Area Coverage Optimization using Networked Mobile Robots with State Estimation

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Abstract

In this paper, we present a solution to the area coverage problem using a team of mobile robots with state estimation. A group of autonomous mobile robots is deployed in a two-dimensional area of interest, for example, where communication among robots is limited or noisy as expected in a real life scenario. Each robot estimates its own state (position and orientation) using noisy range and bearing information received from other robots in its operating range. The area of interest is then divided into multiple sub-area using a voronoi tessellation. Using the classical Lloyd's algorithm, each robot employs distributed action command to move towards the centroid of the voronoi cell that it belongs yielding the maximum coverage of the area. Here we emphasize that the network of autonomous robots deployed in the environment is unknown a priori. A set of computer experiments is conducted to validate the fact that the area coverage is still possible under noisy communication among robots deployed in a two-dimensional area.

Introduction

Due to technological advancements in the field of robotics and autonomous systems, a group of networked autonomous agents can be deployed in a spatial area for solving a large class of area coverage problems (Cortes et al. 2004). In the past two decades, numerous coverage optimization and/or control techniques had been developed for addressing a large class of area coverage problems using multiple autonomous agents deployed in two-dimensional spatial environments (Cortes et al. 2004; Wang and Hussein 2010; Leonard and Olshevsky 2013; Pimenta et al. 2013; Miah et al. 2017). In most cases, autonomous robots are deployed in dynamic or non-stationary environment (Zhou and Ho 2022). It is worth noting that most of the existing technique to address such a large class of area coverage problems in the literature assumed the fact that the communication/data packet sharing among robots/agents is perfect or noise-free. Note that, agents' imprecise location and/or noisy measurement information have been taken into account in various existing area coverage algorithms, see (Papatheodorou et al. 2018; Dou et al. 2018; Mahboubi, Vaezi, and Labeau 2017; Breitenmoser and Sukhatme 2014), for example. Numerous existing techniques on multi-agent systems, in general, are either on paper that present theoretical results or validated using numerical results only. However, it is indeed the case that teams of mobile robots/agents are being practically deployed recently in many civilian and/or military applications, where communication data packets are assumed to perfect or noise free. In this paper, we present a partial solution to the area coverage problem using a group of autonomous mobile robots under the assumption that the communication among robots is noisy and intermittent. We emphasize that positions of robots deployed in the environment are unknown a priori, which is not the case for many existing algorithms suggested in the literature. See (Miah et al. 2015; 2013; Soleymani, Miah, and Spinello 2019), for instance, and some references therein.

There are a wide variety of applications that may use this distribution of mobile robots. For instance, military usage of securing perimeters, search and rescue, and cooperative estimation (Hu et al. 2013; Zhu and Ren 2020; Spinello and Stilwell 2014). The perimeter securing may also be applied to civilian usage as well, being used to measure an environment over time for erosion and possibly to detect tectonic plate movement from the Earth. The study of robot area coverage and design of such systems has been extensively covered in the past by various other applications. The area coverage problem has been addressed using unmanned aerial vehicles and unmanned ground vehicles (Agarwal and Akella 2022). Another study addressed the emergence of new mobile targets and their influence on the risk function (Miah et al. 2017). Therein, authors also addressed non-uniform coverage control problems with time-varying density. As stated previously, most of these studies have mainly been validated using computer simulations only. Therefore, the current work bridges the gap between the existing simulation work that addresses the area coverage problem and a few coverage optimization algorithms that are validated using practical robots (Miah and Knoll 2018) taking into account noise-free communication among robots.

Without loss of generality, this paper addresses a two-dimensional area coverage problem using a team of autonomous car-like mobile robots deployed in an environment. Here we use the words robots and agents interchange-

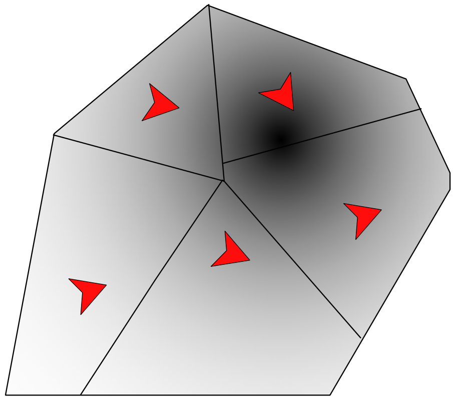


Figure 1: A team of mobile robots operating in a two-dimensional area to optimally configure themselves for maximizing the coverage.

