Qualitative Choice Logic For Modeling Experts Recommendations of Antibiotics

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Abstract

Qualitative Choice Logic (QCL) is a logic-based formalism for preference handling. The logic adds to classical propositional logic a new connective called ordered disjunction (\( \bar{\vee} \)). \( x \bar{\vee} y \) intuitively means: if possible \( x \), but if \( x \) is not possible then at least \( y \). The aim of this paper is to explore QCL in healthcare domain, particularly for modeling experts reasoning for providing recommendations of antibiotics. We show that QCL is fully adapted for modeling this problem.

Qualitative Choice Logic, Preference learning, Antibiotic prescriptions

Introduction

Clinical Practice Guidelines (CPGs) aim at improving the quality of health care by providing best practices for diagnosis and treatments. However, physicians have difficulties to understand the (often implicit) expert recommendations. In the domain of antibiotic prescription, depending on the clinical situation and the properties of antibiotics (e.g. side effects), these later can be recommended or not for prescription. Each antibiotic has a rank of recommendation indicating its degree of preference for prescription. Namely, recommended antibiotics can be prescribed in \( 1^{st} \), \( 2^{nd} \), \( 3^{rd} \) or \( 4^{th} \) line of treatment (Tsopra, Lamy, and Sedki 2018). Recommended antibiotics in \( 1^{st} \) line of treatment (resp. \( 2^{nd} \), \( 3^{rd} \) or \( 4^{th} \)) have a rank of recommendation 1 (resp. 2, 3, 4). Not recommended antibiotics have a rank 0. The higher this rank, the less preferable the antibiotic. Antibiotics having the rank 0 can not be prescribed.

Starting from a data base that contains a set of antibiotics, their features and their ranks of recommendation, our objectif is to learn a model that represents the closest possible experts reasoning and strategies to provide recommendations of antibiotics. Namely, the model allows to predict the correct rank of recommendation for each new antibiotic. The idea of preference learning (Fürnkranz and Hüllermeier 2010) is to learn and construct a preference model from observed preference information. Methods of preference learning can be quantitative (Burges et al. 2005; Tsopra, Lamy, and Sedki 2018) or qualitative (Jiang et al. 2008; de Amo et al. 2015).

Notations

Let \( V \) be a finite set of propositional variables. An interpretation \( I \) is defined as a set of propositional variables such that \( v \in I \) if and only if \( v \) is set to true by \( I \). If \( I \) satisfies a formula \( \phi \), we write \( I \models \phi \), otherwise, we write \( I \not\models \phi \). A model of a formula \( \phi \) is an interpretation \( I \) that satisfies the formula. \( \text{var}(\phi) \) denotes the variables of a formula \( \phi \). \(|.|\) denotes the cardinality of a set.

Antibiotic data base

We used a data base containing the antibiotics, their features, and their rank of recommendations as defined in CPGs. This data base was validated by antibiotic experts according to a Delphi process (Tsopra, Lamy, and Sedki 2018). The base contains recommended and not recommended antibiotics. Among recommended antibiotics, there are those that are preferred to others regarding their rank of recommendation. In fact, there are 5 recommendation ranks in the data base: \( R=\{1, 2, 3, 4, 0\} \). The higher this rank, the less preferable the antibiotic. So, Antibiotics having rank 1 are preferred to those having rank 2 and so on. Antibiotics having rank 0...
are not preferred since they are not recommended. Note that there are some clinical situations where their greatest rank of recommendation can be equal to 3 (such as the case given in Experimental results section). In (Tsopra, Lamy, and Sedki 2018), it is showed that two categories of features exist:

1. Necessary features are mandatory for prescribing the antibiotic. These features ensure that an antibiotic is both safe for the patient, and able to cure the infection. We define $V_{ness}$, the set of necessary features.

2. Preference features indicate which antibiotic is preferred to another, in a given clinical situation. These features make it possible to choose one antibiotic through a list of antibiotics that could be prescribed to cure a patient. We define $V_{pref}$, the set of preference features.

For example, the absence of contraindications may be a necessary feature while the low rate of adverse effects may be a preference feature. Table 1 gives the list of necessary and preference features.

