Qualitative Choice Logic For Modeling Experts Recommendations of Antibiotics

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Abstract

Qualitative Choice Logic (QCL) is a logic-based formalism for preference handling. The logic adds to classical propositional logic a new connective called ordered disjunction ($\vec{\times}$). $x \vec{\times} y$ intuitively means: if possible x, but if x is not possible then at least y. The aim of this paper is to explore QCL in healthcare domain, particularly for modeling experts reasoning for providing recommendations of antibiotics. We show that QCL is fully adapted for modeling this problem.

Qualitative Choice Logic, Preference learning, Antibiotic prescriptions

Introduction

Clinical Practice Guidelines (CPGs) aim at improving the quality of health care by providing best practices for diagnosis and treatments. However, physicians have difficulties to understand the (often implicit) expert recommendations. In the domain of antibiotic prescription, depending on the clinical situation and the properties of antibiotics ($e.g.$ side effects), these later can be recommended or not for prescription. Each antibiotic has a rank of recommendation indicating its degree of preference for prescription. Namely, recommended antibiotics can be prescribed in 1^{st}, 2^{nd}, 3^{rd} or 4^{th} line of treatment (Tsopra, Lamy, and Sedki 2018). Recommended antibiotics in 1^{st} line of treatment (resp. 2^{nd}, 3^{rd} or 4^{th}) have a rank of recommendation 1 (resp. 2, 3, 4). Not recommended antibiotics have a rank 0. The higher this rank, the less preferable the antibiotic. Antibiotics having the rank 0 can not be prescribed.

Starting from a data base that contains a set of antibiotics, their features and their ranks of recommendation, our objectif is to learn a model that represents the closest possible experts reasoning and strategies to provide recommendations of antibiotics. Namely, the model allows to predict the correct rank of recommendation for each new antibiotic. The idea of preference learning (Fürnkranz and Hüllermeier 2010) is to learn and construct a preference model from observed preference information. Methods of preference learning can be quantitative (Burges et al. 2005; Tsopra, Lamy, and Sedki 2018) or qualitative (Jiang et al. 2008; de Amo et al. 2015).

Each antibiotic has a set of features and a rank of recommendation indicating its degree of preference. To solve this problem, we propose using Qualitative Choice Logic (QCL) (Brewka, Benferhat, and Le Berre 2004). More precisely, we aim to learn a QCL model from the antibiotics data base. The method presented here extends the one presented in (Sedki, Lamy, and Tsopra 2020) for learning $PQCL$ logic. Learning QCL and $PQCL$ are similar on several points since $PQCL$ logic presented in (Sedki, Lamy, and Tsopra 2020) is an extension of QCL for handling prioritized preferences. However, this paper is focused more on modeling antibiotic recommendation with QCL, it presents new contributions such as taking into account two types of features: i) necessary that are expert's knowledge modeled as classical formulas with QCL and ii) preference features.

We start with presenting some details of the antibiotic prescription problem. Then, we give some important elements of QCL. The third section describes the proposed method for learning the QCL model for antibiotic prescription. In section 4, we present a case study on a clinical situation in Antibioterapy. Finally, we conclude the paper.

Notations

Let V be a finite set of propositional variables. An interpretation I is defined as a set of propositional variables such that $v \in I$ if and only if v is set to true by I. If I satisfies a formula ϕ, we write $I \models \phi$, otherwise, we write $I \not\models \phi$. A model of a formula ϕ is an interpretation I that satisfies the formula. $var(\phi)$ denotes the variables of a formula ϕ. $|.|$ denotes the cardinality of a set.

Antibiotic data base

We used a data base containing the antibiotics, their features, and their rank of recommendations as defined in CPGs. This data base was validated by antibiotic experts according to a Delphi process (Tsopra, Lamy, and Sedki 2018). The base contains recommended and not recommended antibiotics. Among recommended antibiotics, there are those that are preferred to others regarding their rank of recommendation. In fact, there are 5 recommendation ranks in the data base: R=$\{1, 2, 3, 4, 0\}$. The higher this rank, the less preferable the antibiotic. So, Antibiotics having rank 1 are preferred to those having rank 2 and so on. Antibiotics having rank 0

are not preferred since they are not recommended. Note that there are some clinical situations where their greatest rank of recommendation can be equal to 3 (such as the case given in Experimental results section). In (Tsopra, Lamy, and Sedki 2018), it is showed that two categories of features exist:

1. Necessary features are mandatory for prescribing the antibiotic. These features ensure that an antibiotic is both safe for the patient, and able to cure the infection. We define V_{ness}, the set of necessary features.
2. Preference features indicate which antibiotic is preferred to another, in a given clinical situation. These features make it possible to choose one antibiotic through a list of antibiotics that could be prescribed to cure a patient. We define V_{pref}, the set of preference features.

