentation Frameworks. For example, time intervals of availability have been added to arguments and relations. As a result, the output of Dung semantics varies over time. In this paper, we consider the situation in which arguments hold with a certain probability distribution during a given interval. To model the uncertain character of events, we propose different notions of temporal conflict between arguments according to the type of availabilities intersection (partial, inclusive, or total). Then, we refine these notions of conflict by a defeat relation, using criterion functions that evaluate an attack's significance according to the probability over time.

## 1 Introduction

Argumentation Theory studies how conclusions can be drawn starting from a given set of facts or premises, and, in the field of Artificial Intelligence, it provides tools for modelling human-fashioned logical reasoning where the available information may be discordant. A simple yet powerful representation of conflicting information is provided by the Abstract Argumentation Frameworks (Dung 1995), or AFs in short, composed of a set of arguments and an attack relation that determine conflict between arguments. Analysing an AF under the lens of the so-called "semantics", one can derive sets of acceptable arguments, i.e. non-conflicting arguments that share specific properties.

To increase the expressiveness of the basic framework and enable the modelling of more realistic situations, AFs have been extended to consider other aspects that can influence the unfolding of the reasoning, like the *time* when the arguments are available (Cobo, Martínez, and Simari 2010; Zhang and Liang 2012; Budán et al. 2012; Budán et al. 2017; Zhu 2020). While the works mentioned above use abstract frameworks, the one in (Augusto and Simari 2001) focuses on structured argumentation and defeasible reasoning. Then, the work in (Budán et al. 2015b) associates attacks with time intervals for abstract and structured frameworks.

Another aspect is the consideration of *probability* in arguments and relations; see (Hunter et al. 2021) for a survey. In the literature, two main perspectives exist towards probabilistic argumentation based on constellations (Li, Oren, and Norman 2011) and epistemic approaches (Thimm 2012). The former methodology considers probabilities as the possibility that an argument or a relation exists or not, which leads to the study of all possible structures (with some complexity problems (Dondio 2014; Bistarelli et al. 2022)). The latter suggests that probability denotes a degree of belief. Our study here is closer to the epistemic approach.

In this paper, we take a further step towards a more expressive AF and consider the situation where the time instant at which a given event occurs may be uncertain (probabilistic). In particular, we assume to only know the probability distribution of the events associated with the arguments. Consider the following example.

**Example 1** *We want to help a murder case using the next four arguments describing events before the victim's death:*[1]

- *argument $a$: witness A reports seeing a fight between the victim and another person between 1 pm and 4 pm (i.e. in the interval $\{1, \ldots, 4\}$);*
- *argument $b$: witness B reports to have seen the victim walking between 2 pm and 7 pm (i.e. $\{2, \ldots, 7\}$);*
- *argument $c$: A surveillance Camera recorded the victim walking at 3 pm (i.e. $\{3\}$);*
- *argument $d$: According to the Doctor, the victim died between 6 pm and 10 pm (i.e. $\{6, \ldots, 10\}$).*

*The attacks between $a$, $b$, $c$ and $d$ are given in Figure 1 (left), which provides a static representation of the events. The arguments' probability distribution over time is represented in Figure 1 (right). In this example, we use a uniform distribution for arguments $a$ and $b$, while argument $d$ is more likely to occur around $8$ pm and follows a normal distribution. Finally, argument $c$ holds with probability $1$ at $3$ pm.*

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[1] Notice that we consider events happening at a time point. Therefore, intervals are represented as sets of time points.

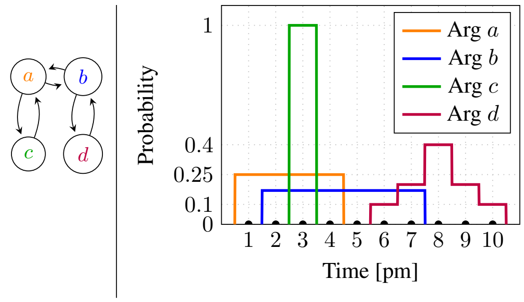


Figure 1: Left: $\mathbf{F}$ describing the events of Example 1; Right: probability distribution over time for the arguments in $\mathbf{F}$.