Table 1: Necessary and preference features (description of features can be found in (Tsopra, Lamy, and Sedki 2018)).

| $V_{ness}$ | naturallyActive, probablyActive, proved, noContraindication |
| $V_{pref}$ | protocol, not precious, side effects, efficacy level, cost, eco risk, taste |

Definition 1. An antibiotic $a$ is recommended if it satisfies both experts knowledge on necessary features and preference features, we have $R(a) \neq 0$. Otherwise, $a$ is not recommended and we have $R(a) = 0$.

An example of expert knowledge is: an antibiotic should have no contraindications and it should be proved.

Example 1. Table 2 gives a simplified example where $V_{ness} = \{v_1, v_2\}$, $V_{pref} = \{v_3, v_4, v_5\}$. $a_1$, $a_2$, $a_3$, $a_5$, $a_6$, $a_7$ are recommended antibiotics while $a_4$ is not recommended.

Table 2: Simple example with 7 antibiotics and 5 features.

<table>
<thead>
<tr>
<th>Antibiotics</th>
<th>$v_1$</th>
<th>$v_2$</th>
<th>$v_3$</th>
<th>$v_4$</th>
<th>$v_5$</th>
<th>$R$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_1$</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$a_2$</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>$a_3$</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$a_4$</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$a_5$</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>$a_6$</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>$a_7$</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>2</td>
</tr>
</tbody>
</table>

Our aim is to learn a qualitative model that allows to assign a correct rank of recommendation of each antibiotic on the basis of the necessary and preference features.

**Qualitative Choice Logic**

*Qualitative Choice Logic* (Brewka, Benferhat, and Le Berre 2004) extends classical logic to represent preferences. It uses two types of connectives: classical connectives ($\land$, $\lor$ and $\neg$) and a new connective $\times$, called ordered disjunction, used to express preferences. Formulas that compose the language of $QCL$ are:

- Basic choice formulas ($BCF$) are ordered disjunctions of classical formulas.
- General choice formulas ($GCF$) are used to express complex preferences and can be obtained from $V$ using connectives $\times$, $\land$, $\lor$, $\neg$.

Example 2. $\phi_1 = v_1 \lor v_2$ is a classical formula. $\phi_2 = v_1 \times v_2 \times v_3$ is a $BCF$ formula that contains three options: preferably $v_1$, but if this is not possible then $v_2$, if this is not possible, then $v_3$. $\phi_3 = (v_1 \times v_2) \land (v_3 \times v_4)$ is a $GCF$ formula.

**Semantic and syntax of $QCL$**

The semantic of a $QCL$ formula is based on two principal functions, satisfaction degree and optionality. The satisfaction degree of a formula given an interpretation is a natural number. The higher this degree, the less preferable the interpretation. The optionality of a formula is a function that assigns to each formula a strictly positive integer. It corresponds to the maximum finite satisfaction degree that this formula can have.

**Definition 2.** The optionality is defined as follows:

1. $\text{opt}(v)=1$, for every $v$ in $V$.
2. $\text{opt}(\phi \times \psi) = \text{opt}(\phi) + \text{opt}(\psi)$.
3. $\text{opt}(\phi \land \psi) = \max(\text{opt}(\phi), \text{opt}(\psi))$.
4. $\text{opt}(\phi \lor \psi) = \max(\text{opt}(\phi), \text{opt}(\psi))$.
5. $\text{opt}(\neg(\phi)) = 1$.

Example 3. From Example 2, we have $\text{opt}(\phi_1)=1$, $\text{opt}(\phi_2)=3$ (from Item 1 and 2 of Definition 2), this means that there exists at least an interpretation which satisfies $\phi_2$ with degree $\leq 3$, but there is no interpretation which satisfies $\phi_2$ with degree $>3$, $\text{opt}(\phi_3)=2$ (from Items 1, 2 and 3 of Definition 2).

In the following, we define the degree of satisfaction of classical and $BCF$ formulas (more details for the inference of $GCF$ formulas are given in (Brewka, Benferhat, and Le Berre 2004)).