For example, the absence of contraindications may be a necessary feature while the low rate of adverse effects may be a preference feature. Table 1 gives the list of necessary and preference features.

Table 1: Necessary and preference features (description of features can be found in (Tsopra, Lamy, and Sedki 2018)).

V_{ness}	naturallyActive, probablyActive, proved, noContraindication
V_{pref}	protocol, not precious, side effects, efficacy level, spect, eco risk, taste

***Definition* 1.** *An antibiotic a is recommended iff it satisfies both experts knowledge on necessary features and preference features, we have $R(a) \neq 0$. Otherwise, a is not recommended and we have $R(a) = 0$.*

An example of expert knowledge is: an antibiotic should have no contraindications and it should be proved.

***Example* 1.** *Table 2 gives a simplified example where V_{ness}=$\{v_1, v_2\}$, V_{pref}=$\{v_3, v_4, v_5\}$. a_1, a_2, a_3, a_5, a_6, a_7 are recommended antibiotics while a_4 is not recommended.*

Table 2: Simple example with 7 antibiotics and 5 features.

$Antibiotics$	v_1	v_2	v_3	v_4	v_5	R
a_1	1	1	1	1	1	1
a_2	1	1	1	1	0	3
a_3	0	1	1	0	1	1
a_4	0	0	1	0	0	0
a_5	1	1	0	1	1	2
a_6	1	1	0	1	0	3
a_7	0	1	0	0	0	2

Our aim is to learn a qualitative model that allows to assign a correct rank of recommendation of each antibiotic on the basis of the necessary and preference features.

Qualitative Choice Logic

Qualitative Choice Logic (Brewka, Benferhat, and Le Berre 2004) extends classical logic to represent preferences. It uses two types of connectives: classical connectives ($\wedge$, $\vee$ and $\neg$) and a new connective $\vec{\times}$, called *ordered disjunction*, used to express preferences. Formulas that compose the language of QCL are:

- Classical formulas (they do not contain $\vec{\times}$).
- Basic choice formulas (BCF) are ordered disjunctions of classical formulas.
- General choice formulas (GCF) are used to express complex preferences and can be obtained from V using connectives $\vec{\times}$, $\wedge$, $\vee$, $\neg$.

***Example* 2.** *ϕ_1=$v_1 \vee v_2$ is a classical formula. ϕ_2=$v_1 \vec{\times} v_2 \vec{\times} v_3$ is a BCF formula that contains three options: preferably v_1, but if this is not possible then v_2, if this is not possible, then v_3. ϕ_3=$(v_1 \vec{\times} v_2) \wedge (v_3 \vec{\times} v_4)$ is a GCF formula.*

Semantic and syntax of QCL

The semantic of a QCL formula is based on two principal functions, satisfaction degree and optionality. The satisfaction degree of a formula given an interpretation is a natural number. The higher this degree, the less preferable the interpretation. The optionality of a formula is a function that assigns to each formula a strictly positive integer. It corresponds to the maximum finite satisfaction degree that this formula can have.

***Definition* 2.** *The optionality is defined as follows:*

1. *opt(v)=1, for every v in V.*
2. *opt($\phi \vec{\times} \psi$)= opt(ϕ)+ opt(ψ).*
3. *opt($\phi \wedge \psi$)= max(opt(ϕ), opt(ψ)).*
4. *opt($\phi \vee \psi$)= max(opt(ϕ), opt(ψ)).*
5. *opt($\neg(\phi)$) = 1.*

***Example* 3.** *From Example 2, we have opt(ϕ_1)=1. opt(ϕ_2)=3 (from Item 1 and 2 of Definition 2), this means that there exists at least an interpretation which satisfies ϕ_2 with degree ≤ 3, but there is no interpretation which satisfies ϕ_2 with degree >3, opt(ϕ_3)=2 (from Items 1, 2 and 3 of Definition 2).*

In the following, we define the degree of satisfaction of classical and BCF formulas (more details for the inference of GCF formulas are given in (Brewka, Benferhat, and Le Berre 2004)).