*Note that one can choose different probability distributions to represent various types of uncertainty.*

Since events can be uncertain over time, the notion of conflict between arguments also needs to be revised. For example, two contradictory arguments, such as "the victim was fighting" and "the victim was walking", may not conflict if they hold at different times.

To deal with temporal and probabilistic aspects of argumentation, we first introduce Temporal Probabilistic Argumentation Frameworks (TPAFs), an extension to classical AFs, and propose a method for deriving conflict between arguments. Then, to evaluate the acceptability of arguments, we provide a set of semantics based on the notion of defence over time. We also study the concept of minimal defence to investigate the conditions under which an argument can be accepted considering a time interval.

## 2 Preliminaries

In this section, we recall the formal definition of an Abstract Argumentation Framework and the related extension-based semantic (Dung 1995).

**Definition 1 (AF)** *An Abstract Argumentation Framework (AF) is a pair $\langle \mathcal{A}, \mathcal{R} \rangle$ where $\mathcal{A}$ is a set of arguments, and $\mathcal{R}$ is a binary relation on $\mathcal{A}$.*

Consider two arguments $a, b$ belonging to an AF. We denote with $(a, b) \in \mathcal{R}$ an attack from $a$ to $b$; we can also say that $b$ is *defeated* by $a$. For $b$ to be *acceptable*, we require that every argument that defeats $b$ is defeated in turn by some other argument of the AF.

Using the notion of defence as a criterion for distinguishing acceptable arguments in the framework, one can further refine the set of selected arguments.

**Definition 2 (Extension-based semantics)** *Let $\langle \mathcal{A}, \mathcal{R} \rangle$ be an AF. A set $E \subseteq \mathcal{A}$ is conflict-free iff $\nexists a, b \in E$ such that $(a, b) \in \mathcal{R}$. A conflict-free subset $E$ is then:* ***admissible****, if each $a \in E$ is defended by $E$;* ***complete****, if it is admissible and $\forall a \in \mathcal{A}$ defended by $E$, $a \in E$;* ***stable****, if it is admissible and attacks every argument in $\mathcal{A} \setminus E$;* ***preferred****, if it is complete and $\subseteq$-maximal;* ***grounded****, if it is complete and $\subseteq$-minimal.*

We also need the notion of time intervals for reasoning on when arguments are available (e.g. (Cobo, Martínez, and Simari 2010)).

**Definition 3 (Temporal interval)** *Let $\mathbf{T}$ be the discrete universe of time points. A temporal interval is a subset $I = \{t_i, \ldots, t_j\}$ of $\mathbf{T}$ with $t_i < t_j$. In particular, $\{t_i\}$ denotes the instant $t_i$.*

**Definition 4 (TAFs)** *A Timed Abstract Argumentation Framework (TAF) is a tuple $\langle \mathcal{A}, \mathcal{R}, Av \rangle$ where $\mathcal{A}$ is a set of arguments, $\mathcal{R}$ is a binary relation on $\mathcal{A}$, and $Av \colon \mathcal{A} \to \wp(\mathbf{T})$ is the availability function for arguments.*[2]

## 3 Temporal Probabilistic Argumentation Frameworks

For reasoning on probabilistic and temporal arguments, we instantiate the generic framework proposed in (Budán et al. 2015a) and define the temporal probabilistic argumentation framework (*TPAF*).

**Definition 5 (TPAF)** *A Temporal Probabilistic Argumentation Framework (TPAF) is a tuple $G = \langle \mathcal{A}, \mathcal{R}, \mathcal{P} \rangle$ such that:*

- *$\mathcal{A}$ is a finite set of arguments;*
- *$\mathcal{R} \subseteq \mathcal{A} \times \mathcal{A}$ is the attack relation;*
- *$\mathcal{P}^I : \mathcal{A} \to [0, 1]$ is the probability distribution of an argument over a time interval $I$.*

**Example 2** *We use the AF $\mathbf{F}$ and the probability distribution of Figure 1 to build a TPAF $\mathbf{G}$. The time points in the considered interval $I = \{1, \ldots, 10\}$ represent hours of the day. We have that $\mathcal{P}^{\{1,\ldots,10\}}(x) = 1$ for all arguments $x$ in $\mathbf{G}$. We can also obtain the probability of their occurrence at a certain instant. For example, $\mathcal{P}^{\{8\}}(d) = 0.4$ in $\mathbf{G}$.*

Note that depending on the user's needs, for example, if the TPAF occurs over a long period, it is helpful to restrict the study of a TPAF to a specific time interval. Thus, in the rest of the article, we will specify the time interval we are working on.

