

Min-Based Conditioning of Possibilistic \mathcal{EL} Ontology

Rim Mohamed and Zied Loukil and Faiez Gargouri

{rim.mohamed,zied.loukil,faiez.gargouri}@isims.usf.tn
University of Sfax, Miracl Laboratory, Isims, Tunisia

Zied Bouraoui

bouraoui@cril.fr
CRIL CNRS & Univ Artois, Lens, France

Abstract

The \mathcal{EL} is a tractable family of lightweight description logics that underlay the *OWL EL* profile. It guarantees the tractability of the reasoning process, especially for concept classification. In particular, such a fragment is widely used for medical applications. This paper investigates the evolution of \mathcal{EL} ontologies when a new piece of information that can be conflicting or attached with a confidence level reflecting its credibility or priority is available. To encode such knowledge, we propose an extension of \mathcal{EL} description logic within the possibility theory, which provides a natural way to deal with ordinal scale reflecting ranking between pieces of knowledge. We then show how such a ranking between axioms is induced from the ontology with the presence of new information and study the evolution process at the semantic level. Finally, we propose a polynomial syntactic counterpart of the evolution process while preserving the consistency of the ontology.

1 Introduction

Description logic (DLs for short) is proved to be the most used formalism for representing and reasoning about static knowledge in various domain areas such as ontology-based data access (Xiao et al. 2018), information and data integration (Goodhue, Wybo, and Kirsch 1992) and the semantics web (Wu, Potdar, and Chang 2008). The DLs use two sets of axioms, first the terminological axioms that encode generic knowledge. Second, the assertional axioms that describe data. DLs provide the foundations of the Web Ontology Language *OWL2*¹, and its profiles *OWL2-QL*, *OWL2-EL* and *OWL2-RL*. Recently, the *OWL2-EL* profile has gained a lot of attention in many ontology applications, in particular, economics and medicine (Achich et al. 2021). It is designed for applications that use a large number of classes or/and relations. The *OWL2-EL* provides powerful class constructors to express the very large biomedical ontology SNOMED CT² and gene ontology (GO)³. This profile is based on a family of lightweight DLs, called \mathcal{EL} (Baader, Brandt, and

Lutz 2008), which guarantees the tractability of the reasoning process, especially for concept classification.

In some applications (e.g., medical applications), ontologies are not static and typically evolve (Wang, Wang, and Topor 2010). For example, with the presence of new medical knowledge coming to revise the old one that one can easily notice with medical knowledge about Covid-19 that didn't stop evolving during the last two years. In the beginning, the symptoms are fever, cough, shortness of breath and fatigue. After that, some studies showed that there exist other symptoms such as muscle or body aches, headache and congestion. At each time, a new study defines new symptoms of this illness, which leads to revising the old ones. The evolution process (or incorporating revising process) of DLs ontology consists in inserting some input information that can be sure (or uncertain) while preserving the consistency of the resulting ontology.

This problem is closely related to the belief revision problem in propositional logic (Benferhat et al. 2002), where the old belief is revised by adding new information. It has been also defined as the knowledge change and was characterized by the well-known AGM postulates (Alchourrón, Gärdenfors, and Makinson 1985). Several works have been proposed to revise the DL ontology (Flouris, Plexousakis, and Antoniou 2005) by adapting the AGM theory to DLs. In (Qi et al. 2008) an extension of kernel-based revision and semi-revision operators to DLs frameworks have been proposed which is closely related to the one proposed by (Hansson 1997) in a propositional logic setting. Recently, some works have been proposed on the evolution of lightweight ontologies. For example, some model-based approaches for revising have been proposed in (Wang, Wang, and Topor 2010). In (Gao, Qi, and Wang 2012), some evolution mechanisms for DL-Lite ontologies have been proposed where the new information is restricted to ABox assertions. In some applications, new knowledge is often provided by several and potentially conflicting sources, which gives rise to a preference between pieces of knowledge reflecting their credibility (Mohamed, Loukil, and Bouraoui 2018). In (Benferhat et al. 2017), a Prioritized Removed Sets Revision (PRSR) is proposed to revise DL-Lite ontology at the assertional level. However, there is to the best of our knowledge, no approach that studies the evolution of \mathcal{EL} ontology when new uncertain information is available.

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¹<https://www.w3.org/TR/owl2-overview/>

²<https://bioportal.bioontology.org/ontologies/SNOMEDCT>

³<http://geneontology.org/>

In this paper, we study the evolution of \mathcal{EL} ontologies when a new piece of information that can be conflicting and attached with a confidence level reflecting its credibility or priority is available. We distinguish several scenarios of evolution depending on i) whether the new information is consistent or not with the original ontology and ii) whether it can be inferred or not from the ontology and to what extent. To take into consideration the weights attached to the ontological axioms, we propose the use of prioritized ontology within the possibility theory setting, which provides a natural way to deal with ordinal, qualitative uncertainty, preferences, and priorities. When the evolution process starts with a flat ontology (a standard ontology without weights attached to axioms), we propose a new approach to induce a prioritized ontology from the original one. According to the different scenarios of input information, we investigate the evolution process at the semantic level by conditioning the ranking (possibility) distribution associated with the interpretations of the ontology. We show that our evolution operators provide meaningful and tractable syntactic counterparts.