**Definition 3.** The satisfaction degree of a formula $\phi$ under an interpretation $I$, denoted by $\text{deg}(I, \phi)$ is defined as follows:

1. Let $\phi$ be a classical formula, $\text{deg}(I, \phi) = 1$ iff $I \models \phi$.
2. Let $\psi = v_1 \times v_2 \times \ldots \times v_n$, $\text{deg}(I, \psi) = k$ iff $I \models v_1 \lor v_2 \lor \ldots \lor v_n$ and $k = \min\{j | I \models v_j\}$. We write $I \models_k \phi$.

And, $\text{deg}(I, \psi) = 0$ iff there is no $k$ such that $I \models v_k$.

For any classical formula $\phi$, its degree is 1 if $\phi$ is satisfied by $I$ and 0 otherwise. Item 2 of Definition 3 states that given a formula $\psi = v_1 \times v_2 \times \ldots \times v_n$, an interpretation $I$ satisfies $\psi$ to a degree $k$, if it satisfies the $k^{th}$ option of $\psi$ (namely $v_k$) and falsifies the first $(k-1)^{th}$ options of $\psi$. Note that an interpretation is a $QCL$ model if it satisfies each formula of a given $QCL$ theory.
Learning the QCL model from the antibiotic data base

In this section, we show that QCL is fully suited for modeling the problem of antibiotics prescription.

**Definition 4.** Antibiotic prescription can be modeled using QCL where:
- each antibiotic is an interpretation,
- the rank of recommendation of each antibiotic corresponds to the satisfaction degree assigned to an interpretation by a QCL formula,
- a recommended antibiotic is considered as a QCL model.

Each antibiotic is recommended if it satisfies both expert knowledge on necessary features and preference features. From QCL point of view, this means that each antibiotic has two ranks of recommendation. The first one indicates if the antibiotic satisfies expert knowledge on necessary features and the second one indicates the preference degree of the antibiotic. Thus, the problem of antibiotic prescription is completed with the following property.

**Property 1.** Let \( D \) be the set of antibiotics, \( a \in D \) then:
- there is a rank of recommendation regarding \( V_{ness} \), denoted by \( R_{ness} \) s.t. \( R_{ness}(a) = 1 \) if \( a \) is recommended and \( R_{ness}(a) = 0 \) if \( a \) is not recommended.
- there is a rank of recommendation regarding \( V_{pref} \), denoted by \( R_{pref} \) s.t. \( R_{pref}(a) = R(a) \neq 0 \) if \( a \) is recommended and \( R_{pref}(a) = 0 \) if \( a \) is not recommended.

More precisely, an antibiotic \( a \) is recommended iff \( R_{ness}(a) = 1 \) and \( R_{pref}(a) \neq 0 \). \( a \) is not recommended otherwise.

**Example 4.** From data of Table 2, we have \( V_{ness} = \{ v_1, v_2 \} \), \( V_{pref} = \{ v_3, v_4, v_5 \} \). \( R_{ness} = \{ 1, 0 \} \), \( R_{pref} = \{ 1, 2, 3, 0 \} \). \( a_1, a_2, a_3, a_5, a_6, a_7 \) are recommended antibiotics while \( a_4 \) is not recommended.

**Proposition 1.** Let \( D \) be the set of antibiotics, \( a \in D \) then:
- There exists a classical formula \( \phi \) with \( \text{var} (\phi) \subseteq V_{ness} \), \( \text{deg}(a, \phi) = R_{ness}(a) = 1 \) if \( a \) is recommended and \( \text{deg}(a, \phi) = R_{ness}(a) = 0 \) if \( a \) is not recommended.
- There exists a formula \( \psi \) with \( \text{var}(\psi) \subseteq V_{pref} \), \( \text{deg}(a, \psi) = R_{pref}(a) \neq 0 \) if \( a \) is recommended and \( \text{deg}(a, \psi) = R_{pref}(a) = 0 \) if \( a \) is not recommended.

**Proof.** A given antibiotic \( a \) is evaluated on the basis of necessary features and preference features. From Property 1, \( a \) has two ranks of recommendation, \( R_{ness} \) and \( R_{pref} \). From Definition 4, \( a \) corresponds to a QCL interpretation and the rank of recommendation corresponds to the satisfaction degree. So, \( a \) has two satisfaction degrees. From QCL point of view, there is a QCL theory that contains two formulas \( \phi \) and \( \psi \) with \( \text{deg}(a, \phi)=R_{ness}(a) \) and \( \text{deg}(a, \psi) = R_{pref}(a) \).