able. We emphasize that the position information of each robot is unknown to other robots. It is important to note that this work assumes each robot to be equipped with on-board sensors that measure range and bearing information from other robots. The 2D area where robots are deployed is known to the team of robots. Each robot estimates its position and orientation (*i.e.*, state) using range and bearing measurements. A robot can also estimate the positions of all other robots in its operating range. Like that, each robot estimates the local network of robots (mapping) that is utilized to partition the environment using Voronoi tessellation (Okabe et al. 2000a). The robots then determine the motion control commands using conventional Lloyd's algorithm (Miah, Fallah, and Spinello 2017) that maximizes the coverage. Figure 1 shows that a team of mobile robots are operating in a two-dimensional area to optimally place themselves to maximize the coverage. The importance of each point in the area is represented by scalar density measure, which will be illustrated later. The goals is to place all the robots in optimal configurations so that the probability, for instance, of detecting a certain event is maximum.

The rest of the paper is organized as follows. Section revisits the conventional Voronoi tessellation technique that each robot executes onboard. The area coverage algorithm with state estimation is briefly illustrated in section . Computer experiments validated the proposed area coverage algorithm are presented in section followed by conclusion and future avenues of this work in section .

Voronoi Tessellation

Motivated by the locational optimization problem (Okabe et al. 2000a) and similar to the coverage optimization using multi-agent systems, this work also employs Voronoi tessellation technique to partition the area of interests, which, without loss of generality, we assume a two-dimensional area, $\mathcal{Q} \subset \mathbb{R}^2$. Let $\mathbb{R}_+$ denotes the set of positive real numbers and $\mathbb{N}$ be the set of natural numbers with $\mathbb{N}_0 = \mathbb{N} \cup \{0\}$. A set of mobile robots $\mathcal{I} = \{1, 2, \dots, n\} \subset \mathbb{N}$ are deployed in a spatial area $\mathbb{Q}$, where the state of each robot is evolved using first-order Euler approximation as

$$x_{k+1}^{[i]} = x_k^{[i]} + T v_k^{[i]} \cos\left(\theta_k^{[i]} + \gamma_k^{[i]}\right) \quad (1a)$$

$$y_{k+1}^{[i]} = y_k^{[i]} + T v_k^{[i]} \sin\left(\theta_k^{[i]} + \gamma_k^{[i]}\right) \quad (1b)$$

$$x_{k+1}^{[i]} = \theta_k^{[i]} + T \frac{v_k^{[i]}}{\ell} \sin\left(\gamma_k^{[i]}\right), \quad (1c)$$

where ith, $i \in \mathbb{N}$, robot is modeled as a car-like mobile robot with position $\mathbf{p}_k^{[i]} = \left[x_k^{[i]}, y_k^{[i]}\right]^T \in \mathcal{Q}$ at time $t = kT$, with $k \in \mathbb{N}_0$, sampling time $T \in \mathbb{R}_0$, orientation $\theta_k^{[i]} \in (-\pi, \pi]$, $v_k^{[i]}$ is the linear speed of the robot applied at time instant k, and $\gamma_k^{[i]}$ is the steering angle.

Following (Okabe et al. 2000a; Cortes et al. 2004), the optimal partition of $\mathcal{Q}$ is the Voronoi partition $\mathcal{V}_k(\mathcal{P}) = \{\mathcal{V}_k^{[1]}, \mathcal{V}_k^{[2]}, \dots, \mathcal{V}_k^{[n]}\}$ generated by the set of robot positions $\mathcal{P} = \left\{\mathbf{p}_k^{[1]}, \mathbf{p}_k^{[2]}, \dots, \mathbf{p}_k^{[n]}\right\}$, is defined as

$$\mathcal{V}_k^{[i]} = \left\{\mathbf{q} \in \mathcal{Q} | \|\mathbf{q} - \mathbf{p}_k^{[i]}\| \le \|\mathbf{q} - \mathbf{p}_k^{[j]}\|, \ \forall i \neq j\right\}. \quad (2)$$

. Note that the area $\mathcal{Q} \equiv \mathcal{V}_k^{[1]} \cup \mathcal{V}_k^{[2]} \cup \dots \cup \mathcal{V}_k^{[n]}$. The partitions $\mathcal{V}_k^{[1]}, \mathcal{V}_k^{[2]}, \dots, \mathcal{V}_k^{[n]}$ are also called Voronoi partitions of the agents (also called generators) $\left\{\mathbf{p}_k^{[1]}, \mathbf{p}_k^{[2]}, \dots, \mathbf{p}_k^{[n]}\right\}$. Two Voronoi partitions are said to be adjacent at time instant k if they share an edge. See (Okabe et al. 2000b) for details. In the following section, we describe how agents move in the area $\mathcal{Q}$ following optimal Voronoi tessellation that maximizes the coverage.