2 The Syntax and Semantics of \mathcal{EL}

In this section, we briefly recall the syntax and semantics of the \mathcal{EL} DL, the fragment underlying *OWL2-EL*.

Syntax. The syntax of \mathcal{EL} is defined upon the three pairwise disjoint sets N_C, N_R, N_I , where N_C denotes a set of atomic concepts, N_R denotes a set of atomic roles and N_I denotes a set of individuals. The \mathcal{EL} concept expressions are built according to the following syntax:

$$C, D \rightarrow \top \mid A \mid C \sqcap D \mid \exists r.C$$

where $A \in N_C, r \in N_R$.

An \mathcal{EL} ontology (or knowledge base) consists of a set of general concept inclusion (GCI) axioms of the form $C \sqsubseteq D$, meaning that C is more specific than D or simply C is subsumed by D , a set of equivalence axioms of the form $C \equiv D$, which is the abbreviation of the two general concept inclusions $C \sqsubseteq D$ and $D \sqsubseteq C$, a set of concept assertions of the form $C(a)$, and a set of role assertions of the form $r(a, b)$. For more details, see for instance (Baader, Brandt, and Lutz 2008).

Semantics. The semantics is given in terms of interpretations $\mathcal{I} = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$ which consist of a non-empty interpretation domain $\Delta^{\mathcal{I}}$ and an interpretation function $\cdot^{\mathcal{I}}$ that maps each individual $a^{\mathcal{I}} \in N_I$ to an element $a^{\mathcal{I}} \in \Delta^{\mathcal{I}}$, each concept $A \in N_C$ to a subset $A^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}}$ and each role $r \in N_R$ to a subset $r^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$. Furthermore, the function $\cdot^{\mathcal{I}}$ is extended in a straightforward way for concept and role expressions as depicted in (Baader, Brandt, and Lutz 2008). An interpretation \mathcal{I} is said to be a model of (or satisfies) a GCI (resp. role inclusion, role composition) axiom, denoted by $\mathcal{I} \models C \sqsubseteq D$ (resp. $\mathcal{I} \models r \sqsubseteq s, \mathcal{I} \models r_1 \circ r_2 \sqsubseteq s$), if $C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$ (resp. $r^{\mathcal{I}} \subseteq s^{\mathcal{I}}, (r_1 \circ r_2)^{\mathcal{I}} \subseteq s^{\mathcal{I}}$). Similarly, \mathcal{I} satisfies a concept (resp. role) assertions, denoted $\mathcal{I} \models C(a)$ (resp. $\mathcal{I} \models r(a, b)$), if $a^{\mathcal{I}} \in C^{\mathcal{I}}$ (resp. $(a^{\mathcal{I}}, b^{\mathcal{I}}) \in r^{\mathcal{I}}$). An interpretation \mathcal{I} is a model of an ontology \mathcal{O} if it satisfies all the axioms of \mathcal{O} . An ontology is said to be consistent if it

has a model. Otherwise, it is inconsistent. An axiom ϕ is entailed by an ontology, denoted by $\mathcal{O} \models \phi$, if ϕ is satisfied by every model of \mathcal{O} . We say that C is subsumed by D w.r.t an ontology \mathcal{O} iff $\mathcal{O} \models C \sqsubseteq D$. Similarly, we say that a is an instance of C w.r.t \mathcal{O} iff $\mathcal{O} \models C(a)$. A concept C is said to be in unsatisfiable w.r.t. \mathcal{O} iff $\mathcal{O} \models C \sqsubseteq \perp$, otherwise C is said to be satisfiable.

3 Prioritized \mathcal{EL} Ontology

The evolution process is the act of adding some information while preserving the consistency of the ontology. Given an \mathcal{EL} ontology, denoted by \mathcal{O} . Let (ϕ, w) be the new input where ϕ is an \mathcal{EL} axiom and w is the weight of ϕ reflecting its priority (credibility). Regarding such information, two situations may happen. The first is the situation where the input is consistent with the ontology. In this case, if the input is fully reliable, namely $w = 1$, then the evolution outcome consists in simply adding the information to the ontology. Now, if ϕ is uncertain, namely $w < 1$, then the evolution process should ensure that ϕ will be inferred from the ontology with its prescribed weight w after revision. Second, in the situation where the input is inconsistent with the ontology, then we need to repair the ontology and add ϕ with its prescribed weight w . To handle qualitative uncertainty of input information, i.e the preference ranking between the provided information according to their level of priority, we use prioritized \mathcal{EL} ontologies within possibility theory (Mohamed, Loukil, and Bouraoui 2018). Section 3.1 provides the syntax and semantics of such framework. Sections 4 and 5 provide evolution process of prioritized ontology when a new information (ϕ, w) is available. In section 3.2, we show how to induce a prioritized ontology in the case where the evolution process starts with a flat ontology, i.e., all its axioms are certain.