When an argument has a probability of occurring equal to zero, it should not be considered in the reasoning process. Therefore, we extract, for each argument, the instants in which its probability is positive.

**Definition 6 (Positive probability over time)** *Let $G = \langle \mathcal{A}, \mathcal{R}, \mathcal{P} \rangle$ be a TPAF, $a \in \mathcal{A}$ an argument, and $I$ a time interval. We define the set of non-null probability of $a$ in $I$ by $\mathcal{T}^I(a) = \{t \in I | \mathcal{P}^{\{t\}}(a) > 0\}$.*

**Example 3** *Consider the TPAF of Example 2. We have that $\mathcal{T}^{\{1,\ldots,4\}}(a) = \mathcal{T}^{\{1,\ldots,10\}}(a) = \{1, \ldots, 4\}$.*

Given the probability over time of arguments, the conflicts are not sure and can be interpreted in different ways according to various notions.[3] In particular, we propose three notions of conflict based on the availability of involved arguments and three criterion functions defining when the conflict is significant, i.e. it is a defeat.

[2]We use $\wp(\mathbf{T})$ to indicate the powerset of $\mathbf{T}$.

[3]For example, in (David, Fournier-S'niehotta, and Travers 2022), various temporal inconsistencies are defined in the Temporal Markov Logic Networks framework.

**Definition 7 (Temporal probabilistic conflicts)** *Let $G = \langle \mathcal{A}, \mathcal{R}, \mathcal{P} \rangle$ be a TPAF, $a, b \in \mathcal{A}$ two arguments and $I$ a time interval. We define a boolean conflict function $\mathtt{CF^I_x} : \mathcal{A} \times \mathcal{A} \rightarrow \{\top, \bot\}$, with $\mathtt{x} \in \{\mathtt{p}, \mathtt{i}, \mathtt{t}\}$ (where $\mathtt{p}$, $\mathtt{i}$ and $\mathtt{t}$ stand for partial, included and total, respectively), which determines a conflict from a to b within I when $(a, b) \in \mathcal{R}$ and:*

- *Partial:* $\mathtt{CF^I_p}(a, b) = \top$ *iff* $\mathcal{T}^I(a) \cap \mathcal{T}^I(b) \neq \emptyset$;
- *Included:* $\mathtt{CF^I_i}(a, b) = \top$ *iff* $\mathcal{T}^I(b) \setminus \mathcal{T}^I(a) = \emptyset$;
- *Total:* $\mathtt{CF^I_t}(a, b) = \top$ *iff* $\mathcal{T}^I(a) = \mathcal{T}^I(b)$.

*Otherwise for any* $\mathtt{x} \in \{\mathtt{p}, \mathtt{i}, \mathtt{t}\}$, $\mathtt{CF^I_x}(a, b) = \bot$.

Note that partial conflict and total conflict are symmetric, while the included conflict is not. Moreover, the notion of $\mathtt{CF^I_t}$ implies the notion of $\mathtt{CF^I_i}$ which implies, in turn, $\mathtt{CF^I_p}$.

The notion of conflict only considers the positive probability over time of the arguments, i.e. we only check if the probability of arguments involved in an attack is positive. We can refine the concept of conflict by using the probability values attached to arguments to establish whether a conflict is significant according to a criterion function. In addition, we use the term *defeat* to refer to a significant conflict.

**Definition 8 (Criterion functions)** *Let $G = \langle \mathcal{A}, \mathcal{R}, \mathcal{P} \rangle$ be a TPAF, $a, b \in \mathcal{A}$ such that $(a, b) \in \mathcal{R}$ and $I$ a time interval. We define a boolean criterion function $\mathtt{CT^I_x} : \mathcal{A} \times \mathcal{A} \rightarrow \{\top, \bot\}$ where $\mathtt{x} \in \{\mathtt{Sg}, \mathtt{Wg}, \mathtt{A}\}$ as follows:*