Area Coverage and State Estimation

The main purpose of this work is to maximize the coverage of the area $\mathcal{Q}$ using a team of mobile robots denoted by $\mathcal{I}$. A system-level block diagram showing the specific inputs and outputs of the proposed multi-agent area coverage algorithm is shown in Figure 2. Here the inputs are spatial area to be covered, $\mathcal{Q}$, resources distribution of the area, and stochastic intermittent communication signals. The outputs are the coverage metric that we maximize and the optimal configurations of the robots. To maximize the coverage, we define the sensory performace function $f(d_k^{[i]})$ with $d_k^{[i]} = \|\mathbf{q} - \mathbf{p}_k^{[i]}\|$ being the Euclidean distance between the ith robot and the point $\mathbf{q} \in \mathcal{Q}$. The interpretation of the

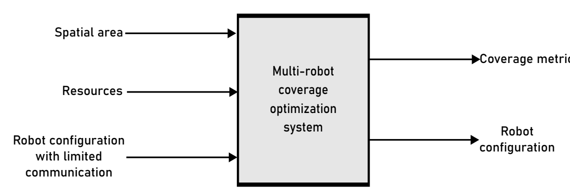


Figure 2: System level block diagram of the area coverage problem in a framework of multi-agent systems.

function $f(\cdot)$ is that the performance of the sensor mounted on the ith robot degrades as the distance $d_k^{[i]}$ increases. Furthermore, a scalar measure $\phi : \mathcal{Q} \to \mathbb{R}_+$ dictating the importance of each point $\mathbf{q} \in \mathcal{Q}$ contributes to the optimal coverage of the area by a team of mobile robots. See Figure 1, where more robots are placed in the area with more density.

Motivated by conventional nonunion area coverage solutions (Miah et al. 2015), let us define the coverage metric

$$H_{\mathcal{V}}(\mathcal{P}) = \sum_{i=1}^{n} \int_{\mathcal{V}_i} f(d_k^{[i]})\phi(q)\mathrm{d}\mathcal{Q}. \quad (3)$$

If the sensor performace function $f(\cdot)$ is strictly decreasing function of its argument, which is the case in this work, then taking the partial derivative of the coverage metric H in (3), gives

$$\frac{\partial}{\partial \mathbf{p}_k^{[i]}}(H) = \int_{\mathcal{V}_i} \frac{\partial}{\partial \mathbf{p}_k^{[i]}} f(d_k^{[i]})\phi(q)\mathrm{d}\mathcal{Q}. \quad (4)$$

Setting the partial derivative in Equation (4) to zero yields that the ith robot position coincides with the generalized Voronoi centroid. A comprehensive treatment of the generalized centroid can be sought in (Miah et al. 2017; Miah, Fallah, and Spinello 2017), which is driven by the definition of the centroidal Voronoi partition (Cortes et al. 2004; Cortes and Bullo 2005). Using the scalar density function $\phi(\cdot)$ and the Voronoi partition $\mathcal{V}$ defined in (2), we define the mass and the Voronoi centroid as follows:

$$M_{\mathcal{V}_k^{[i]}} = \int_{\mathcal{V}_{k^{[i]}}} \mathbf{q}\phi(\mathbf{q})\mathrm{d}\mathcal{Q} \quad \text{and} \quad (5a)$$

$$\mathbf{c}_{\mathcal{V}_k^{[i]}} = \frac{1}{M_{\mathcal{V}_k^{[i]}}} \int_{\mathcal{V}_{k^{[i]}}} \mathbf{q}\phi(\mathbf{q})\mathrm{d}\mathcal{Q} \quad (5b)$$

It is guaranteed that the nonuniform coverage by a team of mobile robots operating in a two-dimensional area is maximum if the robots asymptotically converge themselves in their generalized centroid (Miah and Knoll 2018). However, without loss of generality, this work assumes uniform density ϕ, therefore, the generalized centroid of the Voronoi partition $\mathcal{V}_k^{[i]}$ boils down to its circumcenter, $\mathbf{c}_k^{[i]}$. At time instant $k = \mathbb{N}$, each robot moves towards the circumcenter using the linear speed modeled by

$$v_k^{[i]} = a\Big(\mathbf{c}_k^{[i]} - \mathbf{p}_k^{[i]}\Big), \quad (6)$$

for some $a > 0$. The steering angle of each car-like robot $\gamma_k^{[i]}$ is determined using

$$\gamma_k^{[i]} = \kappa_p\Big(\theta_{k,\mathrm{ref}}^{[i]} - \theta_k^{[i]}\Big) \in (-\frac{\pi}{2}, \frac{\pi}{2}), \quad (7)$$

for some $\kappa > 0$ and $\theta_{k,\mathrm{ref}}^{[i]}$ is the reference angle of the vector pointing to the circumcenter $\mathbf{c}_k^{[i]}$ from the position of the ith robot. For the sake of simplicity, the actuator commands for the ith robot is denoted using the vector $\mathbf{u}_k^{[i]} = \Big[v_k^{[i]}, \gamma_k^{[i]}\Big]^T$.