3.1 Syntax and Semantics of Prioritized \mathcal{EL} Ontology

Syntax. A possibilistic \mathcal{EL} ontology, denoted by \mathcal{O}_π is a set of possibilistic axioms of the form (ϕ_i, w_i) , where ϕ is an \mathcal{EL} axiom and $w \in]0, 1]$ its certainty degree. Note that the higher the degree w , the more certain is the formula. Note that the axioms with w_i 's equal to '0' are not explicitly represented in the ontology. Moreover, when all the degrees are equal to 1, \mathcal{O}_π coincides with a standard \mathcal{EL} ontology \mathcal{O} .

Definition 1 Let \mathcal{O}_π be a possibilistic \mathcal{EL} ontology. We call the w -cut (resp. strict w -cut), denoted by $\mathcal{O}_{\pi \geq w}$ (resp. $\mathcal{O}_{\pi > w}$), the sub-base \mathcal{O}_π contains the set of axioms having degree greater or equal (resp. strictly greater) than w .

When the ontology is inconsistent, in that case, we assign a degree of inconsistency as follows:

Definition 2 The inconsistency degree of \mathcal{O}_π is syntactically defined as follows: $Inc(\mathcal{O}_\pi) = \max\{w : \mathcal{O}_{\pi \geq w} \text{ is inconsistent}\}$.

Semantics. The semantics of \mathcal{O}_π ontology is defined by the possibility distribution. Let \mathcal{L} be \mathcal{EL} description language and $\Omega = \{\mathcal{I}_1, \dots, \mathcal{I}_n\}$ be a universe of discourse consisting

of a set of \mathcal{EL} interpretations. A possibility distribution is the main block of possibility theory, denoted by π and it is a mapping from Ω to the unit interval $[0, 1]$. The possibility distribution $\pi(\mathcal{I})$ represents the degree of compatibility of \mathcal{I} with the available knowledge. More specifically, when $\pi(\mathcal{I}) = 0$ this means that the interpretation \mathcal{I} is impossible. Otherwise, $\pi(\mathcal{I}) = 1$ means that it is totally possible (i.e. fully consistent with available knowledge).

Definition 3 *The possibility distribution associated with \mathcal{O}_π is obtained as follows:*

$$\forall \mathcal{I} \in \Omega, \pi(\mathcal{I}) = \begin{cases} 1 & \text{if } \forall (\phi_i, w_i) \in \mathcal{O}_\pi, \mathcal{I} \models \phi_i \\ 1 - \max\{w_i : (\phi_i, w_i) \in \mathcal{O}_\pi, \mathcal{I} \not\models \phi\} & \\ \text{otherwise.} & \end{cases}$$

An interpretation \mathcal{I} is a model of \mathcal{O}_π if it satisfies all the axioms of the ontology. In this case $\pi(\mathcal{I}) = 1$, which means that the possibility distribution $\pi_{\mathcal{O}_\pi}$ is normalized. Otherwise, the distribution is called sub-normalized. Given the possibility distribution π , two measures are defined, the possibility degree $\Pi_\pi(\phi) = \max_{\mathcal{I} \in \Omega} \{\pi(\mathcal{I}) : \mathcal{I} \models \phi\}$ evaluates the extent to which ϕ is consistent with the available information encoded by π . The necessity measure, $N_\pi(\phi) = 1 - \max_{\mathcal{I} \in \Omega} \{\pi(\mathcal{I}) : \mathcal{I} \not\models \phi\}$ evaluates to what extent ϕ is certainly entailed from the available knowledge encoded by π . Syntactically, an axiom ϕ_i has w_i as its certainty degree, means that $N(\phi_i) \geq w_i$.

3.2 Inducing Prioritized \mathcal{EL} Ontology

Suppose that we have a flat \mathcal{EL} ontology \mathcal{O} , i.e., an \mathcal{EL} ontology where all the axioms are certain. In the following, we propose a new method for introducing the priority relation between axioms using the notion of conflict matrix when the new information ϕ is inconsistent with \mathcal{O} .