***Definition* 3.** *The satisfaction degree of a formula ϕ under an interpretation I, denoted by $deg(I, \phi)$ is defined as follows:*

1. *Let ϕ be a classical formula, $deg(I, \phi)$ =1 iff $I \models \phi$.*
2. *Let ψ=$v_1 \vec{\times} v_2 \vec{\times} \ldots \vec{\times} v_n$, $deg(I, \psi)$=k iff $I \models v_1 \vee v_2 \vee \ldots \vee v_n$ and k = min$\{j \mid I \models v_j\}$. We write $I \models_k \phi$.*

   And, $deg(I, \psi) = 0$ iff there is no k such that $I \models v_k$.

For any classical formula ϕ, its degree is 1 if ϕ is satisfied by I and 0 otherwise. Item 2 of Definition 3 states that given a formula ψ=$v_1 \vec{\times} v_2 \vec{\times} \ldots \vec{\times} v_n$, an interpretation I satisfies ψ to a degree k, if it satisfies the k^{th} option of ψ (namely v_k) and falsifies the first $(k-1)^{th}$ options of ψ. Note that an interpretation is a QCL model if it satisfies each formula of a given QCL theory.

Learning the QCL model from the antibiotic data base

In this section, we show that QCL is fully suited for modeling the problem of antibiotics prescription.

***Definition* 4.** *Antibiotic prescription can be modeled using QCL where:*

- *each antibiotic is an interpretation,*
- *the rank of recommendation of each antibiotic corresponds to the satisfaction degree assigned to an interpretation by a QCL formula.*
- *a recommended antibiotic is considered as a QCL model.*

Each antibiotic is recommended iff it satisfies both expert knowledge on necessary features and preference features. From QCL point of view, this means that each antibiotic has two ranks of recommendation. The first one indicates if the antibiotic satisfies expert knowledge on necessary features and the second one indicates the preference degree of the antibiotic. Thus, the problem of antibiotic prescription is completed with the following property.

***Property* 1.** *Let D be the set of antibiotics, $a \in D$ then:*

- *there is a rank of recommendation regarding V_{ness}, denoted by $Rnes$ s.t. $Rnes(a) = 1$ if a is recommended and $Rnes(a) = 0$ if a is not recommended.*
- *there is a rank of recommendation regarding V_{pref}, denoted by $Rpref$ s.t. $Rpref(a) = R(a) \neq 0$ if a is recommended and $Rpref(a) = 0$ if a is not recommended.*

More precisely, an antibiotic a is recommended iff $Rnes(a) = 1$ and $Rpref(a) \neq 0$. a is not recommended otherwise.

***Example* 4.** *From data of Table 2, we have V_{ness}=$\{v_1, v_2\}$, V_{pref}=$\{v_3, v_4, v_5\}$. $Rnes$=$\{1, 0\}$, $Rpref$=$\{1, 2, 3, 0\}$. a_1, a_2, a_3, a_5, a_6, a_7 are recommended antibiotics while a_4 is not recommended.*

***Proposition* 1.** *Let D be the set of antibiotics, $a \in D$ then:*

- *There exists a classical formula ϕ with $var(\phi) \subseteq V_{ness}$, $deg(a, \phi) = Rnes(a) = 1$ if a is recommended and $deg(a, \phi) = Rnes(a) = 0$ if a is not recommended.*
- *There exists a formula ψ with $var(\psi) \subseteq V_{pref}$, $deg(a, \psi) = Rpref(a) \neq 0$ if a is recommended and $deg(a, \psi) = Rpref(a) = 0$ if a is not recommended.*

Proof. A given antibiotic a is evaluated on the basis of necessary features and preference features. From Property 1, a has two ranks of recommendation, R_{ness} and R_{pref}. From Definition 4, a corresponds to a QCL interpretation and the rank of recommendation corresponds to the satisfaction degree. So, a has two satisfaction degrees. From QCL point of view, there is a QCL theory that contains two formulas ϕ and ψ with $deg(a, \phi)$=$Rnes(a)$ and $deg(a, \psi) = Rpref(a)$.