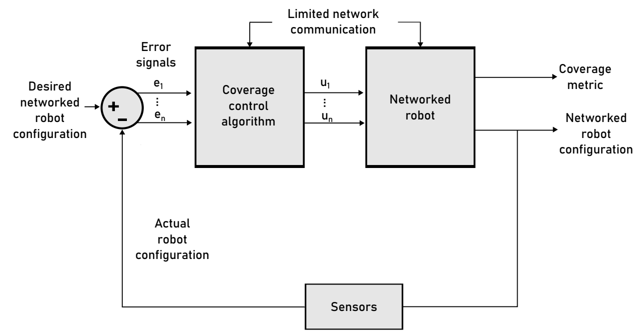


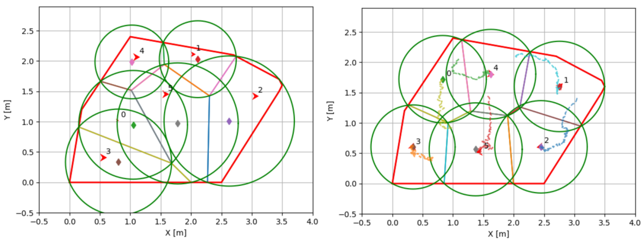
Figure 3: Subsystem level block diagram of the area coverage algorithm showing the major components for the overall implementation.

The actuator commands of the robots are then updated as $k \to \infty$. Note that the Voronoi partition is created at every sampling time instant k. Like that the average coverage is maximized as time $k \to \infty$. Figure 3, shows the subsystem-level block diagram of the overall implementation of the proposed area coverage problem. At time instant $k \in \mathbb{N}$, each robot get sensory measurements from other robots to estimate the position and orientation of itself and other robots. Each robot then determines the Voronoi partition based on the estimated positions of the network of robots, which then are used to compute circumcenters of the partitions. The distance between the ith robot and its circumcenter is the error signal, which is then used to compute the linear speed $v_k^{[i]}$. The orientation error of the vector between the robot and its circumcenter is used to compute the steering angle $\gamma_k^{[i]}$.

It is important to note that the position and orientation information are not assumed to be known in this work. Therefore, the position and orientation of each robot/agent will be estimated using range and bearing information. This work assumes that the each robot is able to measure noisy range (*i.e.,* line-of-sight distance, for example) and bearing of other robots. We then implement a local Extended Kalman filter simultaneous localization and mapping (EKF-SLAM) algorithm onboard each robot. Therefore, each robot implements a simulation localization and mapping (SLAM) algorithm to locally estimate the network for determining the Voronoi partition at each time instant k. More details on EKF-SLAM algorithm can be sought in (Miah, Knoll, and Hevrdejs 2018). The EKF-SLAM algorithm is locally implemented onboard each robot deliberately taking into account the real scenario where the sensory measurements are noisy in nature. The performance of the area coverage algorithm using noisy sensory measurements is illustrated in the following section.

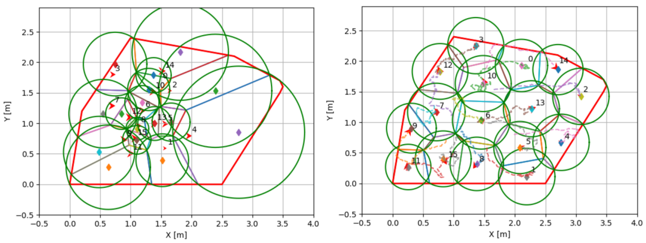
Results and Validation

The performance of the area coverage algorithm is evaluated by distributing mobile robots of group sizes $n = 6$ and $n = 16$ in a two-dimensional convex boundary plane $\mathcal{Q}$. The boundary vertices for the plane are given by the points $(0, 0)$, $(2.5, 0)$, $(3.45, 1.5)$, $(3.5, 1.6)$, $(3.45, 1.7)$,