Definition 4 *Let \mathcal{M} be a matrix that contains the set of ontological axioms. The conflict matrix \mathcal{M} presents the conflict relations between axioms. If $M_{ij} = 0$, then there is a conflict between the i^{th} axiom and j^{th} axiom. Otherwise, i.e., $M_{ij} = 1$, then the axioms can occur together.*

Example 1 *Considering the following \mathcal{EL} ontology and its conflict matrix.*

$$\begin{aligned} ax_1 &= \text{DeltaSymptom} \sqsubseteq \text{CoronaVirus}, & ax_2 &= \text{OmicronSymptom} \sqsubseteq \text{CoronaVirus} \\ ax_3 &= \text{Diarrhea} \sqsubseteq \text{InfluenzaSymptom}, & ax_4 &= \text{breathlessness} \sqsubseteq \text{OmicronSymptom} \\ ax_5 &= \text{Diarrhea} \sqcap \text{InfluenzaSymptom} \sqsubseteq \perp, & ax_6 &= \text{InfluenzaSymptom} \sqsubseteq \text{RespDisease} \\ ax_7 &= \exists \text{highTemperature}. \text{Diarrhea} \sqsubseteq \text{OmicronSymptom}, \\ ax_8 &= \text{DeltaSymptom} \sqsubseteq \text{RespDisease} \\ ax_9 &= \text{DeltaSymptom} \sqsubseteq \text{InfluenzaSymptom}, & ax_{10} &= \text{DeltaSymptom} \sqcap \text{OmicronSymptom} \sqsubseteq \perp \end{aligned}$$

To induce the stratified \mathcal{EL} ontology, we will first transform the conflict matrix into a stochastic matrix where the sum of the elements of each row is equal to 1. Therefore, to obtain the stable vector V from the stochastic matrix, we will consider first that the importance is equally distributed between

	ax_1	ax_2	ax_3	ax_4	ax_5	ax_6	ax_7	ax_8	ax_9	ax_{10}
ax_1	1	0	1	1	1	1	1	0	1	1
ax_2	0	1	1	1	1	1	1	0	1	1
ax_3	1	1	1	1	0	1	1	1	1	1
ax_4	1	1	1	1	1	1	1	1	1	1
ax_5	1	1	0	1	1	1	1	1	0	1
ax_6	1	1	1	1	1	1	1	1	1	1
ax_7	1	1	1	1	1	1	1	1	1	1
ax_8	1	1	1	1	1	1	1	1	1	1
ax_9	1	1	1	1	1	1	1	1	1	1
ax_{10}	0	0	1	1	1	1	1	0	1	1

Table 1: Conflict Matrix of Example 1

axioms, namely $V = (1/n, 1/n, \dots, 1/n)$. Then, the axioms that have more conflict relations will give their important value to those axioms. More formally, we multiply the vector V by the vectors of axioms until the values of V do not change. Note that the axiom that has more conflict relation will have less importance. At the end, we obtain a vector rank V that contains the importance of each axiom presented in the ontology, namely $X = (R_{ax_1}, R_{ax_2}, \dots, R_{ax_n})$.

Example 2 *In the following example, we present the stochastic matrix of the ontology presented in Example 1 and the obtained vector rank. The obtained vectorRank $V = ([R_{ax_1} = 0.0735, R_{ax_2} = 0.0735, R_{ax_3} = 0.0985, R_{ax_4} = 0.1127, R_{ax_5} = 0.1017, R_{ax_6} = 0.1127, R_{ax_7} = 0.1127, R_{ax_8} = 0.0735, R_{ax_9} = 0.1002, R_{ax_{10}} = 0.1127])$. Based on these values, the axioms $\{ax_4, ax_6, ax_7, ax_{10}\}$ are in the same level, and they are the most important ones since they have the most important value, however, $\{ax_1, ax_2, ax_8\}$ are the least reliable axioms.*

Based on the importance values, we can define the prioritized ontology as follows:

Definition 5 *A prioritized \mathcal{EL} ontology, denoted by \mathcal{O}_π is defined as follows: $\mathcal{O}_\pi = S_1 \cup S_2, \dots, \cup S_n$. Where S_1 contains least reliable axioms and S_n , the most important ones.*

In the following sections, we study evolution at the semantic and syntactic levels of the prioritized \mathcal{EL} ontology.

4 Semantics Evolution of Prioritized \mathcal{EL} Ontology

The revision process results from the effect of accepting new information. Let \mathcal{O}_π be the prioritized \mathcal{EL} ontology and $\pi_{\mathcal{O}_\pi}$ be its attached possibility distribution obtained by Definition 3.

Revision at semantic level takes as input the original possibility distribution $\pi_{\mathcal{O}_\pi}$ and the new information ϕ or (ϕ, w) and transforms $\pi_{\mathcal{O}_\pi}$ into a revised possibility distribution $\pi'_{\mathcal{O}_\pi} = \pi_{\mathcal{O}_\pi}(\cdot | (\phi))$ (resp. $\pi'_{\mathcal{O}_\pi} = \pi_{\mathcal{O}_\pi}(\cdot | (\phi, w))$).

A revised possibility distribution $\pi'_{\mathcal{O}}$ is admissible for revising the initial possibility distribution $\pi_{\mathcal{O}}$ with the new information ϕ if it satisfies the following properties:

- **P1:** $\pi'(\mathcal{I}) = 1$
- **P2:** if $\pi(\mathcal{I}) = 0$ then $\pi'(\mathcal{I}) = 0$
- **P3** $\Pi'(\phi) = 1$ and $N'(\phi) \geq w$

The first property ensures that each revised possibility distribution should be normalized. **P2** means that impossible interpretation remains impossible after conditioning. **P3**

stipulates that the reliable input should be inferred from the ontology with at least its prescribed necessity.