- *Weak greater:* $\mathtt{CT^I_{Wg}}(a, b) = \top$ *iff* $\forall t \in I$ *such that* $\mathcal{P}^{\{t\}}(a) \times \mathcal{P}^{\{t\}}(b) > 0$, $\mathcal{P}^{\{t\}}(a) > \mathcal{P}^{\{t\}}(b)$;
- *Strong greater:* $\mathtt{CT^I_{Sg}}(a, b) = \top$ *iff* $\exists t \in I$ *such that* $\mathcal{P}^{\{t\}}(a) \times \mathcal{P}^{\{t\}}(b) > 0$ *and* $\mathcal{P}^{\{t\}}(a) > \mathcal{P}^{\{t\}}(b)$;
- *Aggressive:* $\mathtt{CT^I_A}(a, b) = \top$ *iff* $\exists t \in I$ *such that* $\mathcal{P}^{\{t\}}(a) \times \mathcal{P}^{\{t\}}(b) > 0$ *and* $\mathcal{P}^{\{t\}}(b) < 1$.

*Otherwise* $\forall \mathtt{x} \in \{\mathtt{Wg}, \mathtt{Sg}, \mathtt{A}\}$, $\mathtt{CT^I_x}(a, b) = \bot$.

The strong greater criterion leads to more frequently identifying a defeat, whereas the weak greater criterion will be more cautious in indicating a significant conflict. Note that the universal quantifier (weak) implies the existential (strong) one, and the greater criteria imply the aggressive.

We define a temporal probabilistic defeat function by combining a notion of conflict and a criterion function.

**Definition 9 (Temporal probabilistic defeat function)** *Let $G = \langle \mathcal{A}, \mathcal{R}, \mathcal{P} \rangle$ be a TPAF, $a, b \in \mathcal{A}$ two arguments, and consider a criterion function $\mathtt{CT}$ and a conflict function $\mathtt{CF}$. We define $\Delta^I_{\mathtt{CT},\mathtt{CF}} : \mathcal{A} \times \mathcal{A} \rightarrow \{\top, \bot\}$ the defeat function determining that $a$ defeats $b$ in the interval $I$, with respect to $\mathtt{CT}$ and $\mathtt{CF}$. In particular, $\Delta^I_{\mathtt{CT},\mathtt{CF}}(a, b) = \top$, iff $\mathtt{CT^I}(a, b) = \mathtt{CF^I}(a, b) = \top$. Otherwise $\Delta^I_{\mathtt{CT},\mathtt{CF}}(a, b) = \bot$.*

**Example 3 (Continued)** *We show below how different temporal probabilistic defeat functions behave according to the partial conflict of Definition 7. Consider arguments $a$ and $b$ of* **G** *and the interval $\{1, \ldots, 7\}$. We have that $b$ does not defeat $a$ within $I$ according to the greater criteria. In fact, $\Delta^{\{1,\ldots,7\}}_{\mathtt{Wg,p}}(b, a) = \Delta^{\{1,\ldots,7\}}_{\mathtt{Sg,p}}(b, a) = \bot$. If we consider the aggressive criterion, instead, we obtain $\Delta^{\{1,\ldots,7\}}_{\mathtt{A,p}}(b, a) = \top$, meaning that $b$ defeats $a$ in $I$.*

For a better understanding of the impact of defeat functions and the restriction of time intervals, let's look at the resulting TPAFs (in Figure 2) according to these parameters.

**Example 3 (Continued)** *We denote by $\Delta^I$-$G$ the graph where the attacks are restricted according to $\Delta$ and the arguments are restricted to the time interval $I$.*

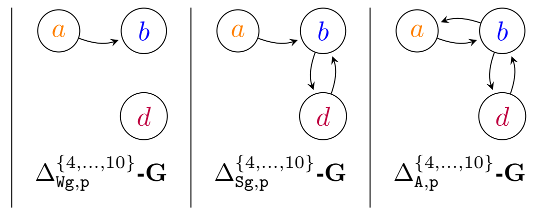


Figure 2: **G** between 4 and 10 according to a $\Delta$

The implications between the different defeat functions can be derived by analysing the relations between conflict and criterion functions. Figure 3 shows the relations between all the defeat functions. In particular, we observe that the strongest (most conflicting) defeat is $\Delta^I_{\mathtt{A,p}}$ and the weakest (less conflicting) defeat is $\Delta^I_{\mathtt{Wg,t}}$.