- If a is recommended then following Definition 1, a satisfies both experts knowledge on necessary features and preference features. So, $Rnes(a)$=1 and $Rpref(a)\neq 0$ (Property 1). From Definition 4, a corresponds to a QCL model, so $deg(a, \phi)$=1 and $deg(a, \psi) \neq 0$. Thus, $deg(a, \phi)$=$Rnes(a)$=1 and $deg(a, \psi)$=$Rpref(a)\neq 0$.
- If a is not recommended then a is not a QCL model since a is not recommended. Thus, we have either $deg(a, \phi) = 0$ or $deg(a, \psi) = 0$. However, $Rnes(a) = 0$ and $Rpref(a) = 0$ since a is not recommended, thus, $deg(a, \phi) = 0$ and $deg(a, \psi) = 0$.

□

From the antibiotics data base, we aim to learn a model $\mathcal{M}_{\mathcal{QCL}}$ defined as follows:

***Definition* 5.** *Let D be the antibiotics data base, $a \in D$ then there is a QCL model, denoted by $\mathcal{M}_{\mathcal{QCL}}$ that can be learned s.t.*

$$\mathcal{M}_{\mathcal{QCL}} = \begin{cases} \phi \quad with \quad var(\phi) \subseteq V_{ness} \\ \psi \quad with \quad var(\psi) \subseteq V_{pref} \end{cases}$$

The learned model contains a classical formula defined on the necessary features and a BCF formula defined on the preference features (here we are interested only to learn a BCF formula but it is possible to learn a GCF formula since in (Brewka, Benferhat, and Le Berre 2004), it is shown that each GCF formula has its equivalent BCF formula). According to the semantic of QCL, the learned model satisfies the following property:

***Property* 2.** *Let $\mathcal{M}_{\mathcal{QCL}}$ be the learned model from the antibiotics data base D s.t. $\phi, \psi \in \mathcal{M}_{\mathcal{QCL}}$, then*

- *opt(ϕ)=$max(Rnes)$=1 with $var(\phi) \subseteq V_{ness}$,*
- *opt(ψ)=$max(Rpref)$ with $var(\psi) \subseteq V_{pref}$.*

Proof. It follows immediately from Definition 2 where it is stated that the optionality of a classical formula is equal to 1 and the optionality of a BCF formula is equal to the greatest degree of satisfaction. From Definition 4, the rank of recommendation of an antibiotic corresponds to the satisfaction degree, thus the optionality of the learned BCF formula is equal to the greatest rank of recommendation in D. □

***Example* 5.** *Data of Table 2 are modeled with QCL in Table 3. From Table 3, we aim to learn the QCL model s.t.*

Table 3: Data of Table 2 modeled with QCL.

$Antibiotics = Interpretations$	v_1	v_2	v_3	v_4	v_5	a classical formula ϕ	a BCF formula ψ
a_1	1	1	1	1	1	1	1
a_2	1	1	1	1	0	1	3
a_3	0	1	1	0	1	1	1
a_4	0	0	1	0	0	0	0
a_5	1	1	0	1	1	1	2
a_6	1	1	0	1	0	1	3
a_7	0	1	0	0	0	1	2

$$\mathcal{M}_{\mathcal{QCL}} = \begin{cases} \phi \quad with \quad var(\phi) \subseteq V_{ness} = \{v_1, v_2\} \\ \psi = x_1 \vec{\times} x_2 \vec{\times} x_3 \quad with \quad var(\psi) \subseteq V_{pref} = \{v_3, v_4, v_5\} \end{cases}$$

In this example, $Rpref$=$\{0, 1, 2, 3\}$, so opt(ψ) = $max(Rpref) = 3$. Thus, we aim to learn a formula ψ=$x_1 \vec{\times} x_2 \vec{\times} x_3$.

Let us first give the following definition:

Definition 6. *• Each single option is built on V and the connective $\neg$.*

• Each conjunctive option is built on V and the connectives $\{\wedge, \neg\}$. The set of conjunctive options is denoted by X_{conj}.

v_1, $\neg v_1$ are examples of single options. $v_1 \wedge \neg v_2$ is an example of conjunctive option.

In the following sections, we detail the method for learning BCF and classical formulas but for lack of space, we limited to single and conjunctive options. Namely, classical formulas are learned on V_{ness} and the connectives $\{\neg, \wedge\}$ and BCF formulas are learned on V_{pref} and the connectives $\{\neg, \wedge, \vec{\times}\}$.