4.1 Min-Based Conditioning

In this section, we first study the case where the input information is fully reliable ($w = 1$). The evolution at the semantic level consists in conditioning the possibility distribution of the ontology (Definition 3) by the new input $(\phi, 1)$. Two possible situations hold: ϕ is consistent or inconsistent with the ontology. In both cases, the revised possibility distribution $\pi'_{\mathcal{O}_\pi}$ is defined as follows:

Definition 6 Let \mathcal{O}_π be the prioritized \mathcal{EL} ontology and ϕ be the new information. The revised possibility distribution $\pi'_{\mathcal{O}_\pi}$ is defined as follows:

$$\pi'_{\mathcal{O}_\pi} = \pi_{\mathcal{O}_\pi}(\cdot | (\phi)) = \begin{cases} 1 & \text{if } \pi(\mathcal{I}) = \Pi(\phi) \text{ and } \mathcal{I} \models \phi \\ \pi(\mathcal{I}) & \text{if } \pi(\mathcal{I}) < \Pi(\phi) \\ 0 & \text{if } \mathcal{I} \not\models \phi \text{ otherwise} \end{cases}$$

This definition guarantees that if ϕ is consistent, then it is fully inferred from the revised ontology. Definition 6 also preserves the weights attached to each axiom.

Example 3 We continue with the prioritized \mathcal{EL} ontology given in Example 1. Let $\mathcal{I}_1, \mathcal{I}_2, \mathcal{I}_3$ be the three interpretations. Considering that \mathcal{I}_1 satisfies all the axioms of the ontology, then $\pi(\mathcal{I}_1) = 1$. The interpretation \mathcal{I}_2 satisfies all the axioms of \mathcal{O}_π but it does not satisfy ax_6 , therefore $\pi(\mathcal{I}_2) = 1 - 0.1127 = 0.8873$. The interpretation \mathcal{I}_3 does not satisfy ax_1, ax_3 and ax_9 , therefore $\pi(\mathcal{I}_3) = 1 - 0.10 = 0.90$. Let $(\phi, 1)$ be new information. Considering that \mathcal{I}_1 is model of ϕ , then $\pi'(\mathcal{I}_1) = 1$ and the two interpretations $\mathcal{I}_2, \mathcal{I}_3$ are not models of ϕ , therefore the revised possibility distribution $\pi'(\mathcal{I}_2) = 0$ and $\pi'(\mathcal{I}_3) = 0$

In the following, we study the conditioning of $\pi_{\mathcal{O}_\pi}$ when the input information is uncertain, namely (ϕ, w) . The conditioning of $\pi_{\mathcal{O}_\pi}$ with (ϕ, w) is defined as follows depending on whether ϕ is consistent or not with the ontology.

Definition 7 Let \mathcal{O}_π be a prioritized \mathcal{EL} ontology and $\pi_{\mathcal{O}_\pi}$ be its associated possibility distribution. Let (ϕ, w) be the uncertain input. The min-based conditioning in prioritized \mathcal{EL} is defined as follows:

$$\forall \mathcal{I} \models \phi, \pi_{\mathcal{O}_\pi}(\cdot |_{\mathcal{m}}(\phi, w)) = \begin{cases} 1 & \text{if } \pi_{\mathcal{O}_\pi}(\mathcal{I}) = \Pi(\phi) \\ 1 - w & \text{if } N_\pi(\phi) \leq \pi_{\mathcal{O}_\pi} \leq 1 - w \\ \pi(\mathcal{I}) & \text{Otherwise} \end{cases}$$

$$\forall \mathcal{I} \not\models \phi, \pi_{\mathcal{O}_\pi}(\cdot |_{\mathcal{m}}(\phi, w)) = \begin{cases} 1 - w & \text{if } \pi_{\mathcal{O}_\pi}(\mathcal{I}) = N_\pi(\phi) \\ 1 - w & \text{if } \pi_{\mathcal{O}_\pi}(\mathcal{I}) > 1 - w \\ \pi(\mathcal{I}) & \text{Otherwise} \end{cases}$$

In Definition 7, accepting the input consists in assigning degree 1 to the most plausible model of ϕ . However, in the case where $N(\phi) > w$, some models of ϕ are forced to level $1 - w$ to ensure inferring ϕ with its prescribed certainty degree. For the countermodels of ϕ , the most plausible interpretation should have a degree equal to $1 - w$ and all interpretations that are more compatible than $1 - w$ are raised to $1 - w$ to maintain the prescribed levels.