$$\begin{array}{ccccc}
\Delta^I_{\mathtt{Wg,t}} & \rightarrow & \Delta^I_{\mathtt{Wg,i}} & \rightarrow & \Delta^I_{\mathtt{Wg,p}} \\
\downarrow & & \downarrow & & \downarrow \\
\Delta^I_{\mathtt{Sg,t}} & \rightarrow & \Delta^I_{\mathtt{Sg,i}} & \rightarrow & \Delta^I_{\mathtt{Sg,p}} \\
\downarrow & & \downarrow & & \downarrow \\
\Delta^I_{\mathtt{A,t}} & \longrightarrow & \Delta^I_{\mathtt{A,i}} & \longrightarrow & \Delta^I_{\mathtt{A,p}}
\end{array}$$

Figure 3: Implications between defeat functions.

For the rest of the paper, we will use $\Delta$ to refer to a generic temporal probabilistic defeat function. Next, we extend conflict-freeness to TPAFs through a defeat function $\Delta$.

**Definition 10 ($\Delta$-conflict-free)** *Let $G = \langle \mathcal{A}, \mathcal{R}, \mathcal{P} \rangle$ be a TPAF, $S \subseteq \mathcal{A}$ a set of arguments and $\Delta^I_{\mathtt{CT},\mathtt{CF}}$ a defeat function. $S$ is $\Delta^I_{\mathtt{CT},\mathtt{CF}}$-conflict-free if and only if $\nexists a, b \in S$ such that $\Delta^I_{\mathtt{CT},\mathtt{CF}}(a, b) = \top$.*

According to a $\Delta$-conflict-free notion, we define the notion of one defence of an argument against another according to a set of arguments able to defend.

**Definition 11 ($\Delta$-SingleDefence of $a$ from $b$ by $S$)** *Given $G = \langle \mathcal{A}, \mathcal{R}, \mathcal{P} \rangle$ be a TPAF, $I$ a time interval, $a, b \in \mathcal{A}$ and $S \subseteq \mathcal{A}$ be a $\Delta$-conflict-free set of arguments within $I$. According to the defeat notion $\Delta$ used for the $\Delta$-conflict-freeness, the $\Delta$ single defence of a from b with respect to S within I, is defined as follows: $\Delta^I$-$\mathtt{1def}(a, b, S) =$*

$$\mathcal{T}^I(a) \cap \bigcup_{c \in \{x | x \in S, \Delta^I(x,b) = \top\}} \mathcal{T}^I(b) \cap \mathcal{T}^I(c)$$

**Example 3 (Continued)** *From the TPAF* **G**, *let us see what is the $\Delta^{\{2,\ldots,7\}}_{\mathtt{Sg,p}}$ single defence of b* *from $a$ and $d$ with respect to the set of arguments $S = \{b,c\}$: $\Delta^{\{2,\dots,7\}}_{\mathtt{Sg,p}}$-$\mathtt{1def}(b,a,S) = \{3\}$ and $\Delta^{\{2,\dots,7\}}_{\mathtt{Sg,p}}$-$\mathtt{1def}(b,d,S) = \{6,7\}$.*

Let us see now when an argument is $\Delta$ defended by a set of arguments in a TPAF at a given time interval.

**Definition 12 ($\Delta$-Defence of $a$ with respect to $S$)** *Let $G = \langle \mathcal{A}, \mathcal{R}, \mathcal{P}\rangle$ be a TPAF, $I$ a time interval and $S$ be a $\Delta$-conflict-free set of arguments within $I$. The $\Delta$-defence for $a$ with respect to $S$, is defined as follows: $\Delta^I$-$\mathtt{def}(a,S) =$*

$$\bigcap_{b\in\{x|\Delta^I(x,a)=\top\}} (\mathcal{T}^I(a) \setminus \mathcal{T}^I(b)) \cup \Delta^I\text{-}\mathtt{1def}(a,b,S)$$

We now define $\Delta$-admissible, $\Delta$-complete, $\Delta$-preferred, $\Delta$-stable and $\Delta$-grounded semantics for TPAFs.