Learning BCF formulas

The learned BCF formula is in the form of ψ=$x_1 \vec{\times} x_2 \vec{\times} x_3 \vec{\times} x_4$ where x_i is a single or a conjunctive option defined on V_{pref} (recall that the BCF formula contains 4 options since the greatest rank of recommendation in the antibiotic database is equal to 4). We proposed a method (Algorithm 1) for generating best options of ψ. Our method is inspired from Apriori algorithm (Agrawal, Imieliński, and Swami 1993) for generating frequent itemsets. Instead of generating all possible options which can be very large ($2^7 - 1$ possible options that can be generated on V_{pref}), we generate only frequent ones which correspond to those exceeding a minimal fixed support and confidence.

Following the semantics of QCL, the idea is that all antibiotics with rank $Rpref_1$=1 should satisfy the option x_1 (even if x_2, x_3 and x_4 are satisfied or not) and all antibiotics with rank $Rpref_2$=2 (resp. $Rpref_3$=3, $Rpref_4$=4, $Rpref_5$=0) should not satisfy the option x_1. So, the option x_1 of the learned formula should be the option that is satisfied by the maximum number of antibiotics having the rank $Rpref_1$=1 (ideally all antibiotics), and falsified by the maximum number of antibiotics with ranks 4, 3, 2 and 0 (ideally all). The option x_2 of the learned formula should be the option that is satisfied by the maximum number of antibiotics having the rank $Rpref_2$=2 and falsified by the maximum number of antibiotics with ranks 4, 3 and 0. The same reasoning for learning the best options x_3 and x_4. Thus, our definition of support and confidence are adapted to our problem as follows.

Definition 7 (Support)**.** *Let $Rpref$=$\{Rpref_1$=1, $Rpref_2$=2...$Rpref_4$=4, $Rpref_5$=0$\}$. The support of an option x for antibiotics a having a recommendation rank $Rpref(a) = Rpref_i$ is defined as:*

$$Supp(x, Rpref_i) = \frac{|\{a \mid a \models x \wedge Rpref(a) = Rpref_i\}|}{|\{a \mid Rpref(a) = Rpref_i\}|}$$

The support of an option x for a rank $Rpref_i$ is defined by the fraction of the number of antibiotics $a \in D$ having a rank $Rpref(a) = Rpref_i$ that satisfy x on the number of antibiotics having rank $Rpref_i$. Its interest increases with its support (ideally 1).

Even if the support of x is high for $Rpref_i$, it is possible that its support for $Rpref_{j \neq i}$ be also high, so there is need to compute its confidence.

Definition 8 (Confidence)**.** *The confidence of an option x for antibiotics $a \in D$ having a rank $Rpref(a) = Rpref_i$ is defined as: $Conf(x, Rpref_i)$=*

$$\frac{|\{(a \mid a \models x \wedge Rpref(a) = Rpref_i) \vee (a \mid a \not\models x \wedge Rpref(a) = Rpref_{i+1})\}|}{|\{a \mid Rpref(a) = Rpref_{i=1...5}\}|}$$

The confidence of an option x for rank $Rpref_i$ indicates the proportion of antibiotics that keep their initial rank if x appears in the learned formulas. An option is interesting for a rank $Rpref_i$ if its confidence is high (ideally 1).

Example 6. *Let us consider data of Table 2 where V_{pref}= $\{v_3, v_4, v_5\}$. Given the option $v_3 \wedge v_4$, we have $Supp(v_3 \wedge v_4, Rpref_1)$=1/2 and $Conf(v_3 \wedge v_4, Rpref_1)$=5/7 which means that this option is not the best for x_1 of the BCF formula that we aim to learn.*

Generating best options on V_{pref} For generating best options, we adapt the approach of association rules (Agrawal, Imieliński, and Swami 1993). The idea is to start with all single options, count their support and find all single frequent options, combine them to form *candidate* 2-conjunctive options, go through data and count their support and find all *frequent* 2-conjunctive options, combine them to form *candidate* 3-conjunctive options and so on. Once frequent options are generated for each rank $Rpref_{i \neq 0}$, we return only those exceeding a minimal confidence θ, called best options ($Best_{Rpref_i}$). Algorithm 1 summarizes these steps. Note that it is not necessary to generate best options for rank 0 since these later do not appear in the learned BCF formula. However, antibiotics with $Rpref_i$=0 are considered for computing the support and confidence of any option.

Depending on the minimal support and confidence, it is possible to have more than one best option for a given rank and then more than one BCF formula and QCL model. However, we evaluate the quality of each learned model by computing its accuracy (Definition 12).