Example 4 Considering again Example 1. Let $(\text{DeltaSymptom} \sqsubseteq \text{RespiratoryDisease}, 0.9)$ be the input. Let $\mathcal{I}_1, \mathcal{I}_2, \mathcal{I}_3$ be the three interpretations presented in Example 3. Let us consider that \mathcal{I}_1 and \mathcal{I}_2 are models of $(\text{DeltaSymptom} \sqsubseteq \text{RespiratoryDisease})$ and \mathcal{I}_3 is not a model. We have $\Pi(\text{DeltaSymptom} \sqsubseteq \text{RespiratoryDisease}) = 1$ and $N_\pi(\text{DeltaSymptom} \sqsubseteq \text{RespiratoryDisease}) = 0.12$. Then the revised possibility distribution is as follows: $\pi'(\mathcal{I}_1) = 1, \pi'(\mathcal{I}_2) = 0.1$ and $\pi'(\mathcal{I}_3) = 0.9$

The following proposition shows that the possibility distribution obtained by Definition 7 satisfies the properties **P1**, **P2** and **P3**.

Proposition 1 Let \mathcal{O}_π be a stratified \mathcal{EL} ontology and $\pi_{\mathcal{O}_\pi}$ its associated possibility distribution. Let (ϕ, w) be the uncertain information. Therefore: $\pi'_{\mathcal{O}_\pi} = \pi_{\mathcal{O}_\pi}(\cdot |_{\mathcal{m}}(\phi, w))$ obtained by the Definition 7 satisfies the logical properties **P1**, **P2** and **P3**.

5 Syntactic Evolution of \mathcal{EL} Ontology

Revision at the syntactic level consists in obtaining a new consistent ontology \mathcal{O}'_π from the original ontology \mathcal{O}_π and the new input (ϕ, w) . Based on the nature of the input, we first study revision when the input information is consistent with the ontology, and then when the new information is inconsistent. We show in particular that syntactic revision follows the semantics evolution process defined in the previous section.

5.1 Revision With Inconsistent Input

Let \mathcal{O}_π be the prioritized ontology and (ϕ, w) be inconsistent input information. One can distinguish two situations on whether (ϕ, w) is inhibited or not by higher priority axioms that contradict it. To obtain the revised ontology \mathcal{O}'_π while ensuring that (ϕ, w) is inferred with its prescribed weight, we proceed according to the following steps:

- Add the new information (ϕ, w) to the prioritized ontology \mathcal{O}_π with the highest possible priority (namely 1).
- Compute the incoherence $w_{inc} = Inc(\mathcal{O}_\pi) \cup \{(\phi, 1)\}$
- Remove all the axioms having a priority level less or equal to w_{inc} .
- Add the new information with its prescribed level to the obtained coherent ontology and adjust the weights.

These steps guarantee that the obtained ontology $\mathcal{O}'_\pi \cup (\phi, w)$ is consistent. The following proposition gives the formal expression of \mathcal{O}'_π for the Definition 7 of conditioning.

Proposition 2 Let \mathcal{O}_π be the prioritized \mathcal{EL} ontology and $\pi_{\mathcal{O}_\pi}$ its associated possibility distribution. Let (ϕ, w) be the new added information and $w_{inc} = Inc(\mathcal{O}_\pi \cup \{(\phi, 1)\})$. Then, the prioritized ontology \mathcal{O}'_π associated with $\pi_{\mathcal{O}_\pi}(\mathcal{I} |_{\mathcal{m}}(\phi, w))$ is: $\mathcal{O}'_\pi = \{(\phi, w)\} \cup \{(\phi_o, w_o) : (\phi_o, w_o) \in \mathcal{O}_\pi \text{ and } w_o > w_{inc}\}$ The associated possibility distribution obtained by min-based conditioning defined in Definition 7 is:

$$\forall \mathcal{I} \in \Omega, \pi'_{\mathcal{O}_\pi}(\mathcal{I}) = \pi_{\mathcal{O}_\pi}(\mathcal{I} |_{\mathcal{m}}(\phi, w))$$

Example 5 Considering the following prioritized \mathcal{EL} ontology $\mathcal{O}_\pi = \{(Delta \sqsubseteq Influenza, 0.4), (InfluenzaSymptom \sqsubseteq RespiratoryDisease, 0.6), (DeltaSymptom \sqcap InfluenzaSymptom \sqsubseteq \perp, 0.3)\}$. Let $(DeltaSymptom \sqsubseteq RespiratoryDisease, 0.9)$ be the input. One can check that $Inc(\mathcal{O}_\pi) = 0.4$ and the axiom $(DeltaSymptom \sqsubseteq RespiratoryDisease)$ is inferred from \mathcal{O}_π with necessity degree equal to 0.4. Then, $\mathcal{O}'_\pi = \{(DeltaSymptom \sqsubseteq InfluenzaSymptom, 0.4), (InfluenzaSymptom \sqsubseteq RespiratoryDisease, 0.6), (DeltaSymptom \sqsubseteq RespiratoryDisease, 0.9)\}$. Consider now the interpretation \mathcal{I}_1 satisfies all the axioms of the ontology, then $\pi(\mathcal{I}) = 1$. The interpretation \mathcal{I}_2 does not satisfy $(DeltaSymptom \sqsubseteq RespiratoryDisease, 0.9)$, then $\pi(\mathcal{I}_2) = 0.1$ and the interpretation \mathcal{I}_3 does not satisfy $(DeltaSymptom \sqsubseteq InfluenzaSymptom, 0.4)$, therefore $\pi(\mathcal{I}_3) = 0.6$. The possibility of the new information is $\Pi(DeltaSymptom \sqsubseteq RespiratoryDisease) = 1$ and $N_\pi(DeltaSymptom \sqsubseteq RespiratoryDisease) = 0.9$. The revised possibility distribution are $\pi'(\mathcal{I}_1) = 1$, $\pi'(\mathcal{I}_2) = 0.1$ and $\pi'(\mathcal{I}_3) = 0.6$.