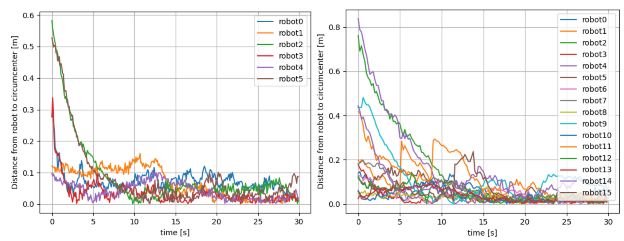
(a) (b)

Figure 4: Area coverage robot trajectory and positioning of a network of $n = 6$ mobile robots using stochastic Kalman filtering at time instant t = (a) 0s and (b) 30s.



(a) (b)

Figure 5: Area coverage robot trajectory and positioning of a network of $n = 16$ mobile robots using stochastic Kalman filtering at time instant t = (a) 0s and (b) 30s.



(a) (b)

Figure 6: Distance from circumcenter of (a) 6 and (b) 16 number of mobile robots using stochastic Kalman filtering position estimates from t = 0s to t = 30s.

$(2.7, 2.1)$, $(1, 2.4)$, and $(0.2, 1.2)$ [m]. Measurements for each robot are taken with a sampling time of $T = 0.1$ [s], and the simulation is run for 30 [s]. The mobile robots are initially distributed such that their positions are not less than 0.1 [m] apart. For both teams of $n = 6$ and $n = 16$ robots, two constraints will be applied and tested. In the first constraint, each robot is assumed to know its position $\mathbf{p}_k^{[i]}$ and the position of any other robot $\mathbf{p}_k^{[j]}$ at any given time $t_k = kT$ with $k \in \mathbb{N}_0$. In the second constraint, each robot is only given a noisy range and bearing measurement for all other robots, and the random noise takes on a Gaussian distribution, for the sake of simplicity. We define $e_i = \left\|\hat{\mathbf{p}}_k^{[i]} - \mathbf{c}_k^{[i]}\right\|$ [m] as the distance between each robot i and its estimated circumcenter $\hat{\mathbf{c}}^{[i]}$ given by the Voronoi tessellation. As the simulation time increases, the error vector e_i converges to 0 meters as each robot approaches its circumcenter.

Coverage Performance with State Estimation

Each robot in the network employ the EKF-SLAM algorithm locally. The control noise and observation noise in Networked State Estimation is determined using the initial covariance matrices $\mathbf{Q}_k$ and $\mathbf{R}_k$, respectively. The parameters for both covariance matrices are tuned $\sigma_R = 0.01$ m, $\sigma_B = 1°$, $\sigma_V = 0.03$ m, and $\sigma_G = 1°$. We run the same set of experiments with $n = 6$ and $n = 16$ robots. Figure 4 shows the trajectory of $n = 6$ robots at time $t = 0$ [s] and $t = 30$ [s]. Figure 5 shows the trajectory of $n = 16$ robots at time $t = 0$ [s] and $t = 30$ [s].

As can be seen, all six robots are optimally placed in the area $\mathcal{Q}$, which guarantees optimal coverage. This is also backed by the error plot shown in Figure 6(a). Initially, robots are far from their respective circumcenters, however, as time goes, all robots tentatively converge to circumcenter with smaller position errors. Due to the sensor noise, this error calculation will continue to fluctuate as robots obtain noisy range and bearing measurements.

As for the experiment with $n = 16$ robots, they start clustered around the center of the region at $t = 0$ [s], then converging to their respective circumcenters at $t = 30$ [s]. The variances of the data in figure 6 comes from the interference of the noise taken in by the sensors. The error calculations for the robots varies from robot to robot and the graphical results for each robot are more noisy than the respective graphs without the use of the Extended Kalman filter.

Conclusion and Future Work

We proposed a solution to the area coverage problem by using a team of mobile robots with state estimation and intermittent and noisy communication. The Voronoi tessellation is employed as an optimal region for area coverage. The distribution for all the mobile robots, follows a Gaussian distribution yielding a maximum coverage for the region. Much of the work that has been previously conducted also utilize the Voronoi tessellation, as well as intermittent communication. With the assumptions that the robots are equipped with onboard sensors, the robots can intermittently communicate using a server that runs a robot operating system, ROS, with each robot transmitting information such as the range and bearing data and other metrics. The proposed work can be applied in a lab setting, using differential drive mobile robots programmed with our algorithm and connected to a laptop running ROS. One potential research path would be to consider a model with a spherical coordinate system which could be applied in an aerial or underwater environment.

Acknowledgments

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