If \( a \) is recommended then following Definition 1, \( a \) satisfies both experts knowledge on necessary features and preference features. So, \( R_{ness}(a)=1 \) and \( R_{pref}(a) \neq 0 \) (Property 1). From Definition 4, \( a \) corresponds to a QCL model, so \( \text{deg}(a, \phi)=1 \) and \( \text{deg}(a, \psi) \neq 0 \). Thus, \( \text{deg}(a, \phi)=R_{ness}(a)=1 \) and \( \text{deg}(a, \psi) = R_{pref}(a) \neq 0 \).

- If \( a \) is not recommended then \( a \) is not a QCL model since \( a \) is not recommended. Thus, we have either \( \text{deg}(a, \phi) = 0 \) or \( \text{deg}(a, \psi) = 0 \). However, \( R_{ness}(a) = 0 \) and \( R_{pref}(a) = 0 \) since \( a \) is not recommended, thus, \( \text{deg}(a, \phi) = 0 \) and \( \text{deg}(a, \psi) = 0 \).

From the antibiotics data base, we aim to learn a model \( \mathcal{M}_{QCL} \) defined as follows:

**Definition 5.** Let \( D \) be the antibiotics data base, \( a \in D \) then there is a QCL model, denoted by \( \mathcal{M}_{QCL} \) that can be learned s.t.
\[
\mathcal{M}_{QCL} = \left\{ \phi \text{ with } \text{var}(\phi) \subseteq V_{ness}, \psi \text{ with } \text{var}(\psi) \subseteq V_{pref} \right\}
\]

The learned model contains a classical formula defined on the necessary features and a BCF formula defined on the preference features (here we are interested only to learn a BCF formula but it is possible to learn a GCF formula since in (Brewka, Benferhat, and Le Berre 2004), it is shown that each GCF formula has its equivalent BCF formula). According to the semantic of QCL, the learned model satisfies the following property:

**Property 2.** Let \( \mathcal{M}_{QCL} \) be the learned model from the antibiotics data base \( D \), \( \phi, \psi \in \mathcal{M}_{QCL} \), then
- \( \text{opt}(\phi)=\max(R_{ness})=1 \) with \( \text{var}(\phi) \subseteq V_{ness} \).
- \( \text{opt}(\psi)=\max(R_{pref}) \) with \( \text{var}(\psi) \subseteq V_{pref} \).

**Proof.** It follows immediately from Definition 2 where it is stated that the optionality of a classical formula is equal to 1 and the optionality of a BCF formula is equal to the greatest degree of satisfaction. From Definition 3, the rank of recommendation of an antibiotic corresponds to the satisfaction degree, thus the optionality of the learned BCF formula is equal to the greatest rank of recommendation in \( D \).

**Example 5.** Data of Table 2 are modeled with QCL in Table 3. From Table 3, we aim to learn the QCL model s.t.

<table>
<thead>
<tr>
<th>Antibiotics</th>
<th>( v_1 )</th>
<th>( v_2 )</th>
<th>( v_3 )</th>
<th>( v_4 )</th>
<th>( v_5 )</th>
<th>( \text{a classical formula } \phi )</th>
<th>( \text{a BCF formula } \psi )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a_1 )</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>( a_2 )</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>( a_3 )</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>( a_4 )</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( a_5 )</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>( a_6 )</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>( a_7 )</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>2</td>
</tr>
</tbody>
</table>

\[
\mathcal{M}_{QCL} = \left\{ \phi \text{ with } \text{var}(\phi) \subseteq V_{ness} = \{v_1, v_2\}, \psi = x_1 \times x_2 \times x_3 \text{ with } \text{var}(\psi) \subseteq V_{pref} = \{v_3, v_4, v_5\} \right\}
\]

In this example, \( R_{pref} = \{0, 1, 2, 3\} \), so \( \text{opt}(\psi) = \max(R_{pref}) = 3 \). Thus, we aim to learn a formula \( \psi = x_1 \times x_2 \times x_3 \).
Let us first give the following definition:

**Definition 6.** • Each single option is built on $V$ and the connective $\bot$.
• Each conjunctive option is built on $V$ and the connectives $\{\land, \bot\}$. The set of conjunctive options is denoted by $X_{\text{conj}}$.