In the following section, we study the syntactic revision when the input is consistent with the original ontology.

5.2 Revision With Consistent Input

Two main situations should be considered when adding a consistent input (ϕ, w) to the original ontology \mathcal{O}_π . The first one holds when the input information is inferred from the ontology with a certain degree of reliability $w \leq 1$. The second is when the input cannot be inferred from \mathcal{O}_π . The revision, in this case, is simply performed with an expansion of the original ontology with (ϕ, w) , namely $\mathcal{O}'_\pi = \mathcal{O}_\pi \cup \{(\phi, w)\}$.

When the input is inferred from the ontology (namely $\mathcal{O}_\pi \models (\phi)$), two situations hold, based on the necessity measure of ϕ (i.e., $N(\phi, w) = w_b$) and the prescribe necessity measure $N'(\phi, w) = w$. In the first situation, when $(w_b > w)$ means that the input is inferred from the prioritized ontology with necessity w_b greater than its prescribed weight w . The second is when $w_b < w$ means that the necessity degree of the inferred axiom is less than its prescribed weight. The revision process is then performed using the following steps:

- Add the assumption that ϕ is false with the highest priority level namely $(w = 1)$.
- Compute the inconsistency of the augmented ontology \mathcal{O}'_π which is equal to w_b . Now if the prescribed level w is greater than w_b , then the revision outcome is simply $\mathcal{O}'_\pi = \mathcal{O}_\pi \cup \{(\phi, w)\}$. In the other scenario, when $w_b < w$ one can either assign the degree w to the axioms having priority between w and w_b or only select the set $\mathcal{S} \in \mathcal{O}_\pi$ of axioms having a priority level between w and w_b and imply ϕ and shifted down their degrees to w . The two procedures lead to inferring ϕ with its prescribed level. But, the second one ensures a minimal change of the ontology because it only change the weights of axioms responsible for inferring ϕ .

We first introduce the formal definition of the revision process using the first procedure and then the second one.

Proposition 3 Let \mathcal{O}_π be the prioritized \mathcal{EL} ontology and $\pi'_{\mathcal{O}}$ its associated possibility distribution. Let (ϕ, w) be the uncertain input. Let \mathcal{O}'_π be the augmented ontology by the assumption that ϕ false. The degree of inconsistency of \mathcal{O}'_π is $w_{inc} = Inc(\mathcal{O}'_\pi)$. The revised $\mathcal{O}'_\pi = \{(\phi, w) \cup \{(\phi_o, w_o) : (\phi_o, w_o) \in \mathcal{O}_\pi \text{ and } w > w_{inc}\} \cup \{(\phi_o, w_o) : (\phi, w) \in \mathcal{O}_\pi \text{ and } w_o < w\} \cup \{(\phi_o, w_o) : (\phi_o, w_o) \in \mathcal{O}_\pi \text{ and } w \leq w_o \leq w_{inc}\}$ and its associated possibility distribution is: $\forall \mathcal{I} \in \Omega, \pi_{\mathcal{O}'_\pi}(\mathcal{I}) = \pi_{\mathcal{O}_\pi}(\mathcal{I}|_m(\phi, w))$ given by Definition 7.

Proposition 3 ensures that the degrees of axioms in \mathcal{O}_π between w and w_{inc} should be minimised to w . However, we can improve the results using the second case, i.e., identifying the set of axioms \mathcal{S} in \mathcal{O}_π that implies ϕ . There exist semantically four sets of interpretations when the input is satisfied:

- $\mathcal{I} \models \mathcal{S}$ and $\mathcal{I} \models \mathcal{O}_\pi \setminus \mathcal{S}$
- $\mathcal{I} \models \mathcal{S}$ and $\mathcal{I} \not\models \mathcal{O}_\pi \setminus \mathcal{S}$
- $\mathcal{I} \not\models \mathcal{S}$ and $\mathcal{I} \models \mathcal{O}_\pi \setminus \mathcal{S}$
- $\mathcal{I} \not\models \mathcal{S}$ and $\mathcal{I} \not\models \mathcal{O}_\pi \setminus \mathcal{S}$

Based on these observations, the following definition provides min-based conditioning of prioritized \mathcal{EL} possibility distribution that improves Definition 7.