Example 7. *Let us consider data of Table 3 where V_{pref}=$\{v_3, v_4, v_5\}$. Table 4 gives frequent single and conjunctive options for $Rpref_1$=1 exceeding minimal support $\sigma = 0.7$. In Table 4, we have C_1=$\{v_3, \neg v_3, v_4, \neg v_4, v_5, \neg v_5\}$. On the basis on the fixed minimal support, from*

Table 4: Best options for $Rpref_1$.

The set C_1 for $Rpref_1 \in Rpref$	
x	$Supp(x, Rpref_1)$
v_3	1
$\neg v_3$	0
v_4	0.5
$\neg_4$	0.5
v_5	1
$\neg v_5$	0

The set F_1 for $Rpref_1$	
x	$Supp(x, Rpref_1)$
v_3	1
v_5	1

The set C_2 for $Rpref_1$	
x	$Supp(x, Rpref_1)$
$v_3 \wedge v_5$	1

The set F_2 for $Rpref_1$	
x	$Supp(x, Rpref_1)$
$v_3 \wedge v_5$	1

F_{Rpref_1}=$F_1 \cup F_2$		
x	$Supp(x, Rpref_1)$	$Conf(x, Rpref_1)$
v_3	1	0.71
v_5	1	0.85
$v_3 \wedge v_5$	1	1

$Best_{Rpref_1}$ with $\theta = 0.9$		
x	$Supp(x, Rpref_1)$	$Conf(x, Rpref_1)$
$v_3 \wedge v_5$	1	1

Algorithm 1 Best options defined on V_{pref}

Input: The antibiotic data base D, $Rpref$=$\{Rpref_1$=1, $Rpref_2$=2...$Rpref_4$=4, $Rpref_5$=0$\}$, a minimal support σ and a minimal confidence θ.
Output: Best options for each rank $Rpref_{i=1...4}$ with support and confidence exceeding σ and θ ($Best_{Rpref_{i=1...4}}$).
C_j: Candidate options of size j from V_{pref}.
F_j: Frequent options of size j.
F_1: Frequent single options.
F_{Rpref_i}: Frequent options for rank $Rpref_{i=1...4}$.
for *each* $Rpref_{i=1...4} \in Rpref$ **do**
  for $j = 1; j \neq \emptyset; j++$ **do**
    C_{j+1}: candidate options generated from F_j
    for *each antibiotic a having* R_i **do**
      for *each* $x \in C_{j+1}$ **do**
        Compute $Supp(x, Rpref_i)$
      end
    end
    F_{j+1}=$\{x \in C_{j+1} \mid Supp(x, Rpref_i) \geq \sigma\}$
  end
  $F_{Rpref i} = \cup_j F_j$
  $Best_{Rpref_i} = \emptyset$
  for *each* $x \in F_{Rpref_i}$ **do**
    if $Conf(x, Rpref_i) \geq \theta$ **then**
      $Best_{Rpref_i}$ =$Best_{Rpref_i} \cup x$
    end
  end
  Return $Best_{Rpref_i}$
end

C_1, *we obtain* F_1=$\{$ v_3, $v_5\}$. *From* F_1, *we obtain the set* C_2=$\{v_3 \wedge v_5\}$. *From* C_2, *we have* F_2 = $\{v_3 \wedge v_5\}$. *Then,* F_{Rpref_1}=$F_1 \cup F_2$=$\{v_3$, v_5, $v_3 \wedge v_5\}$. *With minimal confidence* θ=0.9, $Best_{Rpref_1}$=$\{v_3 \wedge v_5\}$. *Thus, the best option for* x_1 *of the learned formula is equal to* $v_3 \wedge v_5$.

Learning the classical formula

To learn the best classical formula, we first define the support and confidence of each option on V_{ness} for all antibiotics with rank $Rnes = 1$.

***Definition* 9.** *The support of an option* x *for antibiotics having* $Rnes(a) = 1$ *is defined as:*

$$Supp(x, 1) = \frac{|\{a \mid a \models x \wedge Rnes(a) = 1\}|}{|\{a \mid Rnes(a) = 1\}|}$$

***Definition* 10.** *The confidence of an option* x *for antibiotics having a rank* $Rnes(a) = 1$ *is defined as:* $Conf(x, 1)$=

$$\frac{|\{(a \mid a \models x \wedge Rnes(a) = 1) \vee (a \mid a \not\models x \wedge Rnes(a) = 0)\}|}{|D|}$$

As said above, here we consider that a classical formula is a single or a conjunctive option defined on V_{ness}, so to learn a best formula, we adapt Algorithm 1 for this purpose (see Algorithm 2).