$v_1, \neg v_1$ are examples of single options. $v_1 \land \neg v_2$ is an example of conjunctive option.

In the following sections, we detail the method for learning $BCF$ and classical formulas but for lack of space, we limited to single and conjunctive options. Namely, classical formulas are learned on $V_{\text{next}}$ and the connectives $\{\neg, \land, \bot\}$ and $BCF$ formulas are learned on $V_{\text{pref}}$ and the connectives $\{\neg, \land\}$.

**Learning BCF formulas**

The learned $BCF$ formula is in the form of $\psi = x_1 \times x_2 \times x_3 \times x_4$ where $x_i$ is a single or a conjunctive option defined on $V_{\text{pref}}$ (recall that the $BCF$ formula contains 4 options since the greatest rank of recommendation in the antibiotic database is equal to 4). We proposed a method (Algorithm 1) for generating best options of $\psi$. Our method is inspired from Apriori algorithm (Agrawal, Imielinski, and Swami 1993) for generating frequent itemsets. Instead of generating all possible options which can be very large ($2^7 - 1$ possible options that can be generated on $V_{\text{pref}}$), we generate only frequent ones which correspond to those exceeding a minimal fixed support and confidence. Following the semantics of $QCL$, the idea is that all antibiotics with rank $R_{\text{pref}} = 1$ should satisfy the option $x_1$ (even if $x_2, x_3$ and $x_4$ are satisfied or not) and all antibiotics with rank $R_{\text{pref}} = 2$ (resp. $R_{\text{pref}} = 3$, $R_{\text{pref}} = 4$, $R_{\text{pref}} = 0$) should not satisfy the option $x_1$. So, the option $x_1$ of the learned formula should be the option that is satisfied by the maximum number of antibiotics having the rank $R_{\text{pref}} = 1$ (ideally all antibiotics), and falsified by the maximum number of antibiotics with ranks 4, 3, 2 and 0 (ideally all). The option $x_2$ of the learned formula should be the option that is satisfied by the maximum number of antibiotics having the rank $R_{\text{pref}} = 2$ and falsified by the maximum number of antibiotics with ranks 4, 3 and 0. The same reasoning for learning the best options $x_3$ and $x_4$. Thus, our definition of support and confidence are adapted to our problem as follows.

**Definition 7** (Support). Let $R_{\text{pref}} = \{R_{\text{pref}} = 1, R_{\text{pref}} = 2, \ldots R_{\text{pref}} = 4, R_{\text{pref}} = 0\}$. The support of an option $x$ for antibiotics $a \in D$ having a recommendation rank $R_{\text{pref}}(a) = R_{\text{pref}}$ is defined as:

$$\text{Supp}(x, R_{\text{pref}}) = \frac{|\{a \mid a \in D \land x \land R_{\text{pref}}(a) = R_{\text{pref}}\}|}{|\{a \mid x \land R_{\text{pref}}(a) = R_{\text{pref}}\}|}$$

The support of an option $x$ for a rank $R_{\text{pref}}$ is defined by the fraction of the number of antibiotics $a \in D$ having a rank $R_{\text{pref}}(a) = R_{\text{pref}}$ that satisfy $x$ on the number of antibiotics having rank $R_{\text{pref}}$. Its interest increases with its support (ideally 1).

Even if the support of $x$ is high for $R_{\text{pref}}$, it is possible that its support for $R_{\text{pref}} \neq R_{\text{pref}}$ is also high, so there is need to compute its confidence.

**Definition 8** (Confidence). The confidence of an option $x$ for antibiotics $a \in D$ having a rank $R_{\text{pref}}(a) = R_{\text{pref}}$ is defined as:

$$\text{Conf}(x, R_{\text{pref}}) = \frac{|\{a \mid a \in D \land x \land R_{\text{pref}}(a) = R_{\text{pref}}\}|}{|\{a \mid x \land R_{\text{pref}}(a) = R_{\text{pref}}\}|}$$

The confidence of an option $x$ for rank $R_{\text{pref}}$ indicates the proportion of antibiotics that keep their initial rank if $x$ appears in the learned formulas. An option is interesting for a rank $R_{\text{pref}}$ if its confidence is high (ideally 1).