**Definition 13 ($\Delta$-Semantics)** *Let $G = \langle \mathcal{A}, \mathcal{R}, \mathcal{P}\rangle$ be a TPAF, $I$ a time interval, $\Delta$ a defeat function and consider a set of arguments $E \subseteq \mathcal{A}$. We say that:*

- *$E$ is a $\Delta^I$-admissible extension of $G$ within $I$, denoted by $E \in \Delta^I$-ad(G) iff for all $a \in E$ it holds that $\mathcal{T}^I(a) = \Delta^I$-$\mathtt{def}(a,E)$;*
- *$E \in \Delta^I$-co(G) (complete) iff $E$ is a $\Delta^I$-admissible extension of $G$ and $E$ contains all the arguments $a$ such that $\mathcal{T}^I(a) = \Delta^I$-$\mathtt{def}(a,E)$;*
- *$E \in \Delta^I$-pr(G) (preferred) iff $E$ is a $\subseteq$-maximal $\Delta^I$-complete extension;*
- *$E \in \Delta^I$-st(G) (stable) iff $E$ is $\Delta^I$-admissible and for all $b \in \mathcal{A} \setminus E$, there exists $a \in E$ such that $a$ defeats $b$, i.e., $\Delta^I(a,b) = \top$;*
- *$E \in \Delta^I$-gr(G) (grounded) iff $E$ is the $\subseteq$-minimal $\Delta^I$-complete extension.*

**Example 3 (Continued)**

*We show in Table 1 a comparison between the different semantics concerning $\Delta^I_{\mathtt{Wg,p}}$, $\Delta^I_{\mathtt{Sg,p}}$ and $\Delta^I_{\mathtt{A,p}}$. In the remainder, we will assume that $I = \{4,\dots,10\}$.*

| | |
|---|---|
| $\Delta^I_{\mathtt{Wg,p}}$-ad($\mathbf{G}$) $=$ $\Delta^I_{\mathtt{Sg,p}}$-ad($\mathbf{G}$) $=$ | $\{\emptyset, \{a\}, \{d\}, \{a,d\}\}$ |
| $\Delta^I_{\mathtt{Wg,p}}$-co($\mathbf{G}$) $=$ $\Delta^I_{\mathtt{Sg,p}}$-co($\mathbf{G}$) $=$ | $\{\{a,d\}\}$ |
| $\Delta^I_{\mathtt{Wg,p}}$-pr($\mathbf{G}$) $=$ $\Delta^I_{\mathtt{Sg,p}}$-pr($\mathbf{G}$) $=$ | $\{\{a,d\}\}$ |
| $\Delta^I_{\mathtt{Wg,p}}$-st($\mathbf{G}$) $=$ $\Delta^I_{\mathtt{Sg,p}}$-st($\mathbf{G}$) $=$ | $\{\{a,d\}\}$ |
| $\Delta^I_{\mathtt{Wg,p}}$-gr($\mathbf{G}$) $=$ $\Delta^I_{\mathtt{Sg,p}}$-gr($\mathbf{G}$) $=$ | $\{\{a,d\}\}$ |
| $\Delta^I_{\mathtt{A,p}}$-ad($\mathbf{G}$) $=$ | $\{\emptyset, \{a\}, \{d\}, \{a,d\}, \{b\}\}$ |
| $\Delta^I_{\mathtt{A,p}}$-co($\mathbf{G}$) $=$ | $\{\emptyset, \{a,d\}, \{b\}\}$ |
| $\Delta^I_{\mathtt{A,p}}$-pr($\mathbf{G}$) $=$ | $\{\{a,d\}, \{b\}\}$ |
| $\Delta^I_{\mathtt{A,p}}$-st($\mathbf{G}$) $=$ | $\{\{a,d\}, \{b\}\}$ |
| $\Delta^I_{\mathtt{A,p}}$-gr($\mathbf{G}$) $=$ | $\{\emptyset\}$ |

Table 1: Semantics on $\mathbf{G}$ over $\{4,\dots,10\}$.

The next theorem show that the $\Delta$ semantics satisfy classical properties that we have in non-temporal frameworks.