Definition 8 Let \mathcal{O}_π be the prioritized \mathcal{EL} ontology and $\pi'_{\mathcal{O}}$ its associated possibility distribution. Let (ϕ, w) be the input information. Let $\mathcal{S} \subseteq \mathcal{O}_\pi$ be the set of axioms that inferred ϕ . Let $w' = \max\{w_o : (\phi_o, w_o) \in \mathcal{O}_\pi \setminus \mathcal{S} \text{ and } \mathcal{I} \not\models \phi\}$. The min-based conditioning is defined as follows:

- $\forall \mathcal{I} \models (\phi \cup \mathcal{S}), \pi(\cdot|_m(\phi, w)) = \begin{cases} 1 & \text{if } \pi(\mathcal{I}) = \Pi(\phi) \\ \pi(\mathcal{I}) & \text{Otherwise} \end{cases}$
- $\forall \mathcal{I} \models \phi \cup (\mathcal{O}_\pi \setminus \mathcal{S}), \mathcal{I} \not\models \mathcal{S},$
 $\pi(\cdot|_m(\phi, w)) = \begin{cases} 1 - w & \text{if } \pi(\mathcal{I}) = N(\phi) \\ \pi(\mathcal{I}) & \text{Otherwise} \end{cases}$
- $\forall \mathcal{I} \models \phi, \mathcal{I} \not\models \mathcal{S}, \mathcal{I} \not\models \mathcal{O}_\pi \setminus \mathcal{S},$
 $\pi(\cdot|_m(\phi, w)) = \begin{cases} 1 - w & \text{if } \pi(\mathcal{I}) = N(\phi) \text{ and } 1 - w' \geq 1 - w \\ 1 - w' & \text{if } \pi(\mathcal{I}) = N(\phi) \text{ and } 1 - w' \leq 1 - w \\ \pi(\mathcal{I}) & \text{Otherwise} \end{cases}$
- $\forall \mathcal{I} \not\models \phi, \pi(\cdot|_m(\phi, w)) = \begin{cases} 1 - w & \text{if } \pi(\mathcal{I}) = N(\phi) \\ 1 - w' & \text{if } \pi(\mathcal{I}) > 1 - w \\ \pi(\mathcal{I}) & \text{Otherwise} \end{cases}$

Interestingly enough, Definition 8 improves Definition 7 while ensuring the same logical properties.

Proposition 4 Let \mathcal{O}_π be a prioritized \mathcal{EL} ontology and $\pi_{\mathcal{O}_\pi}$ its associated possibility distribution. Let (ϕ, w) be the input. Therefore: $\pi_{\mathcal{O}'_\pi} = \pi_{\mathcal{O}_\pi}(\cdot|_m(\phi, w))$ obtained by the Definition 8 satisfies the logical properties **P1**, **P2** and **P3**

Proposition 5 Let \mathcal{O}_π be the prioritized \mathcal{EL} ontology and $\pi_{\mathcal{O}_\pi}$ be its associated possibility distribution. Let (ϕ, w) be the uncertain input. Considering that \mathcal{O}_1 be the augmented possibility ontology obtained by add the assumption ϕ false to \mathcal{O}_π . Let $w_{inc} = Inc(\mathcal{O}_1)$. The revised prioritized \mathcal{EL}

ontology, denoted \mathcal{O}'_π is defined as: $\mathcal{O}'_\pi = \{(\phi, w)\} \cup \{\mathcal{O}_\pi \setminus \mathcal{S}\} \cup \{(\phi_o, w_o) : (\phi_o, w_o) \in \mathcal{S} \text{ and } w_o > w_{inc}\} \cup \{(\phi, w) : (\phi, w_{inc}) \in \mathcal{S} \text{ and } w_{inc} = w_o\}$.

The associated possibility distribution ($\pi'_{\mathcal{O}'_\pi}$) obtained by the min-based conditioning defined in Definition 8 is as follows:

$$\forall \mathcal{I} \in \Omega, \pi'_{\mathcal{O}'_\pi}(\mathcal{I}) = \pi_{\mathcal{O}_\pi}(\mathcal{I}|_m(\phi, w))$$

6 Conclusion

This paper investigates the evolution of \mathcal{EL} ontologies when a new piece of information that can be conflicting or attached with a confidence level reflecting its credibility or priority is available. To encode such knowledge, we propose an extension of \mathcal{EL} description logic within the possibility theory, which provides a natural way to deal with ordinal scale reflecting ranking between pieces of knowledge. We then show how such a ranking between axioms is induced from the ontology with the presence of new information and study the evolution process at the semantic level. Finally, we propose a polynomial syntactic counterpart of the evolution process while preserving the consistency of the ontology. In future work, we plan to consider the product-based conditioning of prioritized \mathcal{EL} possibility distribution.

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