**Example 6.** Let us consider data of Table 2 where $V_{\text{pref}} = \{v_3, v_4, v_5\}$. Given the option $v_3 \land v_4$, we have $\text{Supp}(v_3 \land v_4, R_{\text{pref}} = 1/2$ and $\text{Conf}(v_3 \land v_4, R_{\text{pref}} = 0/7)$ which means that this option is not the best for $x_1$ of the $BCF$ formula that we aim to learn.

**Generating best options on $V_{\text{pref}}**

For generating best options, we adapt the approach of association rules (Agrawal, Imielinski, and Swami 1993). The idea is to start with all single options, count their support and find all single frequent options, combine them to form candidate 2-conjunctive options, go through data and count their support and find all frequent 2-conjunctive options, combine them to form candidate 3-conjunctive options and so on. Once frequent options are generated for each rank $R_{\text{pref}} \neq 0$, we return only those exceeding a minimal confidence $\theta$, called best options ($\text{Best}_{R_{\text{pref}}}$). Algorithm 1 summarizes these steps. Note that it is not necessary to generate best options for rank 0 since these later do not appear in the learned $BCF$ formula. However, antibiotics with $R_{\text{pref}} = 0$ are considered for computing the support and confidence of any option. Depending on the minimal support and confidence, it is possible to have more than one best option for a given rank and then more than one $BCF$ formula and $QCL$ model. However, we evaluate the quality of each learned model by computing its accuracy (Definition 12).

**Example 7.** Let us consider data of Table 3 where $V_{\text{pref}} = \{v_3, v_4, v_5\}$. Table 4 gives frequent single and conjunctive options for $R_{\text{pref}} = 1$ exceeding minimal support $\sigma = 0.7$. In Table 4, we have $C_1 = \{v_3, \neg v_3, v_4, \neg v_4, v_5, \neg v_5\}$. On the basis of the fixed minimal support, from
Algorithm 1 Best options defined on V_{pref}

Input: The antibiotic data base D, R_{pref1}={R_{pref1}=1, R_{pref2}=2, \ldots, R_{pref_{f2}}=1, R_{pref_{f2}}=0}, a minimal support \sigma and a minimal confidence \theta

Output: Best options for each rank R_{pref1}=1, \ldots, 4 with support and confidence exceeding \sigma and \theta (Best_{R_{pref1}=1, \ldots, 4}).

C_j: Candidate options of size j from V_{pref}.
F_j: Frequent options of size j.
F_i: Frequent single options.
F_{R_{pref1}}: Frequent options for rank R_{pref1}=1, \ldots, 4.

for each R_{pref1}=1, \ldots, 4 do
  for j = 1; j \neq 0; j + + do
    C_{j+1}: candidate options generated from F_j
    for each antibiotic a having R_{i} do
      Compute Supp(x, R_{pref1})
    end
    F_{j+1} = \{x \in C_{j+1} | Supp(x, R_{pref1}) \geq \sigma\}
  end
end

Best_{R_{pref1}} = \bigcup F_j

for each x \in F_{R_{pref1}} do
  if Conf(x, R_{pref1}) \geq \theta then
    Best_{R_{pref1}} = \text{Best}_{R_{pref1}} \cup x
  end
end

Return Best_{R_{pref1}}

Learning the classical formula

To learn the best classical formula, we first define the support and confidence of each option on V_{ness} for all antibiotics with rank R_{ness} = 1.

Definition 9. The support of an option x for antibiotics having R_{ness}(a) = 1 is defined as:

\[ \text{Supp}(x) = |\{a \mid a \models x \land R_{ness}(a) = 1\}| \]

Definition 10. The confidence of an option x for antibiotics having a rank R_{ness}(a) = 1 is defined as:

\[ \text{Conf}(x, a) = |\{a \mid a \models x \land R_{ness}(a) = 1\}| \]

As said above, here we consider that a classical formula is a single or a conjunctive option defined on V_{ness}, so to learn a best formula, we adapt Algorithm 1 for this purpose (see Algorithm 2).

Example 8. Let us consider data of Table 3 where V_{ness} = \{v_1, v_2\}. Table 3 gives frequent options for rank R_{ness} = 1 exceeding minimal support \sigma = 0.5 and minimal confidence \theta = 0.9. From results of Table 3, the best classical formula is: \phi = v_2. This means that each antibiotic having v_2 false is not recommended and each antibiotic a having v_2 true, it can be recommended if the rank inferred by the learned BCF formula \psi is different to 0.

Algorithm 2 Best classical formula defined on V_{ness}

Input: The database D, R_{ness} = \{1, 0\}, a minimal support \sigma and a minimal confidence \theta.

Output: Best classical formula with support and confidence exceeding \sigma and \theta (Best_{formula}).

C_j: Candidate options of size j from V_{ness}.
F_j: Frequent options of size j.
F_i: Frequent single options.
F_{R_{ness}}: Frequent options for rank R_{ness} = 1.

for j = 1; j \neq 0; j + + do
  C_{j+1}: candidate options generated from F_j
  for each antibiotic a having R_{ness} = 1 do
    for each x \in C_{j+1} do
      Compute Supp(x, 1)
    end
  end
  F_{j+1} = \{x \in C_{j+1} | Supp(x, 1) \geq \sigma\}
end

Best_{formula} = \bigcup F_j
for each x \in F_{R_{ness}} do
  if Conf(x, R_{ness} = 1) \geq \theta then
    Best_{formula} = Best_{formula} \cup x
  end
end

Return Best_{formula}

Table 5: Best classical formulas.

<table>
<thead>
<tr>
<th>\text{Best}<em>{R</em>{ness}=1}</th>
<th>\text{Best}<em>{R</em>{ness}=1}</th>
</tr>
</thead>
<tbody>
<tr>
<td>v_1</td>
<td>v_1</td>
</tr>
<tr>
<td>v_2</td>
<td>v_2</td>
</tr>
<tr>
<td>v_1 \land v_2</td>
<td>v_1 \land v_2</td>
</tr>
</tbody>
</table>

In order to evaluate the quality of the learned model, there is need to verify the rank of each antibiotic inferred by each formula.

Definition 11. Let D be the antibiotic database, a \in D, M_{QCL} be the learned model from D, \phi be a classical formula and \psi = x_1 \land x_2 \land x_3 \land x_4 be a BCF formula s.t. \phi, \psi \in M_{QCL}. The rank of recommendation inferred by M_{QCL} for a is R(M_{QCL}, a) = k iff:

- a \models \phi and,
- a \models x_1 \lor x_2 \lor x_3 \lor x_4 and k = min(j \mid a \models x_j)

R(M_{QCL}, a) = 0 iff a \not\models \phi.

Definition 11 states that given a learned model M_{QCL} that contains a BCF formula \psi = x_1 \land x_2 \land x_3 \land x_4 and an classical formula \phi, the rank of a is equal to k, if a satisfies
Table 6: The learned preference model $M_{QCL}$ in pharyngitis. Note that in this case, the greatest recommendation rank is equal to 3. So, the length of the obtained BCF formulas is equal to 3.

<table>
<thead>
<tr>
<th>Learned preference models</th>
<th>Accuracy</th>
</tr>
</thead>
<tbody>
<tr>
<td>naturallyActive ∧ probablyActive ∧ proved ∧ noContraindication</td>
<td>1</td>
</tr>
<tr>
<td>(Proto ∧ Precious ∧ ¬SideEff ∧ Efficacy ∧ Spect ∧ ¬RiskResi) × (Proto ∧ ¬SideEff) × (Proto ∧ Efficacy)</td>
<td>1</td>
</tr>
<tr>
<td>naturallyActive ∧ probablyActive ∧ proved ∧ noContraindication</td>
<td>1</td>
</tr>
<tr>
<td>(Proto ∧ ¬SideEff ∧ Efficacy ∧ Spect ∧ ¬RiskResi) × (Proto ∧ ¬SideEff) × (Proto ∧ Efficacy)</td>
<td>1</td>
</tr>
<tr>
<td>naturallyActive ∧ probablyActive ∧ proved ∧ noContraindication</td>
<td>1</td>
</tr>
<tr>
<td>(Proto ∧ Efficacy ∧ Spect ∧ ¬RiskResi) × (Proto ∧ ¬SideEff) × (Proto ∧ Efficacy).</td>
<td>1</td>
</tr>
<tr>
<td>naturallyActive ∧ probablyActive ∧ proved ∧ noContraindication</td>
<td>1</td>
</tr>
<tr>
<td>(Proto ∧ Spect ∧ ¬RiskResi) × (Proto ∧ ¬SideEff) × (Proto ∧ Efficacy).</td>
<td>1</td>
</tr>
</tbody>
</table>