***Example* 8.** *Let us consider data of Table 3 where* V_{ness}=$\{v_1$, $v_2\}$. *Table 5 gives frequent options for rank* $Rnes = 1$ *exceeding minimal support* $\sigma = 0.5$ *and minimal confidence* $\theta = 0.9$. *From results of Table 5, the best classical formula is:* ϕ=v_2. *This means that each antibiotic having* v_2 *false is not recommended and each antibiotic* a *having* v_2 *true, it can be recommended if the rank inferred by the leaned* BCF *formula* ψ *is different to* 0.

Algorithm 2 Best classical formula defined on V_{ness}

Input: The database D, $Rnes$= $\{1, 0\}$, a minimal support σ and a minimal confidence θ.
Output: Best classical formula with support and confidence exceeding σ and θ ($BestFormula$).
C_j: Candidate options of size j from V_{ness}.
F_j: Frequent options of size j.
F_1: Frequent single options.
F_{Rnes}: Frequent options for rank $Rnes = 1$.
for $j = 1; j \neq \emptyset; j++$ **do**
  C_{j+1}: candidate options generated from F_j
  for *each antibiotic a having* $Rnes = 1$ **do**
    for *each* $x \in C_{j+1}$ **do**
      Compute $Supp(x, 1)$
    end
  end
  F_{j+1}=$\{x \in C_{j+1} \mid Supp(x, 1) \geq \sigma\}$
end
$F_{Rnes=1} = \cup_j F_j$
$BestFormula$=$\emptyset$
for *each* $x \in F_{Rnes=1}$ **do**
  if $Conf(x, Rnes = 1) \geq \theta$ **then**
    $BestFormula$ =$BestFormula \cup x$
  end
end
Return $BestFormula$

Table 5: Best classical formulas.

The set C_1 for $Rnes = 1$	
x	$Supp(x, 1)$
v_1	0.66
$\neg v_1$	0.33
v_2	1
$\neg v_2$	0

The set F_1 for $Rnes = 1$	
x	$Supp(x, 1)$
v_1	0.66
v_2	1

The set C_2 for $Rnes = 1$	
x	$Supp(x, 1)$
$v_1 \wedge v_2$	0.66

The set F_2 for $Rnes = 1$	
x	$Supp(x, 1)$
$v_1 \wedge v_2$	0.66

$F_{Rnes=1}$=$F_1 \cup F_2$		
x	$Supp(x, 1)$	$Conf(x, 1)$
v_1	0.66	0.71
v_2	1	1
$v_1 \wedge v_2$	0.66	0.71

$BestFormula$ with $\theta = 0.9$		
x	$Supp(x, 1)$	$Conf(x, 1)$
v_2	1	1

In order to evaluate the quality of the learned model, there is need to verify the rank of each antibiotic inferred by each formula.

***Definition* 11.** *Let* D *be the antibiotic database,* $a \in D$, $\mathcal{M}_{QCL}$ *be the learned model from* D, ϕ *be a classical formula and* ψ=$x_1 \vec{\times} x_2 \vec{\times} x_3 \vec{\times} x_4$ *be a* BCF *formula s.t.* ϕ, $\psi \in \mathcal{M}_{QCL}$. *The rank of recommendation inferred by* $\mathcal{M}_{QCL}$ *for* a *is* $R(\mathcal{M}_{QCL}, a)$= k *iff:*

- $a \models \phi$ *and,*
- $a \models x_1 \vee x_2 \vee x_3 \vee x_4$ *and* k=$min(j \mid a \models x_j)$

$R(\mathcal{M}_{QCL}, a)$= 0 *iff* $a \not\models \phi$.

Definition 11 states that given a learned model $\mathcal{M}_{QCL}$ that contains a BCF formula ψ=$x_1 \vec{\times} x_2 \vec{\times} x_3 \vec{\times} x_4$ and an classical formula ϕ, the rank of a is equal to k, iff a satisfies

Table 6: The learned preference model $\mathcal{M}_{QCL}$ in pharyngitis. Note that in this case, the greatest recommendation rank is equal to 3. So, the length of the obtained BCF formulas is equal to 3.