**Theorem 1 (Relation between semantics)** *Let $G = \langle \mathcal{A}, \mathcal{R}, \mathcal{P}\rangle$ be a TPAF. Then:*

1. *There always exists only one $\Delta$-grounded extension.*
2. *Let $E \subseteq \mathcal{A}$. Then, $E$ is $\subseteq$-maximal $\Delta$-admissible iff $E$ is a $\subseteq$-maximal $\Delta$-complete extension;*
3. *A $\Delta$-preferred extension is also a $\Delta$-complete extension;*
4. *A $\Delta$-stable extension is also a $\Delta$-preferred extension;*
5. *The $\Delta$-grounded extension is a subset of all $\Delta$-preferred and $\Delta$-stable extensions.*

Let us now define the notion of sceptical acceptability according to a $\Delta$ semantics.

**Definition 14 ($\Delta$-Skeptical acceptability)** *Let $G = \langle \mathcal{A}, \mathcal{R}, \mathcal{P}\rangle$ be a TPAF, $I$ an interval, and let $\{E_1,\dots,E_n\}$ be the set of $\Delta^I$-extensions of $G$, with respect to a semantics between: admissible (ad), complete (co), preferred (pr), stable (st) and grounded (gr). An argument $a \in \mathcal{A}$, is $\Delta^I$-skeptical acceptable under $\Delta^I$-s, denoted by $a \in \Delta^I$-sk-s(G) where $s \in \{ad, co, pr, st, gr\}$, iff $\forall E \in \{E_1,\dots,E_n\}, a \in E$.*

**Example 3 (Continued)** *We compare in Table 2, the different semantics using $\Delta^I_{\mathtt{Wg,p}}$ , $\Delta^I_{\mathtt{Sg,p}}$ and $\Delta^I_{\mathtt{A,p}}$ on $\mathbf{G}$ between 4 and 10. As seen in Table 1, since semantics based on the* Wg *and* Sg *criteria have the same extensions, their sceptical arguments are also identical. Finally, since each argument can defend itself, there is no sceptically accepted argument for the semantics using the criterion* A.

| | | | |
|---|---|---|---|
| $\Delta^I_{\mathtt{Wg/Sg,p}}$-sk-ad($\mathbf{G}$) $=$ | $\emptyset$ | $\Delta^I_{\mathtt{A,p}}$-sk-ad($\mathbf{G}$) $=$ | $\emptyset$ |
| $\Delta^I_{\mathtt{Wg/Sg,p}}$-sk-co($\mathbf{G}$) $=$ | $\{a,d\}$ | $\Delta^I_{\mathtt{A,p}}$-sk-co($\mathbf{G}$) $=$ | $\emptyset$ |
| $\Delta^I_{\mathtt{Wg/Sg,p}}$-sk-pr($\mathbf{G}$) $=$ | $\{a,d\}$ | $\Delta^I_{\mathtt{A,p}}$-sk-pr($\mathbf{G}$) $=$ | $\emptyset$ |
| $\Delta^I_{\mathtt{Wg/Sg,p}}$-sk-st($\mathbf{G}$) $=$ | $\{a,d\}$ | $\Delta^I_{\mathtt{A,p}}$-sk-st($\mathbf{G}$) $=$ | $\emptyset$ |
| $\Delta^I_{\mathtt{Wg/Sg,p}}$-sk-gr($\mathbf{G}$) $=$ | $\{a,d\}$ | $\Delta^I_{\mathtt{A,p}}$-sk-gr($\mathbf{G}$) $=$ | $\emptyset$ |

Table 2: Skeptical arguments on $\mathbf{G}$ over $\{4,\dots,10\}$, where Wg/Sg means Wg or Sg.

## 4 Conclusion

The ability to model and reason with probability on events occurrence is crucial for addressing real-world argumentation problems. The framework we propose captures the temporal probabilistic nature of arguments and provides a tool for drawing conclusions starting from a set of conflicting facts/events for which the placement in time is uncertain. We propose different criteria to determine if the arguments are in conflict and if the conflict is significant enough to represent a defeat. Based on this graph restriction process (as in Figure 2), we then apply the semantics to calculate the acceptability of the arguments.

In the future, we first want to investigate the relationships between the proposed semantics and the classical ones (Dung 1995). Then, we plan to carry on this work by examining other aspects of argumentation that relate uncertainty to the notion of time. The current proposal considers events lasting only a single instant (e.g. "the victim died between 6 pm and 10 pm"). A natural extension would be to allow events with a duration in time (e.g. "the victim has been walking between 2 pm and 7 pm"). In this case, we could use a probability measure to express the likelihood of an event taking place over a time interval.

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