\(\phi\) and the \(k\)th option of \(\psi\) (i.e. \(x_k\)) is true and the preceding ones \((x_1, x_2, ..., x_{k-1})\) are false. If \(\phi\) does not satisfy \(\phi\) then the rank of \(a\) is equal to 0 (even if \(a\) satisfies \(\psi\) or not).

We introduce the accuracy measure as follows:

**Definition 12 (Accuracy).** Given a database \(D\) and a learned model \(M_{QCL}\). The accuracy of \(M_{QCL}\) is:

\[
\text{Accuracy}(M_{QCL}) = \frac{|\{a \in D \mid R(M_{QCL}, a) = R(a)\}|}{|D|}
\]

**Case study**

This section presents experimental results of our method in the domain of antibiotic prescription. Examples of the learned models for pharyngitis are given in Table 6. Note that more than one \(M_{QCL}\) model are learned because there are many best options with the same support and confidence for each rank. In addition, each learned model contains the same classical formula which is equal to \(\text{naturallyActive} \land \text{probablyActive} \land \text{proved} \land \text{noContraindication}\). This means that if an antibiotic does not satisfy this formula, it should not be prescribed and thus it is not recommended.

Examples of best options for \(R_{pref} = 1\) are (\(\text{Proto} \land \text{Precious} \land \neg \text{SideEff} \land \text{Efficacy} \land \text{Spect}\)), (\(\text{Proto} \land \neg \text{RiskResi}\)) and (\(\text{Proto} \land \text{Spect}\)) with \(\sigma = 1\) and \(\theta = 1\). Best options for \(R_2 = 2\) is (\(\text{Proto} \land \neg \text{SideEff}\)) with \(\sigma = 1\) and \(\theta = 0.7\). Best options for \(R_3 = 3\) are (\(\text{Proto} \land \text{Efficacy}\)), (\(\text{Proto} \lor \text{Spect}\)) and \(\text{Proto}\) with \(\sigma = 1\) and \(\theta = 0.35\). These results highlight that experts prefer to recommend (i) in rank 1 the antibiotics having short duration protocol (\(\text{Proto}\)) and narrow spectrum (\(\text{Spect}\)) with either [low risk of ecological adverse effects (\(\neg \text{RiskResi}\)), non-precious class (\(\text{Precious}\)), low risk of side effects (\(\neg \text{SideEff}\)), high efficacy (\(\text{Efficacy}\))], or the antibiotics having \(\text{Proto}\) and \(\neg \text{RiskResi}\) with either (\(\text{Spect}, \text{Precious}, \neg \text{SideEff}, \text{Efficacy}\)), (ii) in rank 2, the antibiotics having short duration protocol (\(\text{Proto}\)) and low risk of side effects (\(\neg \text{SideEff}\)), (iii) in rank 3 the antibiotics with \(\text{Proto}\) alone or with [\(\text{Efficacy}\) or good taste (\(\text{Taste}\))]. Finally, each antibiotic that does not satisfy the classical formula has rank 0 (not recommended). In this example, we detected two antibiotics with the same attributes but with two different rank of recommendation, so we retrieve them. The higher accuracy is 1 since the recommendation rank computed for each antibiotic using the learned models is equal to its initial rank.

**Conclusion**

We proposed modeling the problem of providing recommendation of antibiotics using \(QCL\). The learned preference model is powerful since it represents experts reasoning and strategies to provide recommendations. This work could be improved on several points: for example visualizing the preference model to be more readable and explainable for the user. Another point concerns the use of constraint programming for generating best options.

**References**


Jiang, B.; Pei, J.; Lin, X.; Cheung, D. W.; and Han, J. 2008. Mining preferences from superior and inferior examples. In KDD, 390–398. ACM.