Learned preference models	Accuracy
$\begin{cases} naturallyActive \wedge probablyActive \wedge proved \wedge noContraindication \\ (Proto \wedge Precious \wedge \neg SideEff \wedge Efficacy \wedge Spect \wedge \neg RiskResi) \vec{\times} (Proto \wedge \neg SideEff) \vec{\times} (Proto \wedge Efficacy) \end{cases}$	1
$\begin{cases} naturallyActive \wedge probablyActive \wedge proved \wedge noContraindication \\ (Proto \wedge \neg SideEff \wedge Efficacy \wedge Spect \wedge \neg RiskResi) \vec{\times} (Proto \wedge \neg SideEff) \vec{\times} (Proto \wedge Efficacy) \end{cases}$	1
$\begin{cases} naturallyActive \wedge probablyActive \wedge proved \wedge noContraindication \\ (Proto \wedge Efficacy \wedge Spect \wedge \neg RiskResi) \vec{\times} (Proto \wedge \neg SideEff) \vec{\times} (Proto \wedge Efficacy) \end{cases}$	1
$\begin{cases} naturallyActive \wedge probablyActive \wedge proved \wedge noContraindication \\ (Proto \wedge Efficacy \wedge Spect) \vec{\times} (Proto \wedge \neg SideEff) \vec{\times} (Proto \wedge Efficacy). \end{cases}$	1
$\begin{cases} naturallyActive \wedge probablyActive \wedge proved \wedge noContraindication \\ (Proto \wedge Spect \wedge \neg RiskResi) \vec{\times} (Proto \wedge \neg SideEff) \vec{\times} (Proto \wedge Efficacy) \end{cases}$	1

ϕ and the kth option of ψ (i.e. x_k) is true and the preceding ones $(x_1, x_2, ..., x_{k-1})$ are false. If a does not satisfy ϕ then the rank of a is equal to 0 (even if a satisfies ψ or not).
We introduce the accuracy measure as follows:

***Definition* 12** (Accuracy)**.** *Given a database D and a learned model $\mathcal{M}_{QCL}$. The accuracy of $\mathcal{M}_{QCL}$. is:*

$$Accuracy(\mathcal{M}_{QCL}) = \frac{|\{a \in D \mid R(\mathcal{M}_{QCL}, a) = R(a)\}|}{|D|}$$

Case study

This section presents experimental results of our method in the domain of antibiotic prescription. Examples of the learned models for pharyngitis are given in Table 6. Note that more than one $\mathcal{M}_{QCL}$ model are learned because there are many best options with the same support and confidence for each rank. In addition, each learned model contains the same classical formula which is equal to $naturallyActive \wedge probablyActive \wedge proved \wedge noContraindication$. This means that if an antibiotic does not satisfy this formula, it should not be prescribed and thus it is not recommended.

Examples of best options for $Rpref_1$=1 are $(Proto \wedge Precious \wedge \neg SideEff \wedge Efficacy \wedge Spect)$, $(Proto \wedge \neg RiskResi)$ and $(Proto \wedge Spect)$ with σ=1and θ=1. Best options for R_2=2 is $(Proto \wedge \neg SideEff)$ with $\sigma = 1$ and $\theta = 0.7$. Best options for R_3=3 are $(Proto \wedge Efficacy)$, $(Proto \vee Taste)$ and $Proto$ with $\sigma = 1$ and $\theta = 0.35$. These results highlight that experts prefer to recommend (i) in rank 1 the antibiotics having short duration protocol ($Proto$) and narrow spectrum ($Spect$) with either [low risk of ecological adverse effects ($\neg RiskResi$), non-precious class ($Precious$), low risk of side effects ($\neg SideEff$), high efficacy ($Efficacy$)], or the antibiotics having $Proto$ and $\neg RiskResi$ with either [$Spect$, $Precious$, $\neg SideEff$, $Efficacy$], (ii) in rank 2, the antibiotics having short duration protocol ($Proto$) and low risk of side effects ($\neg SideEff$), (iii) in rank 3 the antibiotics with $Proto$ alone or with [$Efficacy$ or good taste ($Taste$)]. Finally, each antibiotic that does not satisfy the classical formula has rank 0 (not recommended). In this example, we detected two antibiotics with the same attributes but with two different rank of recommendation, so we retrieve them. The higher accuracy is 1 since the recommendation rank computed for each antibiotic using the learned models is equal to its initial rank.

Conclusion

We proposed modeling the problem of providing recommendation of antibiotics using QCL. The learned preference model is powerful since it represents experts reasoning and strategies to provide recommendations. This work could be improved on several points: for example visualizing the preference model to be more readable and explainable for the user. Another point concerns the use of constraint programming for generating best options